APPENDIX A

Derivation of the interest rate rule

Recall from Chapter 5 that, in our model, we distinguish between three classes of variable: the first set consists of the four variables determined contemporaneously with r_t^b (namely, p_t^o, e_t, r_t^*, and r_t); the second set, denoted $\mathbf{w}_t$ contains output and inflation, which we shall assume are the variables of direct concern to the monetary authorities; and the third set, denoted $\mathbf{q}_t$, consists of the remaining variables. Hence, we have

$$\mathbf{z}_t = \begin{pmatrix} p_t^o \\ \mathbf{y}_t \end{pmatrix}, \quad \mathbf{y}_t = \begin{pmatrix} e_t \\ r_t^* \\ r_t \\ \mathbf{w}_t \\ \mathbf{q}_t \end{pmatrix}, \quad \mathbf{w}_t = \begin{pmatrix} y_t \\ \Delta p_t \end{pmatrix}, \quad \mathbf{q}_t = \begin{pmatrix} p_t - p_t^* \\ h_t - y_t \\ y_t^* \end{pmatrix}.$$

The assumptions discussed in Section 5.1 of the text imposes a structure on the parameter matrices of (5.2),

$$\mathbf{A}\,\Delta\mathbf{z}_t = \widetilde{\mathbf{a}} - \widetilde{\boldsymbol{\alpha}}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \widetilde{\boldsymbol{\Gamma}}_i \Delta\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_t, \tag{A.1}$$

as follows:[1]

$$\widetilde{\mathbf{a}} = \begin{pmatrix} \delta_o \\ \widetilde{a}_e \\ \widetilde{a}_{r^*} \\ \widetilde{a}_r \\ \widetilde{\mathbf{a}}_w \\ \widetilde{\mathbf{a}}_q \end{pmatrix}, \quad \widetilde{\boldsymbol{\alpha}} = \begin{pmatrix} 0 \\ \widetilde{\alpha}_e \\ \widetilde{\alpha}_{r^*} \\ \widetilde{\alpha}_r \\ \widetilde{\boldsymbol{\alpha}}_w \\ \widetilde{\boldsymbol{\alpha}}_q \end{pmatrix}, \quad \widetilde{\boldsymbol{\Gamma}}_i = \begin{pmatrix} \delta_{i,o} \\ \widetilde{\boldsymbol{\Gamma}}_{e,i} \\ \widetilde{\boldsymbol{\Gamma}}_{r^*,i} \\ \widetilde{\boldsymbol{\Gamma}}_{r,i} \\ \widetilde{\boldsymbol{\Gamma}}_{w,i} \\ \widetilde{\boldsymbol{\Gamma}}_{q,i} \end{pmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{o,t} \\ \varepsilon_{e,t} \\ \varepsilon_{r^*,t} \\ \varepsilon_{r,t} \\ \boldsymbol{\varepsilon}_{w,t} \\ \boldsymbol{\varepsilon}_{q,t} \end{pmatrix}$$

[1] In fact, for expositional purposes, we make the further assumption that exchange rates are determined prior to foreign interest rates in what follows.

and

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\widetilde{\psi}_e & 1 & 0 & 0 & 0 & 0 \\ -\widetilde{\psi}_{r^*} & a_{r^*e} & 1 & 0 & 0 & 0 \\ -\widetilde{\psi}_r & a_{re} & a_{rr^*} & 1 & 0 & 0 \\ -\widetilde{\boldsymbol{\psi}}_w & \mathbf{A}_{we} & \mathbf{A}_{wr^*} & \mathbf{A}_{wr} & \mathbf{A}_{ww} & \mathbf{A}_{wq} \\ -\widetilde{\boldsymbol{\psi}}_q & \mathbf{A}_{qe} & \mathbf{A}_{qr^*} & \mathbf{A}_{qr} & \mathbf{A}_{qw} & \mathbf{A}_{qq} \end{pmatrix}.$$

The corresponding reduced form equation, given in (5.1), is

$$\Delta \mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{s-1} \boldsymbol{\Gamma}_i \Delta \mathbf{z}_{t-i} + \mathbf{v}_t, \tag{A.2}$$

where $\widetilde{\mathbf{a}} = \mathbf{A}\mathbf{a}$, $\widetilde{\boldsymbol{\alpha}} = \mathbf{A}\boldsymbol{\alpha}$, $\widetilde{\boldsymbol{\Gamma}}_i = \mathbf{A}\boldsymbol{\Gamma}_i$, $\boldsymbol{\varepsilon}_t = \mathbf{A}\mathbf{v}_t$ and

$$\mathbf{a} = \begin{pmatrix} \delta_o \\ a_e \\ a_{r^*} \\ a_r \\ \mathbf{a}_w \\ \mathbf{a}_q \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_e \\ \alpha_{r^*} \\ \alpha_r \\ \boldsymbol{\alpha}_w \\ \boldsymbol{\alpha}_q \end{pmatrix}, \quad \boldsymbol{\Gamma}_i = \begin{pmatrix} \delta_{o,i} \\ \boldsymbol{\Gamma}_{e,i} \\ \boldsymbol{\Gamma}_{r^*,i} \\ \boldsymbol{\Gamma}_{r,i} \\ \boldsymbol{\Gamma}_{w,i} \\ \boldsymbol{\Gamma}_{q,i} \end{pmatrix}, \quad \mathbf{v}_t = \begin{pmatrix} v_{o,t} \\ v_{e,t} \\ v_{r^*,t} \\ v_{r,t} \\ \mathbf{v}_{w,t} \\ \mathbf{v}_{q,t} \end{pmatrix}.$$

A.1 The relationship between policy instruments and targets

To derive the monetary authorities' reaction function, we need an expression that explains the consequences of changes in the policy instrument, r_t^b, on the target variables, $\Delta \mathbf{w}_t$. The policy instrument affects the targets via the market interest rate, r_t, so we first focus attention on the block in the structural model of (A.1) relating the targets to the market interest rate. This block is given by the rows of (A.1) concerned with the determination of $\Delta \mathbf{w}_t$:

$$\begin{aligned} &-\widetilde{\boldsymbol{\psi}}_w \Delta p_t^o + \mathbf{A}_{we}\Delta e_t + \mathbf{A}_{wr^*}\Delta r_t^* + \mathbf{A}_{wr}\Delta r_t + \mathbf{A}_{ww}\Delta \mathbf{w}_t + \mathbf{A}_{wq}\Delta \mathbf{q}_t \\ &= \widetilde{\mathbf{a}}_w - \widetilde{\boldsymbol{\alpha}}_w\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{s-1} \widetilde{\boldsymbol{\Gamma}}_{w,i}\Delta \mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_{w,t}. \end{aligned} \tag{A.3}$$

Using the reduced form model of (A.2), we can replace the terms involving Δp_t^o, Δe_t, Δr_t^*, and $\Delta \mathbf{q}_t$ in (A.3) to obtain an expression relating the targets to the market interest rate which involves only lagged information and news becoming available at time t in the form of structural shocks. Specifically, the reduced form model of (A.2) provides expressions for the oil price, exchange rate, foreign interest rate and

variables in $\mathbf{q}_t$ as follows:

$$\Delta p_t^o = \delta_o + \sum_{i=1}^{p-1} \delta_{o,i}\Delta \mathbf{z}_{t-i} + v_{o,t}, \tag{A.4}$$

$$\Delta e_t = a_e - \alpha_e\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{e,i}\Delta \mathbf{z}_{t-i} + v_{e,t} \tag{A.5}$$

$$\Delta r_t^* = a_{r^*} - \alpha_{r^*}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{r^*,i}\Delta \mathbf{z}_{t-i} + v_{r^*,t} \tag{A.6}$$

and

$$\Delta \mathbf{q}_t = \mathbf{a}_q - \boldsymbol{\alpha}_q\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{q,i}\Delta \mathbf{z}_{t-i} + \mathbf{v}_{q,t}. \tag{A.7}$$

Substituting (A.4)–(A.7) into (A.3) yields the structural relationship between the targets and the market rate:

$$\mathbf{A}_{wr}\Delta r_t + \mathbf{A}_{ww}\Delta \mathbf{w}_t = \mathbf{a}_{ww} + \boldsymbol{\alpha}_{w\xi}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{wz,i}\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_{ww,t}, \tag{A.8}$$

where

$$\begin{aligned} \mathbf{a}_{ww} &= \widetilde{\mathbf{a}}_w + \widetilde{\boldsymbol{\psi}}_w\delta_o - \mathbf{A}_{we}a_e - \mathbf{A}_{we}a_{r^*} - \mathbf{A}_{wq}\mathbf{a}_q \\ \boldsymbol{\alpha}_{w\xi} &= -\widetilde{\boldsymbol{\alpha}}_w + \mathbf{A}_{we}\boldsymbol{\alpha}_e + \mathbf{A}_{wr^*}\boldsymbol{\alpha}_{r^*} + \mathbf{A}_{wq}\boldsymbol{\alpha}_q \\ \boldsymbol{\Gamma}_{wz,i} &= \widetilde{\boldsymbol{\Gamma}}_{w,i} + \widetilde{\boldsymbol{\psi}}_w\delta_{o,i} - \mathbf{A}_{we}\boldsymbol{\Gamma}_{e,i} - \mathbf{A}_{wr^*}\boldsymbol{\Gamma}_{r^*,i} - \mathbf{A}_{wq}\boldsymbol{\Gamma}_{q,i} \\ \boldsymbol{\varepsilon}_{ww,t} &= \boldsymbol{\varepsilon}_{w,t} + \widetilde{\boldsymbol{\psi}}_w v_{o,t} - \mathbf{A}_{we}v_{e,t} - \mathbf{A}_{wr^*}v_{r^*,t} - \mathbf{A}_{wq}\mathbf{v}_{q,t}. \end{aligned}$$

