11

Probability event forecasting with the UK model

In this chapter, we consider the application of the probability forecasting techniques introduced in Chapter 7 to our model of the UK economy. A number of macroeconomic modelling teams in the UK have recently begun to provide further information on the uncertainties surrounding their forecasts of key macroeconomic variables. It is widely acknowledged that it is important to provide this information on the precision of the forecasts in order to enable policy-makers to motivate and justify actions based on the forecasts, and to help a more balanced evaluation of the forecasts by the public.[1] However, it remains rare for forecasters, policy-makers or private, to provide the detailed information on the range of potential outcomes that agents might find useful in decision-making and policy analysis. One explanation of this relates to the difficulty in measuring the uncertainties associated with forecasts in the large mainstream macroeconomic models typically employed. A second explanation is the perceived difficulty in conveying the outcomes of complicated macroeconomic models in a simple and easily understood form.

Our compact modelling approach, however, provides a practical framework for probability forecasting. The model is theoretically coherent, fits the UK historical aggregate time series data reasonably well (as argued in earlier Chapters) and yet the model is small enough to allow for a large variety of probability forecasting problems of interest to be analysed without encountering difficult computational problems. In what follows we shall

[1] For example, the Bank of England now routinely publishes a range of outcomes for its inflation and output growth forecasts (see Britton *et al.* (1998), or Wallis (1999)); the National Institute use their model to produce probability statements alongside their central forecasts (their methods are described in Blake (1996), and Poulizac *et al.* (1996)); and in the financial sector, J.P. Morgan presents 'Event Risk Indicators' in its analysis of foreign exchange markets.

focus on events that particularly interest the monetary authorities namely the inflation rate remaining within a given target band and the economy going into recession over various time frames. We consider these events both individually and jointly. Although only a small number of events are considered, we shall show that these probability event forecasts can convey a considerable amount of information on the uncertainties surrounding a forecast, and correspond with those which the public uses in judging policy-makers' performance.

11.1 An updated version of the core model

In principle, probability forecasts can be computed using any macroeconometric model, although the necessary computations would become prohibitive in the case of most large-scale macroeconometric models, particularly if the objective of the exercise is to compute the probabilities of joint events at long forecast horizons. At the other extreme, the use of small unrestricted VAR models, while computationally feasible, may not be satisfactory for the analysis of forecast probabilities over the medium term. Our VAR model of order 2, involving nine variables, represents an intermediate alternative that is well suited to the generation of probability forecasts. In what follows, therefore we work with a model of the same form as that presented in Chapter 9. However, in order to evaluate the forecasting performance of the model, we extend the dataset, so that it covers the period 1965q1–2001q1, as compared to 1965q1–1999q4 discussed in the earlier chapters and work with updated versions of the core model.[2]

As a reminder we reproduce the model specification below. Under the assumption that oil prices are 'long-run forcing', efficient estimation of the parameters can be based on the following *conditional* error correction model:

$$\Delta \mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_{yi}\Delta\mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo}\Delta p_t^o + \mathbf{u}_{yt}, \quad (11.1)$$

where $\mathbf{y}_t = (e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'$, $\mathbf{a}_y$ is an 8×1 vector of fixed intercepts, $\boldsymbol{\alpha}_y$ is an 8×5 matrix of error correction coefficients, $\{\boldsymbol{\Gamma}_{yi}, i = 1, 2, \ldots, p-1\}$ are 8×9 matrices of short-run coefficients, $\boldsymbol{\psi}_{yo}$ is an 8×1 vector representing the impact effects of changes in oil prices on $\Delta\mathbf{y}_t$, and

[2] The description of the empirical work of this chapter elaborates that provided in Garratt *et al.* (2003b).

$\mathbf{u}_{yt}$ is an 8×1 vector of disturbances assumed to be *i.i.d.*$(0, \boldsymbol{\Sigma}_y)$, with $\boldsymbol{\Sigma}_y$ being a positive definite matrix. For forecasting purposes, we specify the process for the change in the oil price to be:

$$\Delta p_t^o = \delta_o + \sum_{i=1}^{p-1}\boldsymbol{\delta}_{oi}\Delta\mathbf{z}_{t-i} + u_{ot}, \quad (11.2)$$

where $\boldsymbol{\delta}_{oi}$ is a 1×9 vector of fixed coefficients and u_{ot} is a serially uncorrelated error term distributed independently of $\mathbf{u}_{yt}$. This specification encompasses the familiar random walk model used in the impulse response analysis in Chapter 10 as a special case and seems quite general for our purposes. Combining (11.1) and (11.2), and solving for $\Delta\mathbf{z}_t$ yields the following reduced form equation which will be used in forecasting:

$$\Delta\mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{z}_{t-i} + \mathbf{v}_t, \quad (11.3)$$

where $\mathbf{a} = \left(\delta_o, \mathbf{a}_y' - a_o\boldsymbol{\psi}_{yo}'\right)'$, $\boldsymbol{\alpha} = \left(0, \boldsymbol{\alpha}_y'\right)'$, $\boldsymbol{\Gamma}_i = \left(\boldsymbol{\delta}_{oi}', \boldsymbol{\Gamma}_{yi}' - \boldsymbol{\delta}_{oi}'\boldsymbol{\psi}_{yo}'\right)'$ and $\mathbf{v}_t = \left(u_{ot}, \mathbf{u}_{yt}' - u_{ot}\boldsymbol{\psi}_{yo}'\right)'$ is the vector of reduced form errors assumed to be *i.i.d.*$(0, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is a positive definite matrix.

11.1.1 *Estimation results and in-sample diagnostics*

Chapter 9 documents the empirical exercise with respect to the core model using data over the period 1965q1–1999q4. The results showed that: (i) a VAR(2) model can adequately capture the dynamic properties of the data; (ii) there are five cointegrating relationships amongst the nine macroeconomic variables; and (iii) the over-identifying restrictions suggested by economic theory, and described in Chapter 9 above, cannot be rejected. For the present exercise, we re-estimated the model on the more up-to-date sample, 1965q1–2001q1. The results continue to support the existence of five cointegrating relations, and are qualitatively very similar to those described in Garratt *et al.* (2003a). For example, the interest rate coefficient in the real money balance equation is estimated to be 75.68 (standard error 35.34), compared to 56.10 (22.28) in the original work, while the coefficient on the time trend is estimated to be 0.0068 (0.0010), compared to 0.0073 (0.0012).

Since the modelling exercise here is primarily for the purpose of forecasting, we next re-estimated the model over the shorter period of 1985q1–2001q1, taking the long-run relations as given. The inclusion of the long-run relations estimated over the period 1965q1–2001q1 in a cointegrating VAR model estimated over the shorter sample period 1985q1–2001q1, is justified on two grounds: (i) as argued by Barassi *et al.* (2001) and Clements and Hendry (2002), the short-run coefficients are more likely to be subject to structural change as compared to the long-run coefficients; and (ii) the application of Johansen's cointegration tests is likely to be unreliable in small samples. Following this procedure, we are able to base the forecasts on a model with well-specified long-run relations, but which is also data-consistent, capturing the complex dynamic relationships that hold across the macroeconomic variables over recent years.

Table 11.1 gives the estimates of the individual error correcting relations of the benchmark model estimated over the 1985q1–2001q1 period.

These estimates show that the error correction terms are important in most equations and provide for a complex and statistically significant set of interactions and feedbacks across commodity, money and foreign exchange markets. The estimated error correction equations pass most of the diagnostic tests and compared to standard benchmarks, fit the historical observations relatively well. In particular, the $\bar{R}^2$ of the domestic output and inflation equations, computed at 0.549 and 0.603, respectively, are quite high. The diagnostic statistics for tests of residual serial correlation, functional form and heteroscedasticity are well within the 90% critical values, although there is evidence of non-normal errors in the case of some of the error correcting equations. Non-normal errors is not a serious problem at the estimation and inference stage, but can be important in Value-at-Risk analysis, for example, where tail probabilities are the main objects of interest. In such cases non-parametric techniques for computation of forecast probabilities might be used. See Chapter 7 for further details.

11.1.2 *Model uncertainty*

The theory-based cointegrating model is clearly one amongst many possible models that could be used to provide probability forecasts of the main UK macroeconomic variables. In order to address the issue of model uncertainty in the analysis that follows we adopt the Bayesian Model Averaging (BMA) framework described in Chapter 7.[3]

[3] The role of model uncertainty in explaining historical inflation data and the various monetary policy stances held in post-war US and UK has been highlighted recently by the work of Cogley and Sargent (2001, 2005) and Cogley *et al.* (2005).

