10

Impulse response and trend/cycle properties of the UK model

One important use of macroeconometric models is to conduct counter-factual experiments in order to interpret previous historical episodes and to help with policy analysis. For example, it is important to learn about the possible impacts of changes in interest rates or oil prices on output and inflation over one or more years into the future. And our understanding of the macroeconomy will be enhanced if we are able to characterise past observations on economic activity as being related to 'trend' growth or as 'cyclical' movements around the trend. In this chapter, we focus on these uses of an estimated macroeconometric model, noting that we need to supplement the model with additional *a priori* assumptions in order to undertake these counter-factual exercises in many cases.

For example, an analysis of the dynamic impact of shocks is typically carried out using impulse response functions that focus on the evolution of the conditional means of the target variables in response to different types of shocks.[1] The estimation of impulse response functions, with respect to shocks applied to observables such as the oil price, does not pose any new technical difficulties and can be conducted using the generalised impulse response approach described in Section 6.1.3. In the case of monetary policy shocks or shocks to technology or tastes, the analysis of dynamic impulses is complicated due to the fact that such shocks are rarely observed directly and must be identified indirectly through a fully articulated macroeconomic model.

In the context of the core model of the UK economy developed in Chapters 4 and 5, we have made a clear distinction between the long-run

[1] Pesaran, Smith and Smith (2005) argue that a probabilistic approach to the analysis of counter-factuals might be more appropriate. Such an analysis is, however, beyond the scope of the present chapter.

structural and long-run reduced form disturbances, denoted by η_{it} and ξ_{it}, respectively, and between the reduced form shocks associated with the reduced form vector error correction model of (5.1) and the structural shocks associated with the structural macroeconomic model of (5.2), denoted u_{it} and ε_{it}, $i = 1, 2, \ldots, 5$. It is the structural innovations that have a clear economic interpretation: the η_{it} measure the deviations from long-run relationships in which the equilibrating pressures are identified by economic theory,[2] while the u_{it} measure the (typically white noise) deviations of target variables from the value suggested by the corresponding decision rule.

The analysis of the dynamic response of the macroeconomy to reduced form shocks provides important insights with which to interpret recent episodes in the UK economy and with which to consider the potential effects of changes abroad or of moderate changes in policy. Such an analysis illustrates and summarises the complex macrodynamics that can be captured by a cointegrating VAR model. The analysis does not rely on identifying assumptions other than those that relate to the long-run properties of the model (about which there is a relatively high degree of consensus) and so is not subject to the Sims critique. Moreover, the use of the Generalised Impulse Responses (GIR) analysis described in Chapter 6 ensures that the analysis is invariant to the ordering of the variables in the VAR. These impulse responses are relatively robust, therefore, and represent our preferred means of illustrating the dynamic properties of the model. In this chapter, we provide impulse responses of this sort relating to foreign output and to foreign interest rates to illustrate the dynamic properties of the macroeconomy.

If we wish to identify the effects of monetary policy shocks, or structural shocks more generally, we require a much more detailed *a priori* modelling of expectations, production and consumption lags, and the short-run dynamics of the technological process and its diffusion across the countries in the international economy. That is, we require further restrictions to be placed on the contemporaneous relationships amongst the variables. This relates to the 'structural' VECM given in equation (5.2) associated with the long-run structural macroeconometric model:

$$\mathbf{A}\,\Delta\mathbf{z}_t = \widetilde{\mathbf{a}} - \widetilde{\boldsymbol{\alpha}}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1}\widetilde{\boldsymbol{\Gamma}}_i\Delta\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_t, \qquad (10.1)$$

[2] The mechanics of the equilibrating processes are not necessarily described by economic theory (involving unspecified adjustment costs, rigidities, coordination issues and so on), but theory explains why the long-run structural disturbances are stationary.