The 'quasi' reduced form linking targets to the market rate, to be used subsequently in the optimisation problem, is then given by

$$\Delta \mathbf{w}_t = \boldsymbol{\Pi}_{wr}\Delta r_t + \boldsymbol{\Pi}_{ww} + \boldsymbol{\Pi}_{w\xi}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Pi}_{wz,i}\Delta \mathbf{z}_{t-i} + \mathbf{v}_{ww,t}, \tag{A.9}$$

where $\boldsymbol{\Pi}_{ww} = \mathbf{A}_{ww}^{-1}\mathbf{a}_{ww}$, $\boldsymbol{\Pi}_{wr} = -\mathbf{A}_{ww}^{-1}\mathbf{A}_{wr}$, $\boldsymbol{\Pi}_{w\xi} = \mathbf{A}_{ww}^{-1}\boldsymbol{\alpha}_{w\xi}$, $\boldsymbol{\Pi}_{wz,i} = \mathbf{A}_{ww}^{-1}\boldsymbol{\Gamma}_{wz,i}$, and $\mathbf{v}_{ww,t} = \mathbf{A}_{ww}^{-1}\boldsymbol{\varepsilon}_{ww,t}$. Expression (A.9) can also be written as

$$\Delta \mathbf{w}_t = \boldsymbol{\Pi}_{wr}\Delta r_t + E\left[\Delta \mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0\right] + \mathbf{v}_{ww,t}, \tag{A.10}$$

where

$$E\left[\Delta \mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0\right] = \mathbf{\Pi}_{ww} + \mathbf{\Pi}_{w\xi}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \mathbf{\Pi}_{wz,i} \Delta \mathbf{z}_{t-i},$$

and represents the growth in the target variables that would occur in time t in the absence of any adjustment to the base interest rate $\left(\Delta r_t^b = 0\right)$ and in the absence of any structural innovations to the system $(\mathbf{v}_{ww,t} = 0)$.

A.2 Deriving the monetary authority's reaction function

The first-order condition for the minimisation of (5.6) in the text, subject to (A.9), is given by

$$E\left[\left(\frac{\partial r_t}{\partial r_t^b}\right)\left(\frac{\partial \mathbf{w}_t}{\partial r_t}\right)' \mathbf{Q}\left(\mathbf{w}_t - \mathbf{w}_t^{\dagger}\right) + \theta\left(\frac{\partial r_t}{\partial r_t^b}\right) \Delta r_t \mid \Im_{t-1}\right] = 0. \qquad \text{(A.11)}$$

Noting from the term structure relationship of (5.4) in the text that $\partial r_t / \partial r_t^b = 1$, and from (A.9) that

$$\frac{\partial \Delta \mathbf{w}_t}{\partial r_t} = \frac{\partial \mathbf{w}_t}{\partial r_t} = \mathbf{\Pi}_{wr},$$

(A.11) provides

$$E\left[\mathbf{\Pi}_{wr}' \mathbf{Q}\left(\mathbf{\Pi}_{wr} \Delta r_t + E\left[\mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0\right] + \mathbf{v}_{ww,t} - \mathbf{w}_t^{\dagger}\right) + \theta \Delta r_t | \Im_{t-1}\right] = 0.$$

Rearranging, and noting from (5.4) that $E\left[\Delta r_t | \Im_{t-1}\right] = r_t^b - r_{t-1} + \rho_{b,t-1}$, we have

$$\left(\theta + \mathbf{\Pi}_{wr}' \mathbf{Q} \mathbf{\Pi}_{wr}\right)\left(r_t^b - r_{t-1} + \rho_{b,t-1}\right) = -\mathbf{\Pi}_{wr}' \mathbf{Q}\left(E\left[\mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0\right] - \mathbf{w}_t^{\dagger}\right),$$

and the systematic component of the interest rate rule denoted by r_t^b is given by

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \mathbf{\Upsilon}'\left(E\left[\mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0\right] - \mathbf{w}_t^{\dagger}\right), \qquad \text{(A.12)}$$

where

$$\mathbf{\Upsilon}' = -\left(\theta + \mathbf{\Pi}_{wr}' \mathbf{Q} \mathbf{\Pi}_{wr}\right)^{-1} \mathbf{\Pi}_{wr}' \mathbf{Q},$$

or, more fully,

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \boldsymbol{\phi}^{\diamond} - \mathbf{\Upsilon}'\left(\mathbf{w}_t^{\dagger} - \mathbf{w}_{t-1}\right) + \boldsymbol{\phi}_r^{\diamond}\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\phi}_{zi}^{\diamond} \Delta \mathbf{z}_{t-i}, \qquad \text{(A.13)}$$

where

$$\boldsymbol{\phi}^{\diamond} = \mathbf{\Upsilon}' \mathbf{\Pi}_{ww}, \quad \boldsymbol{\phi}_r^{\diamond} = \mathbf{\Upsilon}' \mathbf{\Pi}_{w\xi},$$
$$\boldsymbol{\phi}_{zi}^{\diamond} = \mathbf{\Upsilon}' \mathbf{\Pi}_{wz,i}, \quad i = 1, 2, \ldots, s-1.$$

Expressions (A.12) and (A.13) are those given for r_t^b in the text.

A.3 Inflation targeting and the base rate reaction function

From equation (5.3) of the text, the relation between the market and base interest rates is given by

$$r_t - r_t^b = \rho_{b,t-1} + a_{rr^*}\left[r_t^* - E\left(r_t^* \mid \Im_{t-1}\right)\right] + a_{re}\left[e_t - E\left(e_t \mid \Im_{t-1}\right)\right] + \widetilde{\psi}_r\left[p_t^o - E\left(p_t^o \mid \Im_{t-1}\right)\right] + \varepsilon_{rt}. \qquad \text{(A.14)}$$

Rearranging and substituting out r_t^b from the monetary authorities' reaction function in (A.12), we obtain

$$\Delta r_t = \mathbf{\Upsilon}'\left(E\left[\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0\right] - \mathbf{w}_t^{\dagger}\right) + a_{rr^*}\left[r_t^* - E\left(r_t^* \mid \Im_{t-1}\right)\right] + a_{re}\left[e_t - E\left(e_t \mid \Im_{t-1}\right)\right] + \widetilde{\psi}_r\left[p_t^o - E\left(p_t^o \mid \Im_{t-1}\right)\right] + \varepsilon_{rt}. \qquad \text{(A.15)}$$

Taking this expression back to the quasi-reduced form expression for $\Delta \mathbf{w}_t$ in (A.10), we obtain

$$\mathbf{w}_t = (\mathbf{I} - \mathbf{\Lambda}) E\left[\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0\right] + \mathbf{\Lambda} \mathbf{w}_t^{\dagger} + \mathbf{v}_{ww,t}^{\diamond},$$

where

$$\mathbf{\Lambda} = -\mathbf{\Pi}_{wr}' \mathbf{\Upsilon}' = \mathbf{\Pi}_{wr}'\left(\theta + \mathbf{\Pi}_{wr}' \mathbf{Q} \mathbf{\Pi}_{wr}\right)^{-1} \mathbf{\Pi}_{wr}' \mathbf{Q},$$

and

$$\mathbf{v}_{ww,t}^{\diamond} = \mathbf{\Pi}_{wr}'\left\{a_{rr^*}\left[r_t^* - E\left(r_t^* \mid \Im_{t-1}\right)\right] + a_{re}\left[e_t - E\left(e_t \mid \Im_{t-1}\right)\right] + \widetilde{\psi}_r\left[p_t^o - E\left(p_t^o \mid \Im_{t-1}\right)\right] + \varepsilon_{rt}\right\} + \mathbf{v}_{ww,t}.$$

This shows that the value of the target variables achieved when the authorities pursue their optimal policy is a weighted average of the level that would be achieved if the base rate is left unchanged and the desired level, plus a random element generated by the structural shocks impacting on the p_t^o, e_t, r_t^* and target variables in time t. The weights on the expected target variable and the desired target variable terms are $(\mathbf{I} - \mathbf{\Lambda})$ and $\mathbf{\Lambda}$, respectively. In the simple case where there is only one target variable (say inflation), so that A'_{wr}, A'_{ww} and Q are scalars in (A.1) and (5.7), and equal to a_{wr}, 1, and q respectively then the weights are simply

$$(\mathbf{I}-\mathbf{\Lambda}) = 1 - \frac{a_{wr}^2 q}{a_{wr}^2 q + \theta} \quad \text{and} \quad \mathbf{\Lambda} = \frac{a_{wr}^2 q}{a_{wr}^2 q + \theta}.$$

In particular, as $q/\theta \to \infty$, so that the cost of the target deviating from its desired level rises relative to the cost of changing the base rate in (5.7) in the text, we have

$$\frac{a_{wr}^2 q}{a_{wr}^2 q + \theta} \to 1$$

and

$$\mathbf{w}_t = \mathbf{w}_t^{\dagger} + \mathbf{v}_{ww,t}^{\diamond}.$$

Hence, abstracting from the unpredictable structural shocks, the target variable tracks the desired level precisely.