Table 11.1 Error correction specification for the over-identified model, 1985q1–2001q1.

Equation	$\Delta(p_t - p_t^*)$	Δe_t	Δr_t	Δr_t^*	Δy_t	Δy_t^*	$\Delta(h_t - y_t)$	$\Delta^2\tilde{p}_t$
$\hat{\xi}_{1t}$	−0.020*	0.136*	0.003	0.0006	0.010	0.002	0.031*	−0.014*
	(0.010)	(0.071)	(0.004)	(0.001)	(0.009)	(0.006)	(0.017)	(0.008)
$\hat{\xi}_{2t}$	−0.775	−2.59	−593†	0.117	0.541	0.063	−1.31	−1.05†
	(0.664)	(4.63)	(0.281)	(0.075)	(0.592)	(0.418)	(1.09)	(0.508)
$\hat{\xi}_{3t}$	0.022	0.073	0.029	−0.003	−0.061	0.057	0.271†	0.087*
	(0.060)	(0.414)	(0.025)	(0.007)	(0.050)	(0.037)	(0.098)	(0.045)
$\hat{\xi}_{4t}$	0.010*	0.003	0.004	−0.001	−0.012†	0.0004	−0.003	0.005
	(0.006)	(0.043)	(0.003)	(0.0007)	(0.005)	(0.004)	(0.010)	(0.005)
$\hat{\xi}_{5t}$	0.131	2.04	0.007	−0.014	0.315	0.060	0.257	1.26
	(0.239)	(1.67)	(0.101)	(0.027)	(0.203)	(0.150)	(0.393)	(0.183)
$\Delta(p_{t-1} - p_{t-1}^*)$	0.275	−0.588	−0.030	0.007	0.136	0.031	−0.066	0.163
	(0.176)	(1.23)	(0.074)	(0.020)	(0.149)	(0.111)	(0.289)	(0.134)
Δe_{t-1}	0.020	0.210	−0.0001	0.0004	0.019	−0.012	0.059	−0.025
	(0.022)	(0.155)	(0.009)	(0.003)	(0.029)	(0.014)	(0.037)	(0.017)
Δr_{t-1}	−0.025	−3.90	0.214	0.053	0.190	0.025	−0.296	0.960†
	(0.404)	(2.81)	(0.171)	(0.046)	(0.342)	(0.254)	(0.665)	(0.309)
Δr_{t-1}^*	−0.839	5.74	−0.120	0.407†	0.784	−0.732	−2.42	1.15
	(1.23)	(8.59)	(0.522)	(0.139)	(1.05)	(0.775)	(2.03)	(0.943)
Δy_{t-1}	−0.090	−1.47	0.009	−0.017	0.439†	0.343†	−0.782†	0.252*
	(0.177)	(1.23)	(0.075)	(0.020)	(0.150)	(0.111)	(0.291)	(0.135)
Δy_{t-1}^*	−0.052	0.489	0.131	0.072†	0.351*	0.184	0.386	0.147
	(0.229)	(1.51)	(0.097)	(0.026)	(0.194)	(0.053)	(0.377)	(0.175)
$\Delta(h_{t-1} - y_{t-1})$	0.023	−0.081	−0.029	−0.001	−0.057	−0.007	−0.255*	−0.023
	(0.086)	(0.588)	(0.036)	(0.010)	(0.073)	(0.053)	(0.141)	(0.066)
$\Delta^2\tilde{p}_{t-1}$	−0.064	0.860	−0.012	−0.008	−0.019	−0.049	−0.194	0.017
	(0.171)	(1.19)	(0.072)	(0.019)	(0.145)	(0.107)	(0.281)	(0.131)
Δp_{t-1}^o	−0.005	0.006	−0.0001	−0.0009	0.012†	0.005	0.006	0.003
	(0.005)	(0.036)	(0.002)	(0.0006)	(0.004)	(0.003)	(0.009)	(0.004)
Δp_t^o	−0.010†	−0.019	0.002	−0.0007	−0.010†	−0.001	−0.001	0.004
	(0.005)	(0.032)	(0.002)	(0.0005)	(0.004)	(0.003)	(0.007)	(0.003)
$\bar{R}^2$	0.365	0.089	0.017	0.476	0.549	0.371	0.378	0.603
$\hat{\sigma}$	0.005	0.032	0.002	0.001	0.004	0.003	0.008	0.003
$\chi^2_{SC}[4]$	4.31	3.16	9.40*	1.91	5.74	7.29	7.40	5.89
$\chi^2_{FF}[1]$	3.04	0.76	3.49*	2.26	0.86	2.31	0.02	0.98
$\chi^2_{N}[2]$	3.53	11.2†	7.13†	0.27	1.91	1.47	33.9†	26.0†
$\chi^2_{H}[1]$	0.01	0.01	1.08	0.01	0.83	0.84	0.17	0.057

Note: The five error correction terms, estimated over the period 1965q1–2001q1, are given by

$$\hat{\xi}_{1,t+1} = p_t - p_t^* - e_t - 4.8566,$$

$$\hat{\xi}_{2,t+1} = r_t - r_t^* - 0.0057,$$

$$\hat{\xi}_{3,t+1} = y_t - y_t^* + 0.0366,$$

$$\hat{\xi}_{4,t+1} = h_t - y_t + \underset{(35.34)}{75.68}\, r_t + \underset{(0.001)}{0.0068}\, t + 0.1283,$$

$$\hat{\xi}_{5,t+1} = r_t - \Delta\tilde{p}_t - 0.0037.$$

Standard errors are given in parentheses. '*' indicates significance at the 10% level, and '†' indicates significance at the 5% level. The diagnostics are chi-squared statistics for serial correlation (SC), functional form (FF), normality (N) and heteroscedasticity (H).

We confine our analysis to the class of VAR(p) models, which nonetheless allows for the existence of range of important sources of uncertainties. The most important sources of uncertainty in this context are the order of the VAR, p, the number of the long-run (or cointegrating) relations, r, the validity of the over-identifying restrictions imposed on the long-run coefficients, and the specification of the oil price equation. Given the limited time series data available, consideration of models with $p = 3$ or more did not seem advisable. We also thought it would not be worthwhile to consider $p = 1$ on the grounds that the resultant equations would most likely suffer from residual serial correlation. Therefore, we confined the choice of the models to be considered in the BMA procedure to exactly identified VAR(2) models with $r = 0, 1, \ldots, 5$, and two alternative specifications of the oil price equation, namely (11.2), and its random walk counterpart,

$$\Delta p_t^o = \delta_o + u_{ot}. \tag{11.4}$$

Naturally, we also included our benchmark model in the set (for both specifications of the oil price equation), thus yielding a total of 14 models to be considered. We shall use these models in the forecast evaluation exercise below investigating the robustness of probability forecasts from the benchmark model to model uncertainty.

To allow for the effect of model uncertainty, we employed the BMA formulae, (7.33) and (7.36), with the weights, w_{iT}, set according to the following three schemes: Akaike, Schwarz and equal weights ($w_{iT} = 1/14$). The first two are computed using (7.35). In the event, only five of the 14 models appeared as plausible candidates according to the AIC and SBC criteria. Using the AIC, only two candidate models were considered plausible: namely, the exactly identifed five cointegrating vector (CV) models with the two alternative oil price specifications. For the estimation period 1985q1–1998q4, the two models had estimated weights of 0.93 and 0.07 (for the model containing the oil equation in (11.2) and that containing the random walk model, respectively). These weights gradually changed to 0.60:0.40 for the estimation period 1985q1–2000q4, following our recursive forecasting procedure, but all other models had zero weights throughout. Using the SBC, the exactly identified models with 5, 4, 3 and 2 cointegrating vectors, each supplemented by the random walk model for oil prices, were chosen as plausible candidates. For the estimation period 1985q1–1998q4, the weights of these four models were 0.07:0.86:0.06:0.01, respectively, but these also changed gradually to

0.00:0.01:0.22:0.77 for the estimation period 1985q1–2000q4. The number of candidate models considered 'best' is relatively small, therefore, according to AIC and SBC, but there is considerable variability in the estimated posterior probabilities of these chosen models with relatively minor changes in the sample sizes.