where $\mathbf{A}$ represents the 9×9 matrix of contemporaneous structural coefficients, $\widetilde{\mathbf{a}} = \mathbf{Aa}$, $\widetilde{\boldsymbol{\alpha}} = \mathbf{A}\alpha$, $\widetilde{\boldsymbol{\Gamma}}_i = \mathbf{A}\boldsymbol{\Gamma}_i$, and $\boldsymbol{\varepsilon}_t = \mathbf{Au}_t$ are the associated structural shocks which are serially uncorrelated and have zero means and the positive definite variance covariance matrix, $\boldsymbol{\Omega} = \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'$. As highlighted in Chapter 5, without *a priori* restrictions on $\mathbf{A}$ and/or $\boldsymbol{\Omega}$, it is not possible to give economic meanings to the estimates of the loading coefficients, $\widetilde{\boldsymbol{\alpha}}$, or to identify economically meaningful impulse response functions to shocks. The simplest example of such restrictions is obtained if a variable is considered weakly exogenous. Here, for example, because the oil price is assumed to be an $I(1)$ weakly exogenous variable, with no contemporaneous feedbacks from the endogenous variables to the oil price, identification of the impulse responses of the shock to oil prices does not pose any new problems. More generally, however, the restrictions on $\mathbf{A}$ that are necessary for identification of these structural effects require a tight description of the decision-rules followed by the public and private economic agents, incorporating information on agents' use of information and the exact timing of the information flows. An example of a set of short-run restrictions of this type was given in Chapter 5, based on a decision-theoretic model intended to capture the behaviour of the monetary authorities, and these would allow us to examine the short-run dynamic responses of the system to an economically meaningful monetary policy shock. In the section below, we describe in detail the steps taken to obtain the impulse response functions under these short-run restrictions. Subsequently, the impulse responses of these monetary policy shocks are presented alongside those obtained in response to a reduced form shock to the interest rate equation to illustrate the differences between the two approaches.

10.1 Identification of monetary policy shocks

The decision problem of the monetary authorities that underlies the identification scheme we adopt here to analyse monetary policy shocks has already been articulated in Section 5.1. The aim is to derive the impulse response functions of the monetary policy shocks, ε_{rt}, of the structural interest rate equation (5.14) described in Section 5.1. This requires the use of certain *a priori* restrictions based on the timing of the availability of information on the variables of interest. Recall from Section 5.1 that the aim of the monetary authorities is to set the market interest rate r_t by setting the base rate r_t^b. The difference between the two, the term premium, is influenced by the unanticipated factors such as oil price shocks, unexpected changes in foreign interest rates and exchange rates. We assume the market interest rate, r_t, and these three variables are determined on a daily basis, whereas the remaining variables are assumed to be much less frequently observed. Hence, we decompose $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t})'$, where $\mathbf{z}_{1t} = (p^o_t, e_t, r^*_t, r_t)'$ and $\mathbf{z}_{2t} = (\Delta p_t, y_t, p_t - p^*_t, h_t - y_t, y^*_t)'$, and partition the structural model (10.1) accordingly:

$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{z}_{1t} \\ \Delta \mathbf{z}_{2t} \end{pmatrix} = \boldsymbol{\mu}_{t-1} + \begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{pmatrix},$$

where

$$\boldsymbol{\mu}_{t-1} = \tilde{\mathbf{a}} - \tilde{\boldsymbol{\alpha}}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \tilde{\boldsymbol{\Gamma}}_i \Delta \mathbf{z}_{t-i},$$

and

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{pmatrix} \backsim i.i.d. \left[\mathbf{0}, \begin{pmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} \end{pmatrix}\right].$$