A.4 Reaction functions and targeting future values of variables

In the text, we consider the case where *future* values of target variables might be the concept of interest to monetary authorities. Consider the simple case in which the monetary authorities care about just one future period, $t+h$ say, and face the optimisation problem

$$\min_{r_t^b} \left\{ E\left[C(\mathbf{w}_{t+h}, r_t) \mid \Im_{t-1}\right]\right\}, \tag{A.16}$$

with

$$C(\mathbf{w}_{t+h}, r_t) = \tfrac{1}{2}\left(\mathbf{w}_{t+h} - \mathbf{w}_{t+h}^{\dagger}\right)' \mathbf{Q} \left(\mathbf{w}_{t+h} - \mathbf{w}_{t+h}^{\dagger}\right) + \tfrac{1}{2}\theta\left(r_t - r_{t-1}\right)^2.$$

Identification of the monetary policy shocks is obtained following the steps described in the previous section. Hence, derivation of the base rate decision rule first requires an expression linking the base rate to the target variable. This is readily obtained on the basis of (A.2), from which we can obtain a model of $\Delta\mathbf{z}_{t+h}$ in terms of $\mathbf{z}_{t+h-1}$, $s-1$ lagged values of $\Delta\mathbf{z}_{t+h}$ and $\mathbf{v}_{t+h}$. Recursive substitution of (A.2) can be used to generate a complex expression expressing $\Delta\mathbf{z}_{t+h}$ in terms of $\mathbf{v}_{t+h}, \mathbf{v}_{t+h-1}, \ldots, \mathbf{v}_{t+1}, \Delta\mathbf{z}_t, \mathbf{z}_{t-1}$, and $s-1$ lagged values of $\Delta\mathbf{z}_t$. Substituting out all of the elements of $\Delta\mathbf{z}_t$ other than Δr_t using the relevant rows of (A.1), we obtain an expression relating $\Delta\mathbf{z}_{t+h}$ to Δr_t along with lagged values of $\mathbf{z}_t$ and combinations of structural shocks dated at time t up to time $t+h$. Finally, we can premultiply $\Delta\mathbf{z}_{t+h}$ and the corresponding expression involving Δr_t by a selection vector choosing the target variables from within $\Delta\mathbf{z}_{t+h}$. This provides a relationship of the form:

$$\Delta\mathbf{w}_{t+h} = \mathbf{\Pi}_{wrh}\Delta r_t + E[\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0] + \mathbf{v}_{wwh,t},$$

where $\mathbf{\Pi}_{wrh}$ is a matrix of parameters capturing the effects of Δr_t on the target variables h periods ahead, $E\left[\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0\right]$ indicates the value of the target variables that would occur in time $t+h$ in the absence of any interest rate adjustment at t and in the absence of any structural innovations to system between t and $t+h$, and $\mathbf{v}_{wwh,t}$ summarises the effects of the structural innovations that do occur.

Given this expression describing the relationship between $\Delta\mathbf{w}_{t+h}$ and Δr_t, minimisation of (A.16) provides the first-order condition

$$E\left[\mathbf{\Pi}'_{wrh}\mathbf{Q}\left(\mathbf{\Pi}_{wrh}\Delta r_t + E\left[\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0\right] + \mathbf{v}_{wwh,t} - \mathbf{w}_{t+h}^{\dagger}\right) + \theta\Delta r_t \mid \Im_{t-1}\right] = 0,$$

and this provides a reaction function of the form

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \mathbf{\Upsilon}'_h\left(E\left[\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0\right] - \mathbf{w}_{t+h}^{\dagger}\right),$$

where $\mathbf{\Upsilon}'_h$ is a function of the parameters of the econometric model and of the preference parameters of the monetary authorities (and $\mathbf{w}_{t+h}^{\dagger}$ is assumed known at time $t-1$). Substitution of the reaction function into the quasi-reduced form expression for $\Delta\mathbf{w}_{t+h}$ provides an expression for $\Delta\mathbf{w}_{t+h}$ as a weighted average of $E\left[\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0\right]$ and $\mathbf{w}_{t+h}^{\dagger}$ plus the effects of structural shocks experienced between t and $t+h$. Further, having derived the base rate reaction function, the structural interest rate equation is derived as in (A.15) above, and monetary policy shocks are identified as changes in the interest rate not explained by unanticipated movements in oil prices, exchange rates and foreign interest rates.

APPENDIX B

Invariance properties of the impulse responses with respect to monetary policy shocks

In this appendix, we provide a proof for footnote 4 of Chapter 10 that, once the position of the monetary policy variable in $\mathbf{z}_t$ is fixed (in our application as the fourth element of $\boldsymbol{\varepsilon}_t$), the impulse response functions of the monetary policy shocks will be invariant to the re-ordering of the variables before and after r_t in $\mathbf{z}_t$.

Since the proof becomes unduly complicated for the case where there are four or more variables in $\mathbf{z}_{1t}$, we consider the simpler case (without loss of generality) where there are only three variables in $\mathbf{z}_{1t}$. In particular, we consider the two different cases: (a) $\mathbf{z}_{1t}^{(a)} = (z_{1t}, z_{2t}, z_{3t})'$ and (b) $\mathbf{z}_{1t}^{(b)} = (z_{2t}, z_{1t}, z_{3t})'$, where z_{3t} is fixed at the last element of $\mathbf{z}_{1t}$, and $\mathbf{z}_{2t} = (z_{4t}, \ldots, z_{mt})'$. We then show that the impact impulse responses of ε_{3t} on $\mathbf{z}_{1t}$ and $\mathbf{z}_{2t}$ are the same under both cases.

Note that the impact impulse responses with respect to the third structural shocks are given by

$$\mathfrak{g}(0, \mathbf{z} : \varepsilon_3) = \frac{E(\varepsilon_{3t}\mathbf{u}_t)}{\sqrt{\omega_{33}}} = \frac{1}{\sqrt{\omega_{33}}} \begin{bmatrix} \mathbf{A}_{11}^{-1} E(\varepsilon_{3t}\boldsymbol{\varepsilon}_{1t}) \\ E(\varepsilon_{3t}\mathbf{u}_{2t}) \end{bmatrix} = \frac{1}{\sqrt{\omega_{33}}} \begin{bmatrix} \mathbf{A}_{11}^{-1}\boldsymbol{\Omega}_{11}\boldsymbol{\tau}_i \\ (\boldsymbol{\tau}_3'\mathbf{A}_{11}\boldsymbol{\Sigma}_{12})' \end{bmatrix}, \tag{B.1}$$

where $\boldsymbol{\varepsilon}_{1t} = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})'$ is a 3×1 vector of structural errors, the reduced form errors, $\mathbf{u}_t = (\mathbf{u}_{1t}', \mathbf{u}_{2t}')'$ are decomposed conformably with $\mathbf{z}_t = (\mathbf{z}_{1t}', \mathbf{z}_{2t}')'$, $\boldsymbol{\Omega}_{11} = \mathrm{Cov}(\boldsymbol{\varepsilon}_{1t})$, $\boldsymbol{\Sigma} = \mathrm{Cov}(\mathbf{u}_t) = \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{12}' & \boldsymbol{\Sigma}_{22} \end{bmatrix}$, and $\boldsymbol{\tau}_3 = (0, 0, 1)'$ is a 3×1 selection vector.

Under this set-up we now have

$$\boldsymbol{\Sigma}_{11} = \mathbf{A}_{11}^{-1}\boldsymbol{\Omega}_{11}\mathbf{A}_{11}^{-1\prime} = \left(\mathbf{A}_{11}^{-1}\boldsymbol{\Omega}_{11}^{\frac{1}{2}}\right)\left(\boldsymbol{\Omega}_{11}^{\frac{1}{2}}\mathbf{A}_{11}^{-1\prime}\right) = \mathbf{P}\mathbf{P}', \tag{B.2}$$

where

$$\mathbf{P} = \begin{bmatrix} p_{11} & 0 & 0 \\ p_{21} & p_{22} & 0 \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

is the 3×3 lower-triangular matrix. Using (B.2), then $\mathbf{\Omega}_{11}$, $\mathbf{A}_{11}^{-1}$ and $\mathbf{A}_{11}$ can be obtained, respectively, as

$$\mathbf{\Omega}_{11} = \begin{bmatrix} \omega_{11} & 0 & 0 \\ 0 & \omega_{22} & 0 \\ 0 & 0 & \omega_{33} \end{bmatrix} = \begin{bmatrix} p_{11}^2 & 0 & 0 \\ 0 & p_{22}^2 & 0 \\ 0 & 0 & p_{33}^2 \end{bmatrix} \quad (B.3)$$

$$\mathbf{A}_{11}^{-1} = \mathbf{P} \times \begin{bmatrix} p_{11} & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{33} \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{p_{21}}{p_{11}} & 1 & 0 \\ \frac{p_{31}}{p_{11}} & \frac{p_{32}}{p_{22}} & 1 \end{bmatrix} \quad (B.4)$$

$$\mathbf{A}_{11} = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{p_{21}}{p_{11}} & 1 & 0 \\ \frac{p_{21}p_{32}}{p_{11}p_{22}} - \frac{p_{31}}{p_{11}} & -\frac{p_{32}}{p_{22}} & 1 \end{bmatrix}. \quad (B.5)$$

Then, (B.1) simplifies to

$$g(0, \mathbf{z} : \varepsilon_3) = \frac{1}{\sqrt{\omega_{33}}} \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \omega_{33} \end{pmatrix} \\ E(\varepsilon_{3t}\mathbf{u}_{2t}) \end{bmatrix}.$$