11.1.3 *Evaluation and comparisons of probability forecasts*

In the evaluation exercise, each of the 14 alternative models was used to generate probability forecasts for a number of simple events over the period 1999q1–2001q1. This was undertaken in a recursive manner, whereby we first estimated all the 14 models over the period 1985q1–1998q4 and computed one-step-ahead probability forecasts for 1999q1, then repeated the process moving forward one quarter at a time, ending with forecasts for 2001q1 based on models estimated over the period 1985q1–2000q4. The probability forecasts were computed for directional events of interest. In the case of $p_t - p_t^*$, e_t, r_t, r_t^* and $\Delta\tilde{p}_t$, we computed the probability that these variables rise next period, namely $\Pr\left[\Delta(p_t - p_t^*) > 0 \mid \Im_{t-1}\right]$, $\Pr\left[\Delta e_t > 0 \mid \Im_{t-1}\right]$), and so on, where $\Im_{t-1}$ is the information available at the end of quarter $t-1$. For the remaining variables, ($y_t, y_t^*, h_t - y_t$ and p_t^o) which are trended, we considered the event that the rate of change of these variables rise from one period to the next, namely $\Pr\left[\Delta^2 y_t > 0 \mid \Im_{t-1}\right]$, $\Pr\left[\Delta^2 y_t^* > 0 \mid \Im_{t-1}\right]$), and so on. The probability forecasts are computed recursively using the parametric stochastic simulation technique which allows for future uncertainty and the non-parametric bootstrap technique which allows for parameter uncertainty, as detailed in Chapter 7. Model uncertainty, as highlighted in the previous section, is allowed for through the three weighting schemes: Akaike, Schwarz and equal weights. The probability forecasts were then evaluated using a number of different statistical techniques.

To evaluate the probability forecasts, we adopted a statistical approach, using a threshold probability of 0.5, so that an event was forecast to be realised if its probability forecast exceeded 0.5.[4] Formal statistical comparisons of forecasts and realisations were made using Kuipers score (KS), Pesaran and Timmermann (PT) (1992) directional (market timing) statistic and the probability integral transform as proposed by Dawid (1984) and

[4] As an alternative, we could conduct a decision-theoretic approach to forecast evaluation as advocated in Granger and Pesaran (2000a,b) and reviewed in Pesaran and Skouris (2001), which bases the evaluation of the probability forecasts on their implied economic value in a specific decision-making context. However, this demands a complete specification of the decision problem and this has been rather rare in macroeconomic policy evaluation.

Table 11.2 Forecast evaluation of the benchmark model.

Variable	Threshold	Future uncertainty				Future and parameter uncertainty			
		UD	DD	DU	UU	UD	DD	DU	UU
p_t^o	$\Delta^2 p_t^o > 0$	0	6	1	2	1	5	1	2
e_t	$\Delta e_t > 0$	5	0	0	4	5	0	0	4
r_t^*	$\Delta r_t^* > 0$	0	3	2	4	2	1	2	4
r_t	$\Delta r_t > 0$	5	0	0	4	5	0	0	4
$\Delta \tilde{p}_t$	$\Delta^2 \tilde{p}_t > 0$	1	3	0	5	2	2	0	5
y_t	$\Delta y_t > 0$	2	2	1	4	2	2	1	4
$p_t - p_t^*$	$\Delta(p_t - p_t^*) > 0$	2	5	2	0	3	4	2	0
$h_t - y_t$	$\Delta^2(h_t - y_t) > 0$	0	4	1	4	0	4	1	4
y_t^*	$\Delta^2 y_t^* > 0$	2	3	2	2	2	3	2	2
Total		17	26	9	29	22	21	9	29
Hit rate		55/81 = 0.679				50/81 = 0.617			

Note: The forecast evaluation statistics are based on one-step-ahead forecasts obtained from models estimated recursively, starting with the forecast of events in 1999q1 based on models estimated over 1985q1–1998q4 and ending with forecasts of events in 2001q1. The events of interest are described in Section 11.1.3. In the column headings the first letter denotes the direction of the forecast (U=up, D=down) and the second letter the direction of the outcome (U=up, D=down). For example, UU indicates an upward movement was correctly forecast. Hit rate is defined as (DD + UU)/(UD + DD + DU + UU).

developed further in Diebold, Gunther and Tay (1998). Table 11.2 reports the incidence of the four possible combinations of our directional forecasts based on the benchmark model. For each variable, nine event forecasts are generated over the period 1999q1–2001q1 (nine quarters), thus providing 81 forecasts for evaluation purposes. These event forecasts are compared with their realisations and grouped under the headings, 'UU', indicating forecasts and realisations are in the same upward direction, 'UD' indicating an upward forecast with a realised downward movement, and so on. High values for UU and DD indicate an ability of the model to forecast upward and downward movements correctly, while high values of UD and DU suggest poor forecasting ability.

The information in Table 11.2 documents the forecasting performance of the benchmark model, and comparable tables of results can be generated based on the probability forecasts obtained from the equal-weighted, AIC-weighted and SBC-weighted averages of the 14 candidate models. Briefly, Table 11.2 shows that for the case of future uncertainty the hit rate is 0.68 versus 0.62 when both parameter uncertainty and future uncertainty are considered. The forecasting performance of these is summarised by KS, defined by $H - F$, where H is the proportion of ups that were

Table 11.3 Diagnostic statistics for the evaluation of benchmark and average model probability forecasts.

Model	Future uncertainty				Future and parameter uncertainty			
	KS	Hit rate	PT	D_n	KS	Hit rate	PT	D_n
Benchmark with (11.2)	0.373	0.679	3.356	0.111	0.269	0.617	2.354	0.136
Benchmark with (11.4)	0.302	0.642	2.701	0.123	0.237	0.605	2.094	0.136
Equal Weights Average	0.259	0.630	2.346	0.062	0.256	0.630	2.322	0.111
AIC Weighted Average	0.302	0.642	2.701	0.160	0.273	0.630	2.451	0.136
SBC Weighted Average	0.207	0.605	1.873	0.111	0.233	0.617	2.109	0.099

Note: The forecast evaluation statistics are based on one-step-ahead forecasts obtained from models estimated recursively, starting with the forecast of events in 1999q1 based on models estimated over 1985q1–1998q4 and ending with forecasts of events in 2001q1. The events of interest are described in Section 11.1.3. The hit rate is defined as is the proportion of ups and downs that were correctly forecast to occur. The KS statistic is the Kuipers score statistic, PT statistic is the Pesaran and Timmermann (1992) test which under the null hypothesis has a standard normal distribution. Finally, D_n is the Kolmogorov–Smirnov statistic where the 5% critical value of D_n for $n = 81$ is equal to 0.149.

correctly forecast to occur, and F is the proportion of downs that were incorrectly forecast.[5] This statistic provides a measure of the accuracy of directional forecasts of the model, with high positive numbers indicating high predictive accuracy. In Table 11.3, we report the KS along with the other forecast evaluation statistics listed above, for the benchmark model, the three average models and the benchmark model replacing the oil equation of (11.2) with the random walk model. Where the probability forecasts take account of future uncertainty only, the KS suggests that the most accurate forecasts are provided by the benchmark model. Allowing for parameter uncertainty in the computation of probability forecasts, however, the KS suggests the benchmark model and the AIC average model produce the most accurate forecasts, although these forecasts perform less well than when just considering future uncertainty.[6]

The Kuipers score is a useful summary measure but does not provide a statistical test of the directional forecasting performance. Pesaran and Timmermann (1992) provide a formal statistical test which, as shown in Granger and Pesaran (2000b), turns out to be equivalent to a test based on

[5] These two proportions are known as the 'hit rate' and 'false alarm rate', respectively. In the case where the outcome is symmetric, in the sense that we value the ability to forecast ups and downs equally, then the score statistic of zero means no accuracy, whilst high positive and negative values indicate high and low predictive power.

[6] These statistics are based on probability forecasts where future uncertainty is taken into account using a parametric procedure. The results are hardly affected if a non-parametric procedure is used instead.

the Kuipers score. The PT statistic is defined by

$$PT = \frac{\widehat{P} - \widehat{P}^*}{\left\{\widehat{V}(\widehat{P}) - \widehat{V}(\widehat{P}^*)\right\}^{\frac{1}{2}}},$$

where $\widehat{P}$ is the proportion of correctly predicted upward movements, $\widehat{P}^*$ is the estimate of the probability of correctly predicting the events under the null hypothesis that forecasts and realisations are independently distributed, and $\widehat{V}(\widehat{P})$ and $\widehat{V}(\widehat{P}^*)$ are the consistent estimates of the variances of $\widehat{P}$ and $\widehat{P}^*$, respectively. Under the null hypothesis, the PT statistic has a standard normal distribution. For the forecasts based on the benchmark model in combination with the estimated oil price equation, (11.2), we obtained PT = 3.356 when only future uncertainty was allowed for, and PT = 2.354 when both future and parameter uncertainties were taken into account. Both of these statistics are statistically significant. The alternative oil price specification of (11.4) yielded corresponding PT test statistics of 2.701 and 2.094, which are significant but marginally less so. The probability forecast results based on the average models were marginally less convincing, with the AIC average having the highest PT of 2.701 for future uncertainty only, but when considering parameter uncertainty as well gives the highest PT of all models of 2.451. These results suggest that the benchmark model performs well under future uncertainty, suggesting the importance of imposing theory-based long-run restrictions for probability forecasting, but that this distinction is removed when both future and parameter uncertainty are considered.