Our primary concern is with identification of the impulse responses associated with the structural equations explaining the four variables in $\mathbf{z}_{1t}$, namely, p^o_t, e_t, r^*_t, r_t. For this purpose, we adopt the following sets of restrictions:

$$\mathbf{A}_{12} = \mathbf{0}, \tag{10.2}$$

$$\mathbf{A}_{11} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -a_{eo} & 1 & 0 & 0 \\ -a_{r^*o} & -a_{r^*e} & 1 & 0 \\ -a_{ro} & -a_{re} & -a_{rr^*} & 1 \end{pmatrix}, \tag{10.3}$$

and assume that the covariance matrix of the structural shocks, $\boldsymbol{\varepsilon}_{1t}$, is diagonal:

$$\boldsymbol{\Omega}_{11} = \begin{pmatrix} \omega_{oo} & 0 & 0 & 0 \\ 0 & \omega_{ee} & 0 & 0 \\ 0 & 0 & \omega_{r^*r^*} & 0 \\ 0 & 0 & 0 & \omega_{rr} \end{pmatrix}. \tag{10.4}$$

The first set of restrictions, (10.2), are justified on the grounds that the variables in $\mathbf{z}_{2t}$ are much less frequently observed than those in $\mathbf{z}_{1t}$, and hence are unlikely to contemporaneously affect them. The lower triangular

form of $\mathbf{A}_{11}$ is motivated by our theoretical derivation of the structural interest rate equation in Section 5.1, plus the assumption that the UK exchange rate has a contemporaneous impact on foreign interest rates and not *vice versa*.[3] The final set of restrictions, (10.4), imposes further identifying restrictions on the structural shocks corresponding to $\mathbf{z}_{1t}$ by assuming that these shocks are orthogonal to each other. For the sub-system containing $\mathbf{z}_{1t}$, the assumptions (10.3) and (10.4) are the familiar type of exact identifying restrictions employed in the literature, and together impose 4^2 restrictions needed for the exact identification of the impulse responses of the shocks to $\boldsymbol{\varepsilon}_{1t}$. However, as demonstrated in Appendix B, the impulse responses associated with $\boldsymbol{\varepsilon}_{1t}$ are invariant to the identification of the rest of the system and, in particular, do not require $\boldsymbol{\Omega}_{12} = 0$, or $\mathbf{A}$ to be a lower-triangular matrix.[4] It is also possible to show that in our set-up the impulse responses of the monetary policy shocks are invariant to a re-ordering of the variables p^o_t, e_t and r^*_t in $\mathbf{z}_{1t}$. Hence, once the position of the monetary policy variable in $\mathbf{z}_t$ is fixed (in our application after p^o_t, e_t and r^*_t), the impulse response functions of the monetary policy shocks will be invariant to the re-ordering of the variables before and after r_t in $\mathbf{z}_t$. (A proof is provided in Appendix B.)

To derive the impulse responses, first recall that the reduced form equation associated with (10.1) is given by:

$$\Delta \mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{z}_{t-i} + \mathbf{u}_t, \tag{10.5}$$

where the reduced form errors can be partitioned as $\mathbf{u}_t = (\mathbf{u}'_{1t}, \mathbf{u}'_{2t})'$ conformably with $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t})'$, and note that

$$\boldsymbol{\Omega}_{11} = \text{Cov}(\boldsymbol{\varepsilon}_{1t}), \ \boldsymbol{\Sigma}_{11} = \text{Cov}(\mathbf{u}_{1t}), \ \mathbf{u}_{1t} = \mathbf{A}_{11}^{-1}\boldsymbol{\varepsilon}_{1t}.$$

Then, under (10.2), $\boldsymbol{\Sigma}_{11} = \mathbf{A}_{11}^{-1}\boldsymbol{\Omega}_{11}\mathbf{A}_{11}'^{-1}$ and the 10 unknown coefficients in $\mathbf{A}_{11}$ and $\boldsymbol{\Omega}_{11}$ can be obtained uniquely from the 10 distinct elements of $\boldsymbol{\Sigma}_{11}$. A consistent estimate of $\boldsymbol{\Sigma}_{11}$ can be computed from the reduced form residuals, $\hat{\mathbf{u}}_{1t}$, namely $\hat{\boldsymbol{\Sigma}}_{11} = T^{-1}\sum_{t=1}^{T} \hat{\mathbf{u}}_{1t}\hat{\mathbf{u}}'_{1t}$. Under the identification scheme in (10.3)–(10.4), the impulse response functions of the effects of a unit shock to the structural errors on $\mathbf{z}_t$ can now be obtained following the approach set out in Koop *et al.* (1996), and discussed further

[3] Recall that we are also assuming that the oil price can contemporaneously affect the macroeconomic variables, but is not itself contemporaneously affected by them.