Furthermore, in the absence of any over-identifying restrictions on the system of equations for $\mathbf{z}_{2t}$, $E(\varepsilon_{3t}\mathbf{u}_{2t})$ can be consistently estimated by

$$T^{-1}\sum_{t=1}^{T} \hat{\varepsilon}_{3t}\hat{\mathbf{u}}_{2t}',$$

where $\hat{\mathbf{u}}_{2t}$ are the reduced form residuals associated with $\mathbf{z}_{2t}$, and

$$\hat{\varepsilon}_{3t} = a_{31}\hat{u}_{1t} + a_{32}\hat{u}_{2t} + \hat{u}_{3t},$$

where $\hat{u}_{1t}$, $\hat{u}_{2t}$, and u_{3t} are the reduced form residuals associated with z_{1t}, z_{2t}, z_{3t} in $\mathbf{z}_{1t}$, respectively. Thus,

$$T^{-1}\sum_{t=1}^{T} \hat{\varepsilon}_{3t}\hat{\mathbf{u}}_{2t}' = a_{31}\left(T^{-1}\sum_{t=1}^{T} \hat{u}_{1t}\hat{\mathbf{u}}_{2t}'\right) + a_{32}\left(T^{-1}\sum_{t=1}^{T} \hat{u}_{2t}\hat{\mathbf{u}}_{2t}'\right) + \left(T^{-1}\sum_{t=1}^{T} \hat{u}_{3t}\hat{\mathbf{u}}_{2t}'\right). \quad (B.6)$$

Hence to prove that the invariance of the (structural) impulse responses of ε_{3t} on $\mathbf{z}_{1t}$ and $\mathbf{z}_{2t}$ to changing the order of z_{1t} and z_{2t} in $\mathbf{z}_{1t}$ as well as to changing the order

of variables in in $\mathbf{z}_{2t}$, we *first* need to establish that p_{33}^2's obtained for cases (a) $\mathbf{z}_{1t} = (z_{1t}, z_{2t}, z_{3t})'$ and (b) $\mathbf{z}_{1t} = (z_{2t}, z_{1t}, z_{3t})'$, are identical, and *then* that $a_{31}^{(a)} = a_{32}^{(b)}$ and $a_{32}^{(a)} = a_{31}^{(b)}$, where superscripts '$(a)$' and '$(b)$' refer to cases (a) and (b), respectively.

First, consider the case (a) with $\mathbf{z}_{1t} = (z_{1t}, z_{2t}, z_{3t})'$. Here we have

$$\mathbf{\Sigma}_{11}^{(a)} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}.$$

Using the relationship in (B.2), it is straightforward to show

$$p_{11}^{(a)} = \sqrt{\sigma_{11}}; \; p_{21}^{(a)} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}}; \; p_{31}^{(a)} = \frac{\sigma_{13}}{\sqrt{\sigma_{11}}};$$

$$p_{22}^{(a)} = \sqrt{\frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{11}}}; \; p_{32}^{(a)} = \sqrt{\frac{\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}}{\sigma_{11}\left(\sigma_{11}\sigma_{22} - \sigma_{12}^2\right)}};$$

$$p_{33}^{(a)} = \sqrt{\frac{\sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}}. \quad (B.7)$$

Turning to $\mathbf{A}_{11}$, and using the above results, we have

$$p_{21}^{(a)}p_{32}^{(a)} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} \times \sqrt{\frac{\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}}{\sigma_{11}\left(\sigma_{11}\sigma_{22} - \sigma_{12}^2\right)}} = \frac{\sigma_{12}}{\sigma_{11}}\sqrt{\frac{\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}}$$

$$p_{11}^{(a)}p_{22}^{(a)} = \sqrt{\sigma_{11}} \times \sqrt{\frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{11}}} = \sqrt{\sigma_{11}\sigma_{12} - \sigma_{12}^2}$$

$$\frac{p_{31}}{p_{11}} = \frac{\sigma_{13}}{\sigma_{11}}$$

so that

$$a_{31}^{(a)} = \frac{p_{21}^{(a)}p_{32}^{(a)}}{p_{11}^{(a)}p_{22}^{(a)}} - \frac{p_{31}^{(a)}}{p_{11}^{(a)}} = \frac{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}, \quad (B.8)$$

and

$$a_{32}^{(a)} = -\frac{p_{32}^{(a)}}{p_{22}^{(a)}} = \frac{\sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}. \quad (B.9)$$

Second, consider the case (b) of $\mathbf{z}_{1t} = (z_{2t}, z_{1t}, z_{3t})'$. Now we have

$$\boldsymbol{\Sigma}_1^{(b)} = \begin{bmatrix} \sigma_{22} & \sigma_{12} & \sigma_{23} \\ \sigma_{12} & \sigma_{11} & \sigma_{13} \\ \sigma_{23} & \sigma_{13} & \sigma_{33} \end{bmatrix},$$

and similarly,

$$p_{11}^{(b)} = \sqrt{\sigma_{22}};\ p_{21}^{(b)} = \frac{\sigma_{12}}{\sqrt{\sigma_{22}}};\ p_{31}^{(b)} = \frac{\sigma_{23}}{\sqrt{\sigma_{22}}};$$

$$p_{22}^{(b)} = \sqrt{\frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{22}}};\ p_{32}^{(b)} = \sqrt{\frac{\sigma_{22}\sigma_{13} - \sigma_{12}\sigma_{23}}{\sigma_{22}\left(\sigma_{11}\sigma_{22} - \sigma_{12}^2\right)}};$$

$$p_{33}^{(b)} = \sqrt{\frac{\sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{13}\sigma_{23}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}}. \quad \text{(B.10)}$$

Therefore, we now have

$$a_{31}^{(b)} = \frac{p_{21}^{(b)}p_{32}^{(b)}}{p_{11}^{(b)}p_{22}^{(b)}} - \frac{p_{31}^{(b)}}{p_{11}^{(b)}} = \frac{\sigma_{12}\sigma_{13} - \sigma_{11}\sigma_{23}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}, \quad \text{(B.11)}$$

$$a_{32}^{(b)} = -\frac{p_{32}^{(b)}}{p_{22}^{(b)}} = \frac{\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}}{\sigma_{11}\sigma_{22} - \sigma_{12}^2}. \quad \text{(B.12)}$$

Comparing (B.7), (B.8) and (B.9) with (B.10), (B.11) and (B.12), we find that

$$p_{33}^{2\,(a)} = p_{33}^{2\,(b)};\ a_{31}^{(a)} = a_{32}^{(b)};\ a_{32}^{(a)} = a_{31}^{(b)},$$

as desired.

This result clearly shows that once the order of the particular structural shock is determined, their impulse responses on the variables in the system are invariant to reordering of other variables before the specific equation of interest.

Finally, from (B.6) it is trivial to show that the structural impulse responses of the shocks to ε_{3t} on the variables in the system are also invariant to reordering of variables in $\mathbf{z}_{2t}$, since if their order is changed, then all the associated VAR parameter estimates are changed such that the structural impulse responses are intact.

APPENDIX C

Data for the UK model

Here we describe the definitions and sources of the variables used to estimate the core model of the UK economy. Our intention is to enable the user to use this appendix in combination with the information provided on the authors' web pages (which contains all the necessary files and data used in the estimation and construction of the core variables) to reproduce our results. The appendix also provides a brief guide on how to construct the *Microfit 4.0* file *ukmod.fit*, which contains all the variables used in the estimation and outlines the steps required to be performed in *Microfit 4.0* to reproduce our estimates.

C.1 Definitions and sources of the core model variables

The core UK model variables are as follows:

[1] y_t: the natural logarithm of UK real per capita domestic output, defined as $[\widetilde{Y}_t/(P_t \times POP_t)]$ in Chapter 4, is computed as:

$$\ln(GDP_t/POP_t),$$

where GDP_t is real gross domestic product, at 1995 market prices (index numbers, 1995 = 100), seasonally adjusted, source: Office of National Statistics (ONS) Economic Trends, code YBEZ. POP_t is total UK population in thousands, source: ONS, Monthly Digest of Statistics, code DYAY, which at the time of collection of the data was available up to 1998. For the 1999 number we extrapolated the 1998 annual number using the average annual growth rate for the period 1993–1997. For the population variable we constructed a quarterly series through linear interpolation of the annual numbers and then converted the quarterly population series to an index number.

[2] p_t: the natural logarithm of the domestic price level is computed as:

$$\ln(P_t),$$

where P_t is the UK Producer Price Index: Output of Manufactured Products (1995 = 100), source: ONS, Economic Trends, code PLLU.

The data used in the estimation are seasonally adjusted versions of p_t or $\ln(P_t)$, where the adjustment is performed using the *Stamp* package (see Harvey, Koopman, Doornik and Shephard, 1995). This involved using a Structural Time Series approach on the first difference of p_t, Δp_t (as we observed a seasonal pattern in the spectral density of Δp_t rather p_t) and then integrating the seasonally adjusted first difference up to compute the seasonally adjusted level. We adopted the *Stamp* manual's recommended version (p. 88) of the basic structural model of a stochastic trend with a stochastic slope, a trigonometric seasonal and an irregular component. A cyclical component was not included in the adjustment procedure. It is worth noting the *Stamp* manual's comment (p. 88) that in practice seasonal components seem to be insensitive to the specification of the trend and the inclusion of a cycle.

[3] $\Delta\tilde{p}_t$: the UK inflation rate is computed as:

$$\ln(P_t^R) - \ln(P_{t-1}^R),$$

where P_t^R is the UK Retail Price Index, All Items (1995 = 100, rebased from 1987 = 100), source: ONS, Economic Trends, code CHAW. As with the Producer Price Index, in the estimation we use a seasonally adjusted version of $\ln(P_t^R)$, where the adjustment is performed using the Structural Time Series procedure described above.