An alternative approach to probability forecast evaluation would be to use the probability integral transforms

$$u(\mathbf{z}_t) = \int_{-\infty}^{\mathbf{z}_t} p_t(\mathbf{x})\, d\mathbf{x}, \qquad t = T+1, T+2, \ldots, T+n,$$

where $p_t(\mathbf{x})$ is the forecast probability density function, and $\mathbf{z}_t$, $t = T+1, T+2, \ldots, T+n$, the associated realisations. Under the null hypothesis that $p_t(\mathbf{x})$ coincides with the true density function of the underlying process, the probability integral transforms will be distributed as $i.i.d.U[0,1]$. This result is due to Rosenblatt (1952), and has been recently applied in time series econometrics by Diebold, Gunther and Tay (1998).[7] In our application, we first computed a sequence of one step ahead probability

[7] Also see Diebold, Hahn and Tay (1999) and Berkowitz (1999).

forecasts (with and without allowing for parameter uncertainty) from the over-identified and exactly identified models for the nine simple events set out above over the nine quarters 1999q1, 1999q2, ..., 2001q1, and hence the associated probability integral transforms, $u(\mathbf{z}_t)$. To test the hypothesis that these probability integral transforms are random draws from $U[0,1]$, we calculated the Kolmogorov–Smirnov statistic,

$$D_n = \sup_x |F_n(x) - U(x)|,$$

where $F_n(x)$ is the empirical cumulative distribution function (CDF) of the probability integral transforms, and $U(x) = x$, is the CDF of $i.i.d.U[0,1]$. Large values of the Kolmogorov–Smirnov statistic, D_n, indicate that the sample CDF is not similar to the hypothesised uniform CDF.[8] For the over-identified benchmark specification, we obtained the value of 0.111 for the Kolmogorov–Smirnov statistic when only future uncertainty was allowed for, and the larger value of 0.136 when the underlying probability forecasts took account of both future and parameter uncertainties. The corresponding statistics for the benchmark model with the alternative oil price specification of (11.4) were 0.123 and 0.136, respectively. All these statistics are well below the 5% critical value of Kolmogorov–Smirnov statistic (which for $n = 81$ is equal to 0.149), and the hypothesis that the forecast probability density functions coincide with the true ones cannot be rejected. We cannot reject the same hypothesis for the average models either but with the noticeable exception of the AIC model. The AIC average model obtains a value of 0.160 for the Kolmogorov–Smirnov statistic when only future uncertainty was allowed for and 0.136 when we include parameter uncertainty. Hence we reject the null that the forecast probability density functions coincide with the true ones when considering future uncertainty but not when we consider both future and parameter uncertainty. This is an interesting results in light of the support given to the AIC from the hit rate, the KS and PT statistics. Overall results do not reject any one model but do provide some evidence, in particular when considering future uncertainty, for supporting the use of the over-identified specification in forecasting. With this in mind, we now proceed to the generation of out-of-sample forecast probabilities of interest using the over-identified benchmark model.

[8] For details of the Kolmogorov–Smirnov test and its critical values see, for example, Neave and Worthington (1992, pp. 89–93).

11.2 Probability forecasts of inflation and output growth

Here we apply the techniques described in Chapter 7 to the updated core model of the UK economy to compute out-of-sample probability forecasts of events relating to inflation targeting and output growth which are of particular interest for the analysis of macroeconomic policy in the UK. Inflation targets have been set explicitly in the UK since October 1992, following the UK's exit from the European Exchange Rate Mechanism (ERM). The Chancellor's stated objective at the time was to achieve an average annual rate of inflation of 2%, while keeping the underlying rate of inflation within the 1–4% range. In May 1997, the policy of targeting inflation was formalised further by the setting up of the Monetary Policy Committee (MPC), whose main objective is to meet inflation targets primarily by influencing the market interest rate through fixing the base rate at regular intervals. Its current remit, as set annually by the Chancellor, is to achieve an average annual inflation rate of 2.0%, based on the Harmonised Index of Consumer Prices (HICP), renamed the Consumer Price Index. In this application we have used the RPI index (as an approximation to the measure previously used by the MPC, the Retail Price Index, excluding mortgage interest payments, RPI-x), where the previous target of 2.5% is argued to be equivalent to the new 2.0% target, as the method of constructing the consumer price index will produce a lower measure of inflation than the RPI method. The previous target range of 1.5–3.5% therefore also remains of interest and constitutes one of the events analysed. Note a feature of the policy framework is that the time horizon over which the inflation objective is to be achieved is not stated.

Inflation rates outside the target range act as a trigger, requiring the Governor of the Bank of England to write an open letter to the Chancellor explaining why inflation had deviated from the target, the policies being undertaken to correct the deviation, and how long it is expected before inflation is back on target. The Bank is also expected to conduct monetary policy so as to support the general economic policies of the government, so far as this does not compromise its commitment to its inflation target.

Since October 1992, the Bank of England has produced a quarterly *Inflation Report* which describes the Bank's assessment of likely inflation outcomes over a two-year forecast horizon. In addition to reviewing the various economic indicators necessary to place the inflation assessment into context, the *Report* provides forecasts of inflation over two year horizons, with bands presented around the central forecast to illustrate the range of inflation outcomes that are considered possible (the so-called fan

charts). The forecasts are based on the assumption that the base rate is left unchanged. Since November 1997, a similar forecast of output growth has also been provided in the *Report*, providing insights on the Bank's perception of the likely outcome for the government's general economic policies beyond the maintenance of price stability. For a critical assessment of the Bank's approach to allowing for model and parameter uncertainties, see Wallis (1999).

The fan charts produced by the Bank of England are an important step towards acknowledging the significance of forecast uncertainties in the decision-making process and this is clearly a welcome innovation. However, the approach suffers from two major shortcomings. First, it seems unlikely that the fan charts can be replicated by independent researchers. This is largely due to the subjective manner in which uncertainty is taken into account by the Bank, which may be justified from a real-time decision-making perspective but does not readily lend itself to independent analysis. Second, the use of fan charts is limited for the analysis of uncertainty associated with joint events. Currently, the Bank provides separate fan charts for inflation and output growth forecasts, but in reality one may also be interested in joint events involving both inflation and output growth, and it is not clear how the two separate fan charts could be used for such a purpose. Here, we address both of these issues using the benchmark long-run structural model and the various alternative models discussed.

In what follows, we present plots of estimated predictive distribution functions for inflation and output growth at a number of selected forecast horizons. These plots provide us with the necessary information with which to compute probabilities of a variety of events, and demonstrate the usefulness of probability forecasts in conveying the future and parameter uncertainties that surround the point forecasts. But our substantive discussion of the probability forecasts focuses on two central events of interest; namely, keeping the rate of inflation within the announced target range of 1.5–3.5% and avoiding a recession. Following the literature, we define a recession as the occurrence of two successive negative quarterly growth rates. See, for example, Harding and Pagan (2002).

11.2.1 *Point and interval forecasts*

Before reporting the probability forecasts, it is worth briefly summarising the point and interval forecasts to help place the probability forecasts in context. Tables 11.4a and 11.4b provide the point forecasts for domestic

Table 11.4a Point and interval forecasts of inflation and output growth (four quarterly moving averages, per cent, per annum).

