The standard error of measurement in CTT/TTT is:

$$SEM = \sigma \cdot \sqrt{1 - Rel}$$

Sigma is the standard deviation in the sample.

Therefore:

$$SEM = \sqrt{VAR(X)} \cdot \sqrt{1 - Rel}$$

$$SEM = \sqrt{VAR(T + E)} \cdot \sqrt{1 - Rel}$$

The variance of a sum is the sum of the variances (provided T and E are independent of one another, so no covariance):

$$SEM = \sqrt{VAR(T) + VAR(E)} \cdot \sqrt{1 - Rel}$$

Reliability is VAR (T) over VAR (X):

$$SEM = \sqrt{VAR(T) + VAR(E)} \cdot \sqrt{1 - \frac{VAR(T)}{VAR(T) + VAR(E)}}$$

$$SEM = \sqrt{VAR(T) + VAR(E)} \cdot \sqrt{\frac{VAR(T) + VAR(E)}{VAR(T) + VAR(E)} - \frac{VAR(T)}{VAR(T) + VAR(E)}}$$

$$SEM = \sqrt{VAR(T) + VAR(E)} \cdot \sqrt{\frac{VAR(T) + VAR(E) - VAR(T)}{VAR(T) + VAR(E)}}$$

$$SEM = \sqrt{VAR(T) + VAR(E)} \cdot \sqrt{\frac{VAR(E)}{VAR(T) + VAR(E)}}$$

$$SEM = \sqrt{\frac{[VAR(T) + VAR(E)] \cdot VAR(E)}{VAR(T) + VAR(E)}}$$

SEM is square root of error variance:

$$SEM = \sqrt{VAR(E)}$$

$$SEM = \sigma_E$$

Thus, the SEM is mathematically independent of the sample homogeneity. With increasing homogeneity, VAR(T) decreases, so the standard deviation of the observed scores also decreases, while 1-reliability increases (because reliability decreases).

These effects cancel out and the SEM is exactly the square root of error variance.