[4] r_t: the domestic nominal interest rate, measured as a quarterly rate is computed as:

$$0.25 \times \ln\left[1 + (R_t/100)\right],$$

where R_t is the 90 day Treasury Bill average discount rate, at an annualised rate, source: ONS, Financial Statistics, code AJNB.

[5] $h_t - y_t$: the natural logarithm of real per capita money stock expressed as a proportion of real per capita income is computed as:

$$\ln(\widetilde{H}_t/\widetilde{Y}_t),$$

where $\widetilde{H}_t$ is the M0 definition of the money stock (end period, £ Million) seasonally adjusted, source: ONS, Financial Statistics and Bank of England. For the period 1969q2–1999q4 we use M0 money stock source: ONS, Financial Statistics, code AVAE. Prior to this period, where no M0 money stock data is available, we project the AVAE series backwards using the quarterly percentage change (where the quarterly data is the average of the monthly data) of estimated circulation of notes and coins with the public as documented in the Bank of England Abstract 1970. Nominal income $\widetilde{Y}_t$, is measured using gross domestic product at market prices (£ Million) and is seasonally adjusted, source: ONS, Economic Trends, code YBHA. Note that $\ln(\widetilde{H}_t/\widetilde{Y}_t) = \ln(h_t/y_t)$ given that P_t and POP_t appear in both the numerator and denominator (see the definitions in Chapter 4).

[6] e_t: the natural logarithm of the UK nominal effective exchange rate is computed as:

$$-\ln(E_t),$$

where E_t is the Sterling Effective Exchange Rate (1995 = 100, rebased from 1990 = 100), source: ONS, Financial Statistics, code AJHX. The ONS define E_t as the foreign price of domestic currency (a rise represents a UK currency appreciation) hence we take minus the logarithm of E_t redefining e_t as the domestic price of foreign currency, as defined in the text.

[7] y_t^*: the natural logarithm of real per capita foreign output, defined as $[\widetilde{Y}_t^*/(P_t^* \times POP_t^*)]$ in Chapter 4 is computed as:

$$\ln(GDP_t^*/POP_t^*),$$

where GDP_t^* is a total OECD Gross Domestic Product Volume Index (1995 = 100), at 1995 market prices, seasonally adjusted, source: OECD, Main Economic Indicators (MEI), code Q00100319. POP_t^* is total OECD population (adjusted by subtracting the populations of Mexico, Poland, Hungary and Czech Republic), source: OECD, Labour Force Statistics, 1967–1987 and 1974–1996. For 1997–1999 we extrapolated the 1996 annual number using the average annual growth rate for the period 1992–1996. For the population variable we constructed a quarterly series through linear interpolation of the annual numbers and then converted the quarterly population series to an index number.

[8] p_t^*: the natural logarithm of the foreign price index is computed as:

$$p_t^* = \ln(P_t^*),$$

where P_t^* is the total OECD Producer Price Index, 1995 = 100, source: OECD, MEI, code Q005045k. Data was available on this series from 1982q1. The data prior to 1982q1 was constructed by backwardly imposing the percentage changes of a separately constructed weighted average index of OECD consumer and producer prices on the 1982q1 figure. As with the previous two price measures, in the estimation we used a seasonally adjusted version of the foreign price variable, where the adjustment is performed using the Structural Time Series procedure described above.

[9] r_t^*: the foreign nominal interest rate, measured as a quarterly rate is computed as:

$$r_t^* = 0.25 \times \ln\left[1 + (R_t^*/100)\right],$$

Data for the UK Model

where R_t^* is a weighted average of foreign annualised interest rates computed as:

$$R_t^* = \sum_{j=1}^{m_r} w_j^r R_{jt},$$

where w_j^r are fixed weights and $m_r = 4$. The countries and weights in brackets are the United States (0.4382), Germany (0.236), Japan (0.2022) and France (0.1236). The weights are taken from the IMFs International Financial Statistics Yearbook 1998, pages x and xi which report Special Deposits Rights (SDR) weights for five countries which in 1996 were for the US 0.39, Germany 0.21, France 0.11, Japan 0.18 and the UK 0.11. Excluding the UK we the recompute the weights to get those reported above.

The annualised interest rates used in the calculation, R_{jt}, are all from the IMFs International Financial Statistics (IFS). For the US we use the three-month Treasury Bill rate (IFS Code Q11160C), for Germany the Money Market Rate (IFS Code Q13460B), for Japan the Money Market Rate (IFS Code Q15860B) and for France the three-month Treasury Bill Rate (IFS Code Q13260C).

[10] p_t^0: the natural logarithm of the oil price is computed as:

$$\ln(POIL),$$

where *POIL* is the Average Price of Crude Oil, in terms of US Dollars per Barrel, source: IMF, IFS, code Q00176AAZ, converted into a 1995 = 100 index.

To construct the *Microfit 4.0* file *ukmod.fit* read in the file *core.fit* into *Microfit 4.0* and run *core.bat*. The resulting file is *ukmod.fit*, which must be saved, where the names used in file, which correspond to the model variables defined above, are the following: $y = y_t$, $p = p_t$, $dpr = \Delta\tilde{p}_t$, $r = r_t$, $hy = (h_t - y_t)$, $e = e_t$, $ys = y_t^*$, $ps = p_t^*$, $rs = r_t^*$, $po = p_t^o$, $pps = (p_t - p_t^*)$, $dpo = p_t^o - p_{t-1}^o$.

All the estimation reported in Chapter 9 is performed in *Microfit 4.0* (the impulse responses, persistence profiles and probability forecasts can be computed using the *Gauss* files provided, see the next appendix describing the *Gauss* files). The results in the paper may be reproduced, using the file *ukmod.fit* in *Microfit 4.0*, through the execution of the following steps:

(i) Choose the multivariate estimation option, select the cointegrating VAR menu and choose option 4, unrestricted intercepts restricted trends.

(ii) Read in the *ukmod.lst*, set the period to be 1965q1–1999q4 and the order of the VAR to be two and estimate.

(iii) Set number of cointegrating vectors to be five ($r = 5$, option 2) and in the following menu select option 6, long-run structural modelling.

(iv) Choose option 4, likelihood ratio test, exactly identify the system by reading in *exiden.equ* and then estimate the cointegrating VAR model subject to the exact identifying restrictions.

(v) Then choose to impose and test the over-identifying restrictions. First using the restrictions contained in *oviden1.equ*, second using *oviden2.equ*.

APPENDIX D

Gauss programs and result files

Much of the estimation and analysis of the UK core model was carried out using Pesaran and Pesaran's (1997) econometric software package *Microfit 4.0,* and *Microfit 4.11*. However, a number of the calculations and computations reported in the book were conducted using a series of *Gauss* programs. For users who prefer the flexibility such programs allow and for those who wish to perform (and adapt) the range of estimation and computations reported in the book, we are making available, through our webpages, the *Gauss* programs we have used in the analysis of the core model in a sequence of files. The content and operation of these files is described below. Note that an updated version of microfit, *Microfit 5.0* (to be published by Oxford University Press in 2006), will be able to compute all the impulse responses and persistence profiles described below.

In total there are eight programs. The first two relate to impulse responses and persistence profiles:

- *GLPS-GIR.g* computes Generalised Impulse Responses (GIRs), Orthogonalised Impulse Responses (OIRs), Persistence Profiles (PPs), and VECM estimation results (with diagnostics), and examines the stability of the VECM system.
- *GLPS-SIR.g* computes impulse responses which result from (exogenous) oil price shocks and (unanticipated) monetary policy shocks, where monetary policy shocks are defined according to the short-run identification scheme developed in Chapter 5.

The next five programs compute and evaluate probability event forecasts. Two are concerned with out-of-sample probability events:

- *GLPS-PFS.g* computes out-of-sample probability event forecasts, *h*-steps ahead, taking into account future uncertainty only.
- *GLPS-PFB.g* computes out-of-sample probability event forecasts, *h*-steps ahead, taking into account future and parameter uncertainty.

The next three programs conduct in-sample forecast evaluation using one-step ahead recursive probability forecasts of directional-changes and events used in the calculation of probability integral transforms over the period 1999q1–2001q1 (nine quarters).

- *GLPS-EVS.g* computes in-sample one-step ahead probability event forecasts taking into account future uncertainty only.
- *GLPS-EVB.g* computes in-sample one-step ahead probability event forecasts taking into account future and parameter uncertainty.
- *GLPS-EV.g* computes forecast evaluation statistics for one-step ahead probability event forecasts: hit ratios, Kuipers Score, Pesaran–Timmermann, Kolmogorov–Smirnov test statistics for probability integral transform. To obtain the results reported in the book, you run this program using as inputs the files produced by first running the two programs above, *GLPS-EVS.g* and *GLPS-EVB.g*.

Finally the eighth program computes the trend decomposition in cointegrating VARs described in Section 10.3.

- *GLPS-DEC.g* computes the permanent and transitory decomposition of all the endogenous variables in the vector $\mathbf{z}_t$ using the estimated VECM core model and estimates of the restricted growth rates, $\mathbf{g}$.

D.1 General comments on the *Gauss* programs

All the programs presuppose that certain results have been obtained already (*e.g.* by *Microfit*, as described at the end of Appendix C). Specifically, they take as inputs: the ML estimates of the long-run cointegrating relationships subject to general linear non-homogeneous restrictions (and their rank); and the estimation results for the exogenous $I(1)$ variable(s) (here an oil price equation).