Forecast horizon	Output growth		Inflation	
	Forecast	Actual	Forecast	Actual
2001q2	1.84 (1.02, 2.65)	2.33	1.80 (1.11, 2.49)	1.92
2001q3	1.30 (−0.13, 2.73)	2.12	1.61 (0.34, 2.88)	1.80
2001q4	1.28 (−0.62, 3.18)	2.11	1.37 (−0.36, 3.11)	1.04
2002q1	1.27 (−1.05, 3.51)	1.60	1.69 (−0.44, 3.82)	1.21
2002q2	1.42 (−1.10, 3.94)	1.49	2.08 (−0.31, 4.47)	1.20
2002q3	1.65 (−1.08, 4.37)	1.88	2.01 (−0.51, 4.52)	1.48
2002q4	1.89 (−1.04, 4.81)	1.97	1.92 (−0.69, 4.52)	2.50
2003q1	2.02 (−1.08, 5.12)	2.06	1.93 (−0.75, 4.60)	3.00

Note: Forecasts are based on the model reported in Table 11.1, combined with an estimate of the oil price equation (11.2). The figures in parentheses are the lower and upper 95% confidence intervals. The four quarterly moving average output growth is defined as $100 \times \ln(GDP_{T+h}/GDP_{T+h-4})$, where GDP_T is the real Gross Domestic Product in 2001q1, which is computed from the forecasts of per capita output, y_{T+h}, assuming a population growth of 0.22% per annum. The four quarterly moving average inflation rate is defined as $100 \times (p_{T+h} - p_{T+h-4})$ where p_T is the natural logarithm of the retail price index in 2001q1.

Table 11.4b Point and interval forecasts of inflation and output growth (quarter on quarter changes, per cent, per annum).

Forecast horizon	Output growth		Inflation	
	Forecast	Actual	Forecast	Actual
2001q2	1.30 (−1.96, 4.55)	2.01	0.28 (−2.49, 3.06)	4.86
2001q3	1.16 (−2.61, 4.91)	2.00	2.22 (−2.05, 6.50)	0.23
2001q4	1.12 (−2.83, 5.07)	1.19	2.31 (−2.40, 7.04)	−0.46
2002q1	1.53 (−2.59, 5.64)	1.19	1.93 (−3.01, 6.87)	0.23
2002q2	1.89 (−2.37, 6.15)	1.58	1.86 (−3.28, 7.00)	4.80
2002q3	2.05 (−2.36, 6.45)	3.54	1.91 (−3.39, 7.21)	1.36
2002q4	2.08 (−2.45, 6.61)	1.56	1.95 (−3.47, 7.37)	3.61
2003q1	2.08 (−2.56, 6.71)	1.56	1.97 (−3.54, 7.49)	2.24

Note: See Notes to Table 11.4a. Output growth is defined as $400 \times \ln(GDP_{T+h}/GDP_{T+h-1})$, while inflation is defined as $400 \times (p_{T+h} - p_{T+h-1})$.

inflation rates and output growth over the period 2001q1–2003q1 together with their 95% confidence intervals.

Table 11.4a presents the four quarterly growth rate forecasts, while Table 11.4b gives the forecasts of annualised quarter-on-quarter growth rates.[9]

[9] It is worth noting that the inflation target is expressed in terms of RPI-x while our model provides forecasts of RPI.

The model predicts the average annual rate of inflation to fall from 2.5% in 2001q1 to 1.8% in 2001q2. This is followed by further falls for the rest of 2001 before returning to approximately 2% to the end of the forecast horizon, 2003q1. These point forecasts are lower than the inflation rates realised during 2000, as illustrated by the historical data on inflation presented in Figure 11.1a. Output growth is predicted to be positive throughout the forecast horizon, falling from an average annual rate of 2.8% in 2000 to 1.3% by the end of 2001, before rising to around 2.0% thereafter (see Table 11.4a). Therefore, based on these point forecasts, we may be tempted to rule out the possibility of a recession occurring in the UK over the 2001–2003 period.

However, these point forecasts are subject to a high degree of uncertainty, particularly when longer forecast horizons are considered. For example, at the two year forecast horizon the point forecast of annual inflation in 2003q1 is predicted to be 1.9%, which is well within the announced inflation target range. But the 95% confidence interval covers the range −0.8% to +4.6%. For the quarter on quarter definition, the uncertainty is even larger, with a range of −3.5% to 7.5% around a point forecast of approximately 2.0%. Similarly, the point forecast of the quarter on quarter annual rate of output growth in 2003q1 is 2.1%, but its 95% confidence

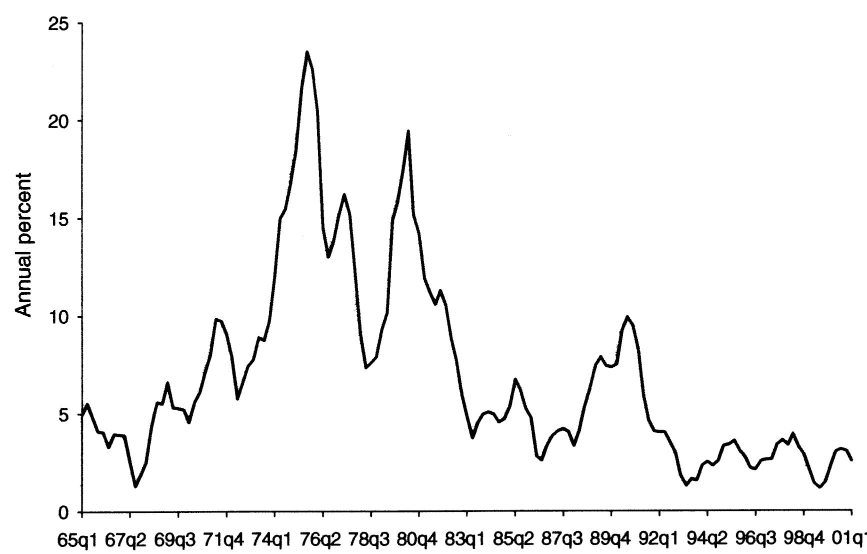

Figure 11.1a Inflation (four-quarter moving average).

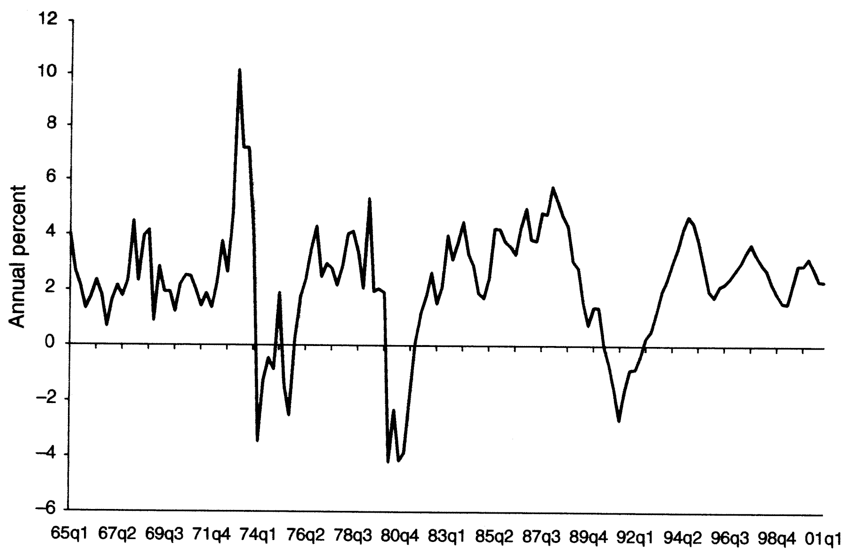

Figure 11.1b Output growth (four-quarter moving average).

interval covers the range −2.6% to +6.7%. As we have noted, it is difficult to evaluate the significance of these forecast intervals for policy analysis and a more appropriate approach is to directly focus on probability forecasts as a method of characterising the various uncertainties that are associated with events of interest. This is the topic that we shall turn to now.

11.2.2 *Predictive distribution functions*

In the case of single events, probability forecasts are best represented by means of probability distribution functions. Figures 11.2a and 11.2b give the estimates of these functions for the four-quarter moving averages of inflation and output growth for the one-quarter, one- and two-year ahead forecast horizons based on the benchmark model (*i.e.* the over-identified version of the cointegrating model, (11.1), augmented with the oil price equation, (11.2)). These estimates are computed using the simulation techniques described in detail in Section 7.3 and take account of both future and parameter uncertainties.