[4] Note also that the impulse response functions of the monetary policy shocks are invariant to the ordering of the variables in $\mathbf{z}_{2t}$.

in Pesaran and Shin (1998).[5] Let $g(n, \mathbf{z} : \varepsilon_i)$, $i = o, e, r^*, r$, be the generalised impulse responses of $\mathbf{z}_{t+n}$ to a unit change in ε_{it}, measured by one standard deviation, namely $\sqrt{\omega_{ii}}$. Then, at horizon n we have

$$g(n, \mathbf{z} : \varepsilon_i) = E\left(\mathbf{z}_{t+n} \mid \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathfrak{I}_{t-1}\right) - E\left(\mathbf{z}_{t+n} \mid \mathfrak{I}_{t-1}\right), \quad i = o, e, r^*, r,$$

where $\mathfrak{I}_{t-1}$ is the information set available at time $t-1$. Since all the shocks are assumed to be serially uncorrelated with zero means, (10.5) provides the following recursive relations in $g(n, \mathbf{z} : \varepsilon_i)$:

$$\Delta g(n, \mathbf{z} : \varepsilon_i) = -\mathbf{\Pi} g(n-1, \mathbf{z} : \varepsilon_i) + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \Delta g(n-1, \mathbf{z} : \varepsilon_i) \quad \text{for } n = 1, 2, \ldots, \tag{10.6}$$

with the initialisation $g(n, \mathbf{z} : \varepsilon_i) = 0$ for $n < 0$, where $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$ and $\Delta g(n, \mathbf{z} : \varepsilon_i) = g(n, \mathbf{z} : \varepsilon_i) - g(n-1, \mathbf{z} : \varepsilon_i)$. In the case of $n = 0$ (*i.e.* the impact effects), we have

$$g(0, \mathbf{z} : \varepsilon_i) = E\left(\Delta \mathbf{z}_t \mid \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathfrak{I}_{t-1}\right) - E\left(\Delta \mathbf{z}_t \mid \mathfrak{I}_{t-1}\right). \tag{10.7}$$

Under (10.1) and conditional on $\mathfrak{I}_{t-1}$, $\left(\varepsilon_{it}, \Delta \mathbf{z}_t'\right)'$ is distributed with mean

$$\begin{pmatrix} 0 \\ \mathbf{A}^{-1}\boldsymbol{\mu}_{t-1} \end{pmatrix},$$

and the covariance matrix

$$\begin{pmatrix} \omega_{ii} & E\left(\varepsilon_{it}\mathbf{u}_t'\right) \\ E\left(\varepsilon_{it}\mathbf{u}_t\right) & E\left(\mathbf{u}_t\mathbf{u}_t'\right) \end{pmatrix}.$$

In the case where, conditional on $\mathfrak{I}_{t-1}$, $\Delta \mathbf{z}_t$ is normally distributed, using familiar results on conditional expectations of multivariate normal densities, we have[6]

$$E\left(\Delta \mathbf{z}_t \mid \varepsilon_{it} = \sqrt{\omega_{ii}}, \mathfrak{I}_{t-1}\right) = \mathbf{A}^{-1}\boldsymbol{\mu}_{t-1} + \frac{E(\varepsilon_{it}\mathbf{u}_t)}{\omega_{ii}}\sqrt{\omega_{ii}}.$$

But under (10.2),

$$\mathbf{u}_t = \begin{pmatrix} \mathbf{A}_{11}^{-1}\boldsymbol{\varepsilon}_{1t} \\ \mathbf{u}_{2t} \end{pmatrix},$$

[5] For more details and the application of the approach to structural simultaneous equation models see Pesaran and Smith (1998).