The initial step in each program loads and defines the data. It also specifies some initial information which is needed for the rest of the program, such as the VAR lag order, the rank and the estimates of cointegrating vectors. Given the estimates of the cointegrating vectors, the program estimates the dynamic short-run parameters. It then combines these results with the estimation results for the exogenous $I(1)$ variable(s), to provide the full system VAR estimation results. These form the basis for an analysis of further short-run dynamics such as impulse responses and forecasts. For the underlying econometric theory, see Chapters 6 and 7 and the related papers by Pesaran, Shin and Smith (2000) and Pesaran and Shin (2002).

D.2 Impulse response and persistence profile programs

The impulse response results for the UK described in Chapter 10 were obtained using the two programs *GLPS-GIR.g* and *GLPS-SIR.g* and reading in the UK dataset given in *ukmod99.dat.* The dataset has the dimension of 148×10 and the variables

are saved in the column order: y_t, y_t^*, r_t, r_t^*, e_t, $h_t - y_t$, p_t^o, Δp_t^o, Δp_t and $p_t - p_t^*$ (see Appendix C for details). The full data period is 1963q1–1999q4 (148 observations), but the program estimates the cointegrating VAR(2) model over the period 1965q1–1999q4 (140 observations) using the Cointegrating VAR Option 4 with unrestricted intercepts and restricted trends.

GLPS-GIR.g

This program computes GIRs, OIRs, PPs, and the estimation results, and analyses stability of the VECM. It also provides an option to compute the empirical confidence intervals for PPs, GIRs and OIRs with respect to reduced form errors, based on the bootstrap re-sampling techniques. In our work, we employ non-parametric re-sampling methods with 2000 replications to allow for parameter uncertainty (see Section 6.4 for further details).

The estimation results in Sections 10.2.2 and 10.2.3, and also those reported in Garratt, Lee, Pesaran and Shin (2000) can be generated using this program. The results reported in Figures 10.3, 10.4, 10.5, 10.6, 10.9 and 10.10 are also computed using this file. The program requires the user to select the shock (to an equation) by specifying the number defining the order of the variable in the $\mathbf{z}_t$ vector (see below for the order). The program assumes the size of the shock is equal to the standard deviation of the selected equation error, and that all the results (except for OIR) are invariant to re-ordering of the variables in the VAR.

After running the program, you will obtain the following five *Gauss* data files (with an *fmt* extension) which contain the results for PPs, GIRs and OIRs. The saved files are: *PPOUT.fmt*, *GIRZOUT.fmt*, *OIRZOUT.fmt*, *GIROUT.fmt*, and *OIROUT.fmt*, respectively.

PPOUT.fmt contains the results for the scaled PPs of the cointegrating relations, which take the value of unity on impact of the shock and tend to zero as the time horizon tends to infinity. The dimensions are $(h+1)$ by $7r$, where h is the number of horizon and r is the number of cointegrating vectors (= 5 in the case of the core UK model). The first r columns (1 to r) are point estimates of the PPs of the $1, \ldots, r$ cointegrating vectors; the next r columns ($r+1$ to $2r$) are empirical means; the next r columns ($2r+1$ to $3r$) are empirical medians; the next r columns ($3r+1$ to $4r$) are empirical 90% lower confidence intervals (CIs); the next r columns ($4r+1$ to $5r$) are empirical 90% upper CIs; and finally, the next r columns ($5r+1$ to $6r$) are empirical 95% lower CIs, whereas the final r columns ($6r+1$ to $7r$) are empirical 95% upper CIs. Note the order of the cointegrating relations for each block (containing r columns) is *PPP*, *IRP*, *OG*, *MME* and *FIP*.

GIRZOUT.fmt (*OIRZOUT.fmt*) contains the GIRs (OIRs) of the r cointegrating relations with respect to selected shocks, referred to as PPs in the text. These are the files which contain the results, when the foreign interest rate, foreign output and domestic interest rate are selected, which are plotted in Figures 10.3, 10.5 and 10.9, respectively. The dimensions and ordering of these result files are exactly the same as those of *PPOUT.fmt*.

The files *GIROUT.fmt* (*OIROUT.fmt*) contain results for GIRs (OIRs) of the m exogenous and endogenous $I(1)$ variables in the system with respect to selected shocks ($m = 9$ in the core UK model). The dimensions are $(h+1)$ by $7m$, where m is number of variables. The first m columns (1 to m) are point estimates of GIRs (OIRs) of $1, \ldots, m$ variables; the next m columns ($m+1$ to $2m$) are empirical means; the next m columns ($2m+1$ to $3m$) are empirical medians; the next m columns ($3m+1$ to $4m$) are empirical 90% lower CIs; and the next m columns ($4m+1$ to $5m$) are empirical 90% upper CIs. The next m columns ($5m+1$ to $6m$) are empirical 95% lower CIs, whereas the final m columns ($6m+1$ to $7m$) are empirical 95% upper CIs. Note the order of the variables for each block (containing m columns) is: p_t^o, e_t, r_t, r_t^*, Δp_t, y_t, $p_t - p_t^*$, $h_t - y_t$ and y_t^* (the numbering for the selection of the shock follows this order).

GLPS-SIR.g

This program computes the Structural Impulse Responses and PPs reported in Figures 10.1, 10.2, 10.7, 10.8, 12.3 and 12.4. For this purpose we decompose variables as $\mathbf{z}_t = (\mathbf{z}_{1t}, \mathbf{z}_{2t})$, where $\mathbf{z}_{1t} = (p_t^o, e_t, r_t^*, r_t)$ and $\mathbf{z}_{2t} = (\Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)$. Note the position of the variable, r_t, determined by the short-run identification scheme, is important for an analysis of monetary policy shocks. Once its position is determined, the impulse responses are invariant to the change of ordering of other variables in the system before and after r_t; see Appendix B for a proof.

As an additional option the program can examine the impact of an (exogenous) intercept shift in the interest rate equation, as an alternative autonomous or exogenous monetary policy shock. The program also provides the empirical mean and confidence intervals for generalised impulse response functions with respect to structural shocks to the oil price, exchange rate, foreign interest rate and domestic interest rate equations as well as an intercept shift in the interest rate equation, based on the bootstrap re-sampling techniques with 2000 replications to allow for parameter uncertainty (see Section 6.4 for further details). In all cases the size of the shock is equal to the standard deviation of the selected equation error. For the case of the intercept shift in the domestic interest equation, the size of the shock is equal to the standard deviation of the domestic interest equation error.

After running the program, you will obtain 10 *Gauss* result files (with an fmt extension). The saved files are *POGIR.fmt*, *POGIRZ.fmt*, *EXGIR.fmt*, *EXGIRZ.fmt*, *RSGIR.fmt*, *RSGIRZ.fmt*, *MPGIR.fmt*, *MPGIRZ.fmt*, *INTIR.fmt* and *INTIRZ.fmt*, respectively. We have then provided estimation results in Sections 10.2.1 and 10.2.4.

The files *POGIRZ.fmt*, *EXGIRZ.fmt*, *RSGIRZ.fmt* and *MPGIRZ.fmt* contain the results for the GIRs of the r cointegrating relations with respect to oil price shocks, exchange rate shocks, foreign interest rate shocks and monetary policy shocks, respectively. The file *INTIRZ.fmt* contains the results for impulse responses of the r cointegrating relations with respect to the autonomous intercept shift in the domestic interest equation. Their dimensions are $(h+1)$ by $7r$. The first r ($= 5$ here) columns (1 to r) are point estimates of the GIRs of $1, \ldots, r$ cointegrating vectors; the next r columns ($r+1$ to $2r$) are empirical means; the next r columns ($2r+1$ to $3r$) are empirical medians; the next r columns ($3r+1$ to $4r$) are empirical 90% lower CIs; the next r columns ($4r+1$ to $5r$) are empirical 90% upper CIs; the next r columns ($5r+1$ to $6r$) are empirical 95% lower CIs; and the final r columns ($6r+1$ to $7r$) are empirical 95% upper CIs. Note the order of the cointegrating relations for each block (containing r columns) is *PPP*, *IRP*, *OG*, *MME* and *FIP*.

POGIR.fmt, *EXGIR.fmt*, *RSGIR.fmt* and *MPGIR.fmt* contain the results for the GIRs of the m variables with respect to oil price shocks, exchange rate shocks, foreign interest rate shocks and monetary policy shocks, respectively. The file *INTIR.fmt* contains the results for the impulse responses of the m variables with respect to the autonomous intercept shift in the domestic interest equation. Their dimensions are $(h+1)$ by $7m$. The first m columns (1 to m) are the empirical means of the GIRs of $1, \ldots, m$ variables; the next m columns ($m+1$ to $2m$) are the empirical means; the next m columns ($2m+1$ to $3m$) are the empirical medians; the next m columns ($3m+1$ to $4m$) are the empirical 90% lower CIs; the next m columns ($4m+1$ to $5m$) are the empirical 90% upper CIs; the next m columns ($5m+1$ to $6m$) are the empirical 95% lower CIs; and the final m columns ($6m+1$ to $7m$) are the empirical 95% upper CIs. Note the order of the impulse responses for each block (containing m columns) is: p^o, e, r^*, r, Δp, y, $p - p^*$, $h - y$ and y^*.

D.3 Programs for computing probability forecasts

The probability forecast programs use the data file, *ukmod01.dat*. This is a 153×9 file which contains data for the extended period 1963q1–2001q1 (153 observations), saved in the column order of y, r, r^*, e, $h - y$, p^o, Δp, $p - p^*$, y^* (the change in oil prices, Δp^o, is defined in the program). We estimate the ML cointegrating vectors for the period 1965q1–2001q1, but estimate the short-run dynamic parameters of the vector error correction model over the shorter sample 1985q1–2001q1.