Figure 11.2a presents the estimated predictive distribution function for inflation for the threshold values ranging from 0% to 5% per annum at

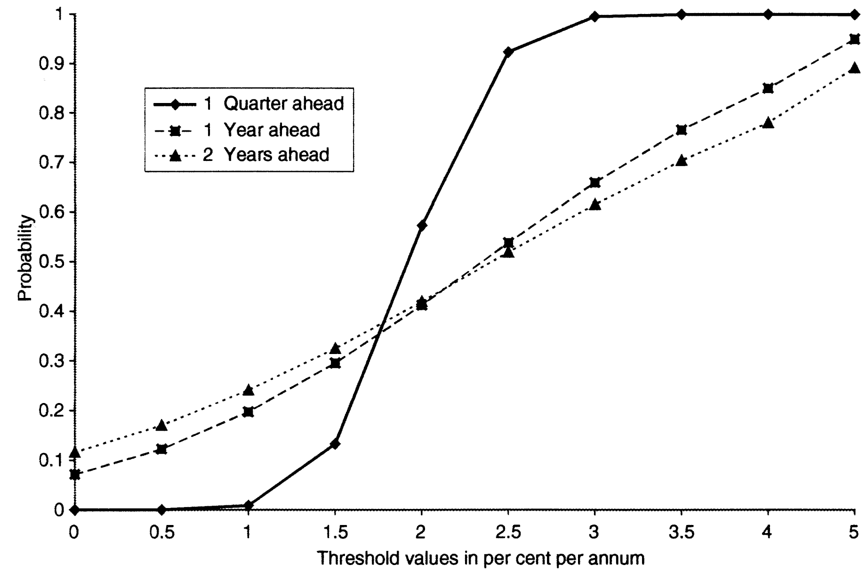

Figure 11.2a Predictive distribution functions for inflation (benchmark model with parameter uncertainty).

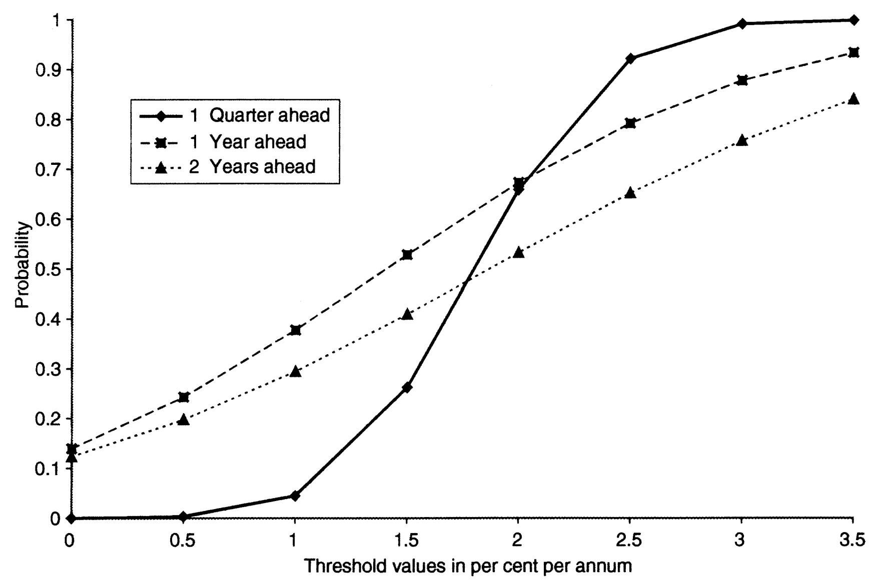

Figure 11.2b Predictive distribution functions for output growth (benchmark model with parameter uncertainty).

the three selected forecast horizons. Perhaps not surprisingly, the function for the one-quarter ahead forecast horizon is quite steep, but it becomes flatter as the forecast horizon is increased. Above the threshold value of 2.0%, the estimated probability distribution functions shift to the right as longer forecast horizons are considered, showing that the probability of inflation falling below thresholds greater than 2.0% declines with the forecast horizon. For example, the forecast probability that inflation lies below 3.5% becomes smaller at longer forecast horizons, falling from close to 100% one quarter ahead (2001q2) to 70% eight quarters ahead (2003q1). These forecast probabilities are in line with the recent historical experience: over the period 1985q1–2001q1, the average annual rate of inflation fell below 3.5% for 53.9% of the quarters, but were below this threshold value throughout the last two years of the sample, 1999q1–2001q1.

Figure 11.2b plots the estimated predictive distribution functions for output growth. These functions also become flatter as the forecast horizon is increased, reflecting the greater uncertainty associated with growth outcomes at longer forecast horizons. These plots also suggest a weakening of the growth prospects in 2001 before recovering a little at longer horizons. For example, the probability of a negative output growth one quarter ahead (2001q2) is estimated to be almost zero, but rises to 14% four quarters ahead (2002q1) before falling back to 12% after eight quarters (2003q1). Therefore, a rise in the probability of a recession is predicted, but the estimate is not sufficiently high for it to be much of a policy concern (at least viewed from the end of our sample period 2001q1).

11.2.3 *Event probability forecasts*

Here we consider three single events of particular interest:

- A : achievement of inflation target, defined as the four-quarterly moving average rate of inflation falling within the range 1.5–3.5%;
- B : recession, defined as the occurrence of two consecutive quarters of negative output growth;
- C : poor growth prospects, defined to mean that the four-quarterly moving average of output growth is less than 1%;

and the joint events $A \cap \overline{B}$ (inflation target is met *and* recession is avoided), and $A \cap \overline{C}$ (inflation target is met *combined* with reasonable growth prospects), where $\overline{B}$ and $\overline{C}$ are complements of B and C.

INFLATION AND THE TARGET RANGE

Two sets of estimates of $\Pr(A_{T+h} \mid \mathfrak{I}_T)$ are provided in Table 11.5a (for $h = 1, 2, \ldots, 8$) and depicted in Figure 11.3 over the longer forecast horizons $h = 1, 2, \ldots, 24$.

The first set relates to π, which only take account of future uncertainty, and the second set relates to $\tilde{\pi}$ which allow for both future and parameter uncertainties. Both π and $\tilde{\pi}$ convey a similar message, but there are nevertheless some differences between them, at least at some forecast horizons, so that it is important that both estimates are considered in practice.

Based on these estimates, and conditional on the information available at the end of 2001q1, the probability that the Bank of England will be able to achieve the government inflation target is estimated to be high in the short run but falls in the longer run, reflecting the considerable uncertainty surrounding the inflation forecasts at longer horizons. Specifically, the probability estimate is high in 2001q2, at 0.87 (0.80) for $\tilde{\pi}$ (π), but it falls rapidly to nearer 0.45 by the end of 2001/early 2002. This fall in the first quarters of the forecast reflects the increasing likelihood of inflation falling below the 1.5% lower threshold (since the probability of observing inflation above the 3.5% upper threshold is close to zero through this

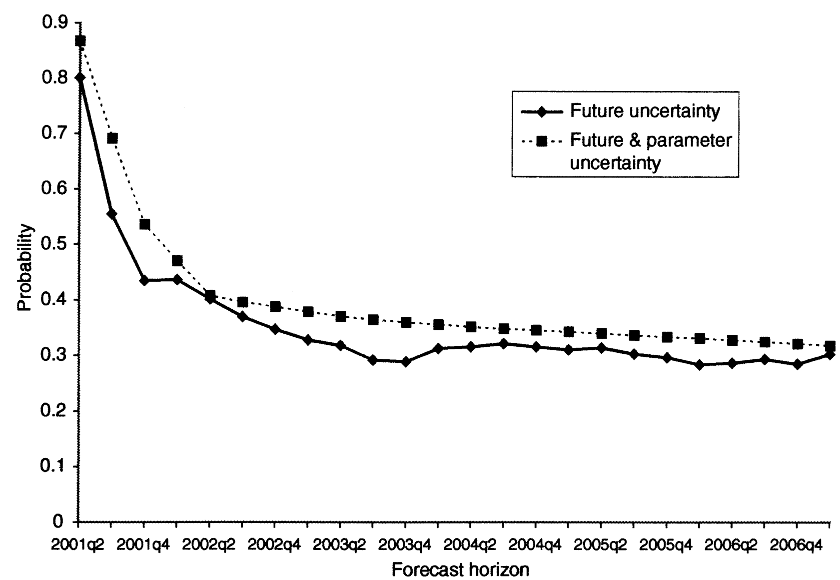

Figure 11.3 Probability estimates of inflation falling within the target range using the benchmark model.

period). Ultimately, though, the estimated probability of achieving inflation within the target range settles to 0.38 (0.35) for $\tilde{\pi}$ (π) in 2003q1. At this longer forecast horizon, the probabilities of inflation falling below and above the target range are 0.32 and 0.30, respectively, using $\tilde{\pi}$ (or 0.42 and 0.23 using π), so these figures reflect the relatively high degree of uncertainty associated with inflation forecasts even at moderate forecast horizons. Hence, while the likely inflation outcomes are low by historical standards and there is a reasonable probability of hitting the target range, there are also comparable likelihoods of undershooting and overshooting the inflation target range at longer horizons.

RECESSION AND GROWTH PROSPECTS

Figure 11.4 shows the estimates of the recession probability, $\Pr(B_{T+h} \mid \Im_T)$ over the forecast horizons $h = 1, 2, \ldots, 24$. For this event, the probability estimates that allow for parameter uncertainty (*i.e.* $\tilde{\pi}$) exceed those that do not (*i.e.* π) at shorter horizons, but the opposite is true at longer horizons. Having said this, however, π and $\tilde{\pi}$ are very similar in size across the different forecast horizons and suggest a very low probability of a recession: based on the $\tilde{\pi}$ estimate, for example, the probability of a recession occurring in 2001q2 is estimated to be around zero, rising to 0.09 in

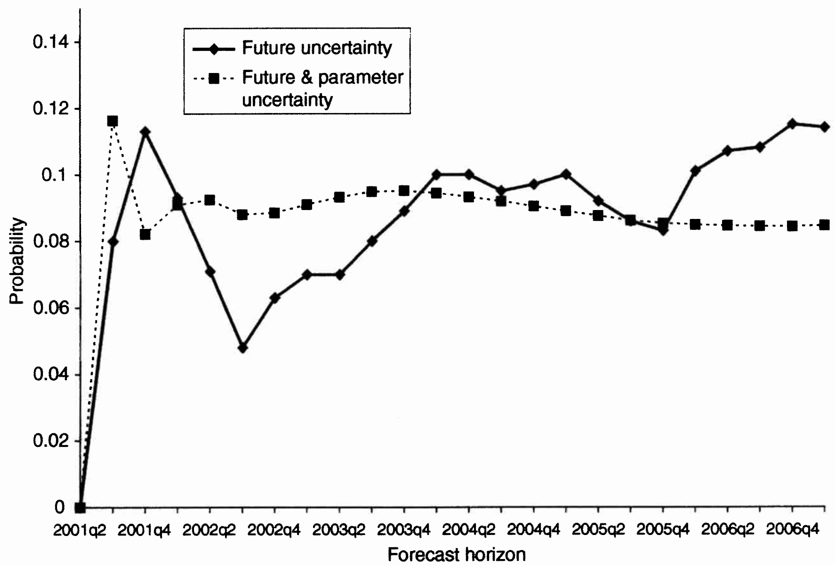

Figure 11.4 Probability estimates of a recession using the benchmark model.

2002q1. However, as shown in Table 11.5b, the probability that the UK faces poor growth prospects is much higher, in the region of 0.35 at the end of 2001, falling to 0.3 in 2003q1 according to the $\tilde{\pi}$ estimates.

Single events are clearly of interest but very often decision-makers are concerned with joint events involving, for example, both inflation and output growth outcomes. As examples here, we consider the probability estimates of the two joint events, $A_{T+h} \cap \overline{B}_{T+h}$, and $A_{T+h} \cap \overline{C}_{T+h}$ over

Table 11.5a Probability forecasts of single events involving inflation.

Forecast horizon	Pr(Inf < 1.5%)		Pr(Inf < 2.5%)		Pr(Inf < 3.5%)		Pr(1.5% < Inf < 3.5%)	
	π	$\tilde{\pi}$	π	$\tilde{\pi}$	π	$\tilde{\pi}$	π	$\tilde{\pi}$
2001q2	0.206	0.135	0.978	0.920	1.000	1.000	0.795	0.865
2001q3	0.437	0.275	0.884	0.732	0.996	0.963	0.560	0.688
2001q4	0.541	0.364	0.849	0.682	0.974	0.899	0.433	0.535
2002q1	0.451	0.292	0.721	0.533	0.893	0.761	0.442	0.469
2002q2	0.367	0.244	0.597	0.441	0.801	0.652	0.434	0.408
2002q3	0.405	0.285	0.611	0.484	0.785	0.683	0.381	0.398
2002q4	0.424	0.315	0.625	0.514	0.792	0.705	0.368	0.390
2003q1	0.422	0.321	0.607	0.515	0.772	0.702	0.351	0.381

Note: The probability estimates for inflation relate to the four quarterly moving average of inflation defined by 400 × ($p_{T+h} - p_{T+h-4}$), where p is the natural logarithm of the retail price index. The probability estimates (π and $\tilde{\pi}$) are computed using the model reported in Table 11.1, where π is the 'Profile Predictive Likelihood' that only takes account of future uncertainty, whereas $\tilde{\pi}$ is the 'Bootstrap Predictive Distribution' function and accounts for both future and parameter uncertainties. The computations are carried out using 2000 replications. See Chapter 7 for computational details.

Table 11.5b Probability forecasts of events involving output growth and inflation.

Forecast horizon	Pr(Recession)	Pr(output growth < 1%)	Pr(1.5% < Inf < 3.5%, No recession)	Pr(1.5% < Inf <3.5%, output growth > 1%)
	$\tilde{\pi}$	$\tilde{\pi}$	$\tilde{\pi}$	$\tilde{\pi}$
2001q2	0.000	0.040	0.865	0.832
2001q3	0.111	0.319	0.629	0.500
2001q4	0.084	0.343	0.499	0.381
2002q1	0.092	0.371	0.426	0.300
2002q2	0.092	0.312	0.373	0.278
2002q3	0.088	0.314	0.365	0.273
2002q4	0.090	0.305	0.358	0.272
2003q1	0.092	0.295	0.350	0.270

Note: The probability estimates for output growth are computed from the forecasts of per capita output, assuming a population growth of 0.22% per annum. Recession is said to have occurred when output growth (measured, quarter on quarter, by 400 × ln(GDP_{T+h}/GDP_{T+h-1})) becomes negative in two consecutive quarters. Also see the notes to Tables 11.4a and 11.5a.

the forecast horizons $h = 1, 2, \ldots, 24$. Probability estimates of these events (based on $\tilde{\pi}$) are presented in Table 11.5b. Both events are of policy interest as they combine the achievement of the inflation target with alternative growth objectives. For the event $A_{T+h} \cap \overline{B}_{T+h}$, the joint probability forecasts are similar in magnitude to those for $\Pr(A_{T+h} \mid \Im_T)$ alone at every time horizon. This is not surprising since the probability of a recession is estimated to be small at most forecast horizons and therefore the probability of avoiding recession is close to one. Nevertheless, the differences might be important since even relatively minor differences in probabilities can have an important impact on decisions if there are large, discontinuous differences in the net benefits of different outcomes. The probability forecasts for $A_{T+h} \cap \overline{C}_{T+h}$ are, of course, considerably less than those for $\Pr(A_{T+h} \mid \Im_T)$ alone.

Figure 11.5 plots the values of the joint event probability over the forecast horizon alongside a plot of the product of the single event probabilities; that is $\Pr(A_{T+h} \mid \Im_T) \times \Pr\left(\overline{B}_{T+h} \mid \Im_T\right)$, $h = 1, 2, \ldots, 24$. This comparison provides an indication of the degree of dependence/independence of the two events. As it turns out, there is a gap between these of just under 0.1

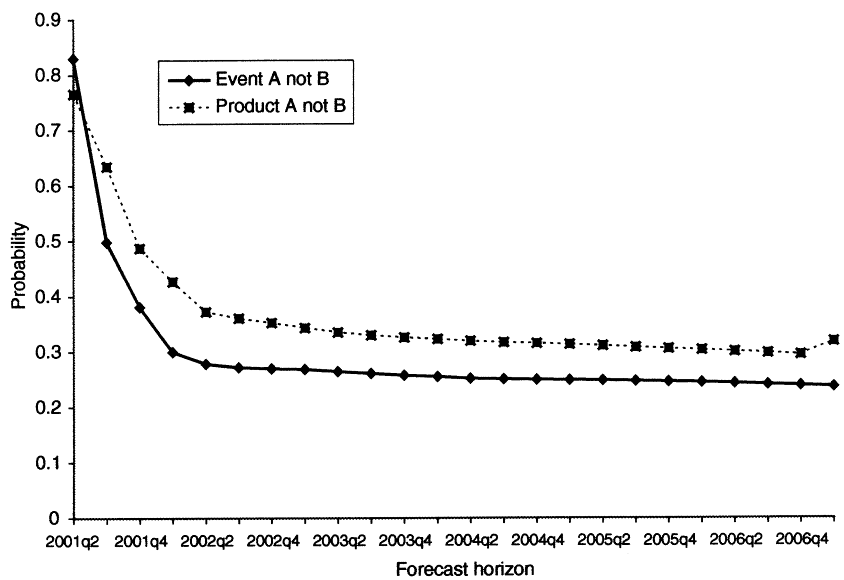

Figure 11.5 Probability estimates of meeting the inflation target without a recession (future and parameter uncertainty).

at most forecast horizons. But the probabilities are relatively close, indicating little dependence between output growth prospects and inflation outcomes. This result is compatible with the long-term neutrality hypothesis that postulates independence of inflation outcomes from output growth outcomes in the long run.