We allow for future and parameter uncertainty separately and jointly and in addition we allow for model uncertainty. We focus on uncertainty regarding the rank of the cointegrating vectors, so we consider the six cases with rank = 0,1,2,3,4,5 where we use exactly identified cointegrating vectors. We also consider our core model, *i.e.* the case where we have five cointegrating relationships which impose the theory based over-identifying restrictions described and tested in Chapter 9. This makes for seven models. For each of the seven models, we examine exogenous uncertainty through the consideration of two different oil price equations, based on (A) the simple random walk with a drift model and (B) the unrestricted VAR(2) specification. Hence in total 14 models are considered.

These models are denoted by OV5A and OV5B for the five cointegrating vectors obtained subject to the theory based over-identifying restrictions, combined with the oil price equations A and B, respectively. Similarly we denote EX5A and EX5B as being five cointegrating vectors obtained subject to the exactly identifying restrictions combined with an oil price equations A and B, respectively. Following this use of notation the remaining 10 models are denoted: EX4A, EX4B, EX3A, EX3B, EX2A, EX2B, EX1A, EX1B, EX0A, EX0B. Note that the models, EX0A and EX0B, have zero cointegrating relations.

The program computes the weights for these models according to the AIC weight scheme described in Chapter 7, but also considers weights based on SBC, HQ, and equal weights of 1/14. See Section 7.3 for more details.

D.3.1 *Programs for computing out-of-sample probability event forecasts*

The two programs, *GLPS-PFS.g* and *GLPS-PFB.g*, compute out-of-sample probability event forecasts based on the h-step ahead forecasts of the nine variables in $\mathbf{z}_t$ and their four-quarter moving averages with $h = 1, \ldots, 24$. Note that the computation algorithms for *GLPS-PFS.g* and *GLPS-PFB.g* are basically the same, where only future uncertainty is allowed for in *GLPS-PFS.g*, whereas both future and parameter uncertainties are allowed for in *GLPS-PFB.g*.

In our UK application, we consider the following seven events:

$E1$: A single event: Pr(four-quarter moving average of inflation $< a\%$), where a is per cent per annum and we use 10 threshold values of $a = (0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 5)$.

$E2$: A single event: Pr(four-quarter moving average of the gross output growth $< a\%$), where gross output growth is the sum of output growth and deterministic population growth, a is per cent per annum and we use 10 threshold values of $a = (-1.5, 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 5)$.

$E3$: A single event: BofE target met, Pr(1.5% < four-quarter moving average of inflation < 3.5%).

$E4$: A single event: recession, Pr(quarterly output growths < 0% for two consecutive quarters).

$E5$: A single event: low growth, Pr(four-quarter moving average of gross output growth < 1%)

$E6$: A joint event: Pr(no recession and BofE target met).

$E7$: A joint event: Pr(high growth and BofE target met).

GLPS-PFS.g (with future uncertainty only)

After running the program, you will obtain the following 18 *Gauss* result files (with an fmt extension). They contain the results of the Probability Event Forecasts based on future uncertainty only, which we have used in obtaining the tables and figures reported in Chapter 11 and Garratt, Lee, Pesaran and Shin (2003, *Journal of American Statistical Association*).

The saved files are *OV5ASPE.fmt*, *OV5BSPE.fmt*, *EX5ASPE.fmt*, *EX5BSPE.fmt*, *EX4ASPE.fmt*, *EX4BSPE.fmt*, *EX3ASPE.fmt*, *EX3BSPE.fmt*,*EX2ASPE.fmt*, *EX2BSPE.fmt*, *EX1ASPE.fmt*, *EX1BSPE.fmt*, *EX0ASPE.fmt*, *EX0BSPE.fmt*, *AVGSPE.fmt*, *AICSPE.fmt*, *SBCSPE.fmt*and *HQSPE.fmt*, respectively.

The dimensions of these *Gauss* result files is the number of horizons (= 24 here) by 25. The first 10 columns (1 to 10) are probability forecasts for event $E1$ for the

10 thresholds; the next 10 columns (11 to 20) are probability forecasts for event $E2$ with 10 thresholds; the 21st column is probability forecasts for event $E3$; the 22nd column is the probability forecasts for event $E4$; the 23rd column is the probability forecasts for event $E5$; the 24th column is the probability forecasts for event $E6$; and, finally, the 25th column is the probability forecasts for event $E7$.

GLPS-PFB.g (with future and parameter uncertainty)

This program is as above but where the Probability Event Forecasts are based on both future and parameter uncertainty. The saved files are *OV5ABPE.fmt*, *OV5BBPE.fmt*, *EX5ABPE.fmt*, *EX5BBPE.fmt*, *EX4ABPE.fmt*, *EX4BBPE.fmt*, *EX3ABPE.fmt*, *EX3BBPE.fmt*, *EX2ABPE.fmt*, *EX2BBPE.fmt*, *EX1ABPE.fmt*, *EX1BBPE.fmt*, *EX0ABPE.fmt*, *EX0BBPE.fmt*, *AVGBPE.fmt*, *AICBPE.fmt*, *SBCBPE.fmt* and *HQBPE.fmt*.

D.3.2 *Programs for computing in-sample probability event forecast evaluation*

The three programs, *GLPS-EVS.g*, *GLPS-EVB.g* and *GLPS-EV.g*, are used to evaluate the probability event forecasts. They compute in-sample forecast evaluation using one-step ahead probability forecasts of directional-change and events used in calculating probability integral transforms, which are obtained using recursive point forecasts over 1999q1–2001q1 (nine quarters).

To replicate the results reported in Chapter 11, first run the programs *GLPS-EVS.g* and *GLPS-EVB.g* and save the output *Gauss* results files. Then run the program *GLPS-EV.g*. The algorithms used in *GLPS-EVS.g* and *GLPS-EVB.g* are essentially the same, although only future uncertainty is allowed in *GLPS-EVS.g* whereas both future and parameter uncertainties are allowed in *GLPS-EVB.g*.

Here we consider the following nine single event probability of directional changes:

$$E1: \Pr\left(\Delta^2 p^o_{T+1} > 0\right) \qquad E2: \Pr\left(\Delta e_{T+1} > 0\right)$$
$$E3: \Pr\left(\Delta r^*_{T+1} > 0\right) \qquad E4: \Pr\left(\Delta r_{T+1} > 0\right)$$
$$E5: \Pr\left(\Delta^2 p_{T+1} > 0\right) \qquad E6: \Pr\left(\Delta^2 y_{T+1} > 0\right)$$
$$E7: \Pr\left(\Delta(p_{T+1} - p^*_{T+1}) > 0\right) \qquad E8: \Pr\left(\Delta^2(h_{T+1} - y_{T+1}) > 0\right)$$
$$E9: \Pr\left(\Delta^2 y^*_{T+1} > 0\right).$$

We also consider the following nine single events for the probability integral transform, which will be used in computing the Kolmogorov–Smirnov test statistic:

$$I1: \Pr\left(\text{forecast of } \Delta^2 p^o_{T+1} > \text{actual } \Delta^2 p^o_{T+1}\right)$$
$$I2: \Pr\left(\text{forecast of } \Delta e_{T+1} > \text{actual } \Delta e_{T+1}\right)$$
$$I3: \Pr\left(\text{forecast of } \Delta r^*_{T+1} > \text{actual } \Delta r^*_{T+1}\right)$$
$$I4: \Pr\left(\text{forecast of } \Delta r_{T+1} > \text{actual } \Delta r_{T+1}\right)$$
$$I5: \Pr\left(\text{forecast of } \Delta^2 p_{T+1} > \text{actual } \Delta^2 p_{T+1}\right)$$
$$I6: \Pr\left(\text{forecast of } \Delta^2 y_{T+1} > \text{actual } \Delta^2 y_{T+1}\right)$$

$$I7: \Pr\left(\text{forecast of } \Delta(p_{T+1} - p^*_{T+1}) > \text{actual } \Delta(p_{T+1} - p^*_{T+1})\right)$$
$$I8: \Pr\left(\text{forecast of } \Delta^2(h_{T+1} - y_{T+1}) > \text{actual } \Delta^2(h_{T+1} - y_{T+1})\right)$$
$$I9: \Pr\left(\text{forecast of } \Delta^2 y^*_{T+1} > \text{actual } \Delta^2 y^*_{T+1}\right).$$

GLPS-EVS.g (with future uncertainty only)

After running the program, you will obtain the following 50 *Gauss* data files (with an fmt extension). They contain the results for (i) the one-step ahead central forecasts (18 files), (ii) root mean square errors (RMSEs) (14 files), (iii) the in-sample Probability Event Forecasts (18 files):

(i) The 18 files for one-step ahead central forecasts with no future and no parameter uncertainties are: *OV5AFOR.fmt, OV5BFOR.fmt, EX5AFOR.fmt, EX5BFOR.fmt, EX4AFOR.fmt, EX4BFOR.fmt, EX3AFOR.fmt, EX3BFOR.fmt, EX2AFOR.fmt, EX2BFOR.fmt, EX1AFOR.fmt, EX1BFOR.fmt, EX0AFOR.fmt, EX0BFOR.fmt, AVGFOR.fmt, AICFOR.fmt, SBCFOR.fmt* and *HQFOR.fmt*. Here the first four letters refer to individual models, and AVG, AIC, SBC and HQ indicate the equal weights, the AIC weights, the SBC weights and the HQ weights, respectively, used in pooling the forecasts.