Figure 11.6 also plots the probability estimates of the joint event $A_{T+h} \cap \overline{B}_{T+h}$, but illustrates the effects of taking into account model uncertainty. The figure shows three values of the probability of the joint event over the forecast horizon, each calculated without taking account of parameter uncertainty. One value is based on the benchmark model, but the other two show the weighted average of the probability estimates obtained from the 14 alternative models described in the model evaluation exercise of the previous section. The weights in the latter two probability estimates are set equal in one of the estimates and are the in-sample posterior probabilities of the models approximated by the Akaike weights in the other. The plots show that estimated probabilities from the benchmark model are, by and large, quite close to the 'equal weights' estimate, but these are both lower than the AIC-weighted average, by more than 0.1 at some forecast horizons. Again, the extent to which these differences are considered large or important will depend on the nature of the underlying decision problem.

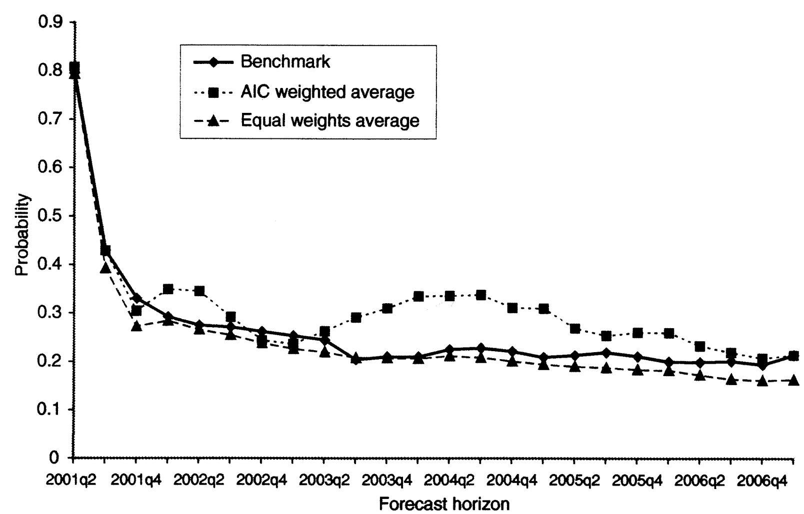

Figure 11.6 Probability estimates of meeting the inflation target without a recession (future uncertainty only).

11.3 A postscript

The elapse of time since the publication of the above forecasts in Garratt *et al.* (2003b) presents us with an opportunity for a real-time out-of-sample forecast evaluation, albeit over a rather short period. In what follows we compare the point and probability forecasts, reported in Tables 11.4a and 11.4b with the realised values of output growth and inflation for the eight quarters 2001q2–2003q1.

The difficulty of producing accurate point forecasts is reflected in the size of the forecast errors but the uncertainty surrounding the point forecasts is so large that in only one case does the realised value exceed the 95% confidence intervals. The less volatile four quarterly moving average changes perform reasonably, with root mean square errors (RMSE) of 0.47 and 0.60 percentage points for output growth and inflation, respectively. The quarter on quarter annual realisations exhibit high volatility, particularly for inflation and as such have larger and more volatile forecast errors. This is reflected in the RMSEs which take the values of 0.72 and 2.43 for output growth and inflation, respectively. On this definition inflation forecasts perform badly. For example, the realised value was 4.86% in 2001q2 as compared to the forecast value of 0.28%.

The probability event forecasts, which use the same distributions as the point and interval forecasts, perform well in terms of predicting specific events and as such convey useful information, not always apparent when using the point forecasts. If we evaluate the probability event forecasts using the threshold probability of 0.5, so that an event was forecast to be realised if its probability forecast exceeded 0.5, then the 'hit rate' (see footnote 5 of this chapter) or percentage of correctly forecasting events, for all the 32 events regarding inflation defined in Table 11.5a is 84% (27 out of 32) for future uncertainty only and 75% (24 out of 32) for future and parameter uncertainty. The hit rate for events associated with output growth (*i.e.* recession defined as two consecutive quarters of negative growth and output growth of <1%) exhibits a hit rate of 100% (16 out of 16). Joint event probability event predictions also perform well with a hit rate of 69% (11 out of 16).

11.4 Concluding remarks

One of the many problems economic forecasters and policy-makers face is conveying to the public the degree of uncertainty associated with point

forecasts. Policy-makers recognise that their announcements, in addition to providing information on policy objectives, can themselves initiate responses which affect the macroeconomic outcome. This means that Central Bank Governors are reluctant to discuss either pessimistic possibilities, as this might induce recession, or more optimistic possibilities, since this might induce inflationary pressures. There is therefore an incentive for policy-makers to seek ways of making clear statements regarding the range of potential macroeconomic outcomes for a given policy, and the likelihood of the occurrence of these outcomes, in a manner which avoids these difficulties.

Here we have argued for the use of probability forecasts as a method of characterising the uncertainties that surround forecasts from a macroeconomic model believing this to be superior to the conventional way of trying to deal with this problem through the use of confidence intervals. We argue that the use of probability forecasts has an intuitive appeal, enabling the forecaster (or users of forecasts) to specify the relevant 'threshold values' which define the event of interest (*e.g.* a threshold value corresponding to an inflation target range 1.5–3.5%). This is in contrast to the use of confidence intervals which define threshold values only implicitly, through the specification of the confidence interval widths, and these values may or may not represent thresholds of interest. A further advantage of the use of probability forecasts compared with the use of confidence intervals and over other more popular methods is the flexibility of probability forecasts, as illustrated by the ease with which the probability of joint events can be computed and analysed. Hence, for example, we can consider the likelihood of achieving a stated inflation target range whilst simultaneously achieving a given level of output growth, with the result being conveyed in a single number. In situations where utility or loss functions are non-quadratic and/or the constraints are nonlinear the whole predictive probability distribution function rather than its mean is required for decision-making. This chapter shows how such predictive distribution functions can be obtained in the case of long-run structural models, and illustrates its feasibility in the case of a small macroeconometric model of the UK.

The empirical exercise provides a concrete example of the usefulness of event probability forecasting both as a tool for model evaluation and as a means for conveying the uncertainties surrounding the forecasts of specific events of interest. The model used represents a small but comprehensive model of the UK macroeconomy which incorporates long-run relationships suggested by economic theory so that it has a transparent and theoretically coherent foundation. The model evaluation exercise not only demonstrates the statistical adequacy of the forecasts generated by the model but also highlights the considerable improvements in forecasts obtained through the imposition of the theory-based long-run restrictions. The predictive distribution functions relating to single events and the various joint event probabilities presented illustrate the flexibility of the functions in conveying forecast uncertainties and, from the observed independence of probability forecasts of events involving inflation and growth, in conveying information on the properties of the model. The model averaging approach also provides a coherent procedure to take account of parameter and model uncertainties as well as the future uncertainty.

12

Global modelling and other applications

The modelling approach described in Chapters 2–7, and adopted in the detailed description of the UK macroeconometric model of Chapters 8–11, is widely applicable and has been recently employed in a variety of studies investigating important macroeconomic issues. We conclude the book with a brief description of a number of these applications. The applications have been chosen to illustrate the flexibility of the modelling approach and the range of topics that can be addressed using these techniques. The *first* group of applications are concerned with the widespread use of the Structural Cointegrating VAR modelling approach, and provides a brief description of a global VAR (GVAR) model, which is aimed at capturing regional interdependencies in the world economy. The GVAR illustrates how the modelling approach advanced in the book can be generalised to build a global model within which the core UK model could, in principle, be subsumed. The *second* area focuses on the increasing use of impulse responses and the ways in which the VAR estimates can be interpreted, commenting on the construction of a high-frequency (monthly) version of the core model which is of particular use in identifying monetary policy shocks. Finally, a *third* area of applications focuses on recent use of probability forecasts, including a description of a measure of 'financial distress' that provides probabilistic statements on events in the UK unsecured credit market, investigated as a 'satellite' of the core UK model.

12.1 Recent applications of the structural cointegrating VAR approach

There has been considerable interest and activity in the application of the Structural Cointegrating VAR approach to macroeconometric modelling