The dimensions of all the above *Gauss* result matrices are the same, the number of in-sample horizons (here nine quarters over 1999q1–2001q1) by 54. The first nine columns (1 to 9) are one-step ahead central forecasts of the level of the nine variables (in the order of $p^o, e, r^*, r, \Delta p, y, p-p^*, h-y, y^*$); the next nine columns (10 to 18) are one-step ahead central forecasts of the four-quarter moving averages of the levels of the nine variables; the columns from 19 to 27 are one-step ahead central forecasts of the first differences; the next nine columns (28 to 36) are one-step ahead central forecasts of the four-quarter moving average of the first differences; columns 37 to 45 are one-step ahead central forecasts of the second differences; and the next nine columns (46 to 54) are one-step ahead central forecasts of the four-quarter moving average of the second differences.

(ii) The 14 files for RMSEs of the one-step ahead central forecasts with no future and no parameter uncertainties are: *OV5ARMSE.fmt, OV5BRMSE.fmt, EX5ARMSE.fmt, EX5BRMSE.fmt, EX4ARMSE.fmt, EX4BRMSE.fmt, EX3ARMSE.fmt, EX3BRMSE.fmt, EX2ARMSE.fmt, EX2BRMSE.fmt, EX1ARMSE.fmt, EX1BRMSE.fmt, EX0ARMSE.fmt* and *EX0BRMSE.fmt*.

The dimensions of all the above *Gauss* result files are the same, the number of in-sample horizon (here nine quarters over 1999q1–2001q1) by 27. The first nine columns (1 to 9) are RMSEs of the one-step ahead central forecasts of the level of the nine variables (in the order of p^o, e, r^*, r, Δp, y, $p - p^*$, $h - y$, y^*); the next 9 columns (10 to 18) are RMSEs of the one-step ahead central forecasts of the first diferences; and the next columns from 19 to 27 are RMSEs of the one-step ahead central forecasts of the second differences.

(iii) The 18 files for the probabilities of directional changes and probability integral transform with future uncertainty only are: *OV5ASPR.fmt, OV5BSPR.fmt, EX5ASPR.fmt, EX5BSPR.fmt, EX4ASPR.fmt, EX4BSPR.fmt, EX3ASPR.fmt, EX3BSPR.fmt, EX2ASPR.fmt, EX2BSPR.fmt, EX1ASPR.fmt, EX1BSPR.fmt, EX0ASPR.fmt, EX0BSPR.fmt, AVGSPR.fmt, AICSPR.fmt, SBCSPR.fmt* and *HQSPR.fmt*.

The dimensions of all the above *Gauss* data files are the same: the number of in-sample horizon (here nine quarters over 1999q1–2001q1) by 36. The first nine columns (1 to 9) are the probabilities of directional changes (see definitions of the events given above and denoted by $E1, \ldots, E9$) for the nine variables (in the order of p^o, e, r^*, r, Δp, y, $p - p^*$, $h - y$, y^*), using one-step ahead central forecasts of the first and second differences; the next nine columns (10 to 18) are the probabilities of directional changes for the nine variables using one-step ahead central forecasts of the four-quarter moving average of the first and second differences; the nine columns (19 to 27) are the probabilities of integral transforms (see definitions of the events given above and denoted by $I1, \ldots, I9$) for the nine variables, using one-step ahead central forecasts of the first and second differences; and the final nine columns (28 to 36) are the probabilities of integral transforms for the nine variables using one-step ahead central forecasts of the four-quarter moving average of the first and second differences.

Finally, we have also saved the two additional data files, *actdat.fmt* and *a4actdat.fmt*, which contain in-sample actual data observations for the first differences and the second differences of the data, and which will be used for comparison with one-step ahead forecasts of directional changes in the program *GLPS-EV.g*.

GLPS-EVB.g (with both future and parameter uncertainties)

After running the program, you will obtain the following 18 *Gauss* result files (with an fmt extension). They contain the results for Probability Event Forecasts for directional changes and probability integral transform with both future and parameter uncertainties. These will be used in the companion file *GLPS-EV.g* to compute various test statistics reported in the tables of Chapter 11.

The 18 files are *OV5ABPR.fmt, OV5BBPR.fmt, EX5ABPR.fmt, EX5BBPR.fmt, EX4ABPR.fmt, EX4BBPR.fmt, EX3ABPR.fmt, EX3BBPR.fmt, EX2ABPR.fmt, EX2BBPR.fmt, EX1ABPR.fmt, EX1BBPR.fmt, EX0ABPR.fmt, EX0BBPR.fmt, AVGBPR.fmt, AICBPR.fmt, SBCBPR.fmt* and *HQBPR.fmt*. The dimensions and ordering of the *Gauss* result files are as described in probability event matrices for *GLPS-EVS.g*.

GLPS-EV.g

This program computes the in-sample forecast evaluation test statistics using the *Gauss* result files saved after running the companion programs, *GLPS-EVS.g* and *GLPS-EVB.g*. The program computes the following statistics:

(i) *UD*, *DD*, *DU* and *UU*, where the first letter denotes the direction of forecasts (*D* for down, *U* for up) and the second the direction of actual outcome.

(ii) The hit ratio defined as: $(DD + UU) / (UD + DD + DU + UU)$.

(iii) The Kuipers Score statistic given by $H - F$, where $H = UU/(UU + UD)$ is the proportion of ups that were correctly predicted to occur, and $F = DU/(DU + DD)$ is the proportion of downs that were incorrectly predicted.

(iv) The Pesaran–Timmerman, test statistic.

(v) The Kolmogorov–Smirnov test statistic.

D.4 Program for computing the decomposition of trends in cointegrating VARs

GLPS-DEC.g

This program provides the decomposition of the underlying $I(1)$ variables into permanent and transitory components as described in Section 10.3. This decomposition can be viewed as a (generalised) multivariate BN decomposition but has an advantage that it is characterised fully in terms of observables and estimated parameters. See also Garratt, Robertson and Wright (2005). The program also computes the more conventional multivariate Beveridge–Nelson trends of the system.

As in the case of the programs for GIRs and PPs, we use the data file, *ukmod99.dat* and the ML estimates of the cointegrating VAR(2) model over 1965q1–1999q4 (140 observations) using the Cointegrating VAR Option 4 with unrestricted intercepts and restricted trends. The program requires as an input estimates of the vector **g**, the trend growth rates (these are computed using a restricted SURE procedure in Chapter 10; see Section 10.3). After running the program, you will obtain nine ASCII files with txt extensions: *po.txt, ex.txt, rs.txt, r.txt, dp.txt, y.txt, pps.txt, hy.txt* and *ys.txt*. They contain summary results for each of the variables (in the order of $p^o, e, r^*, r, \Delta p, y, p-p^*, h-y, y^*$). These files can be easily be read into the *Excel* program for constructing tables and figures. The dimensions of the result files are 140 (the sample size) by 6. In each case, the first column contains the actual data, the second column the permanent component, the third column the transitory component, the fourth column the de-trended data, the fifth column the deterministic (permanent) trend and the sixth column the stochastic (permanent) trends.

Bibliography

Abadir, K. M., K. Hadri and E. Tzavalis (1999), 'The Influence of VAR Dimensions on Estimator Biases', *Econometrica*, 67, 163–181.

Abbot, A. J. and G. De Vita (2002), 'Long-Run Price and Income Elasticities of Demand for Hong Kong Exports: A Structural Cointegrating VAR Approach', *Applied Economics*, 34, 1025–132.

Adolfson, M., S. Laseen, J. Linde and M. Villani (2005), 'Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through', *Sveriges Riksbank Working Paper Series*, no 179.

Akerlof, G., W. Dickens and G. Perry (1996), 'The Macroeconomics of Low Inflation', *Brookings Papers on Economic Activity*, 1, 1–76.

Akusuwan, M. (2005), *A Small Quarterly Macroeconometric Model for the Thai Economy: A Structural Cointegrating VAR Approach*. PhD. Thesis, Faculty of Economics, University of Cambridge.

Allen, C., S. G. Hall and J. Nixon (1994), 'The New London Business School Model of the UK Economy', *Center for Economic Forecasting Discussion Paper* No. 18–94.

Anderson, R. G., D. L. Hoffman and R. H. Rasche (2002), 'A Vector Error-Correction Forecasting Model of the US Economy', *Journal of Macroeconomics*, 24, 569–598.

Ando, A. and F. Modigliani (1969), 'Econometric Evaluation of Stabilisation Policies', *American Economic Review*, 59, 296–314.

Astley, M. and A. Garratt (1996), 'Interpreting Sterling Exchange Rate Movements', *Bank of England Quarterly Bulletin*, November, 394–404.

Baillie, R. and P. McMahon (1989), *The Foreign Exchange Market: Theory and Econometric Evidence*. Cambridge University Press: Cambridge.

Bakhshi, H., A. Haldane and N. Hatch (1997), 'Some Costs and Benefits of Price Stability in the UK', *Bank of England Working Paper* No. 97.

Banerjee, A., J. Dolado, J. W. Galbraith and D. F. Hendry (eds.) (1993), *Co-integration, Error Correction and the Econometric Analysis of Non-Stationary Data*. Oxford University Press: Oxford.

Barassi, M. R., G. M. Caporale and S. G. Hall (2001), 'Testing for Changes in the Long-Run Causal Structure of Cointegrated Vector Auto Regressions', *Imperial College Discussion Paper*, ICMS13.