[6] This result provides an optimal linear approximation when the errors are not normally distributed.

and hence, using (10.7), we have

$$g(0, \mathbf{z} : \varepsilon_i) = \frac{E\left(\varepsilon_{it}\mathbf{u}_t\right)}{\sqrt{\omega_{ii}}} = \frac{1}{\sqrt{\omega_{ii}}}\begin{bmatrix} \mathbf{A}_{11}^{-1}E\left(\varepsilon_{it}\boldsymbol{\varepsilon}_{1t}\right) \\ E\left(\varepsilon_{it}\mathbf{u}_{2t}\right) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11}^{-1}\mathbf{\Omega}_{11}^{\frac{1}{2}}\boldsymbol{\tau}_i \\ E\left(\frac{\varepsilon_{it}\mathbf{u}_{2t}}{\sqrt{\omega_{ii}}}\right) \end{bmatrix}, \tag{10.8}$$

where $\boldsymbol{\tau}_i$ is a 4×1 selection vector for $i = o, e, r^*, r$. For the oil price shock $\boldsymbol{\tau}_o = (1, 0, 0, 0)'$, and for the monetary policy shock the selection vector is defined by $\boldsymbol{\tau}_r = (0, 0, 0, 1)'$. Under the identification restrictions (10.3) and (10.4), a consistent estimate of $\mathbf{A}_{11}^{-1}\mathbf{\Omega}_{11}^{\frac{1}{2}}$ can be obtained by the lower triangular Choleski factor of $\hat{\mathbf{\Sigma}}_{11}$. To consistently estimate $E\left(\frac{\varepsilon_{it}\mathbf{u}_{2t}}{\sqrt{\omega_{ii}}}\right)$, we note that, under the same restrictions, ω_{ii}, $i = o, e, r^*, r$, and the unknown elements of $\mathbf{A}_{11}$ can also be consistently estimated using $\hat{\mathbf{\Sigma}}_{11}$. It, therefore, remains to obtain a consistent estimate of $E\left(\varepsilon_{it}\mathbf{u}_{2t}\right)$. Recall that $\boldsymbol{\varepsilon}_{1t} = \mathbf{A}_{11}\mathbf{u}_{1t}$. Hence $E\left(\varepsilon_{it}\mathbf{u}_{2t}\right)$ can be consistently estimated by the ith row of

$$T^{-1}\sum_{t=1}^{T}\hat{\mathbf{A}}_{11}\hat{\mathbf{u}}_{1t}\hat{\mathbf{u}}_{2t}',$$

where $\hat{\mathbf{A}}_{11}$ is a consistent estimate of $\mathbf{A}_{11}$. It is clear that the impulse response functions of shocks to the structural errors, ε_{it}, $i = o, e, r^*$, and r, are invariant to the way the structural coefficients associated with the second block, $\mathbf{z}_{2t}$, in (10.1) are identified.

10.2 Estimates of impulse response functions

We now report the estimates of impulse response functions of the endogenous variables of the core model. We begin by describing the impulse responses to an oil price shock, which is obtained on the relatively uncontentious assumption that oil prices are weakly exogenous. We then present the impulse responses to a foreign output and foreign interest equation shock, illustrating the use of the GIR techniques. And we then present the impulse responses to a monetary policy shock, obtained under the short-run identifying restrictions and using the method described in Section 10.1 above. We also compare the responses to monetary policy shocks directly with those to an interest rate equation shock. The macroeconomic analyses of the effects of these shocks have been of special interest and help provide further insights into the short-run dynamic properties of our model. We shall also consider the time profile of the effects of shocks on the long-run relationships. Recall that despite the integrated properties of the underlying variables, the effects of shocks on the long-run relations can only be temporary and should eventually disappear. But it is interesting to see how long such effects are likely to last. These types of impulse response functions are referred to as 'persistence profiles' and, as shown in Pesaran and Shin (1996), they shed light on the equilibrating mechanisms embedded within the model.

To compute all the impulse response functions analysed, we need an estimate of the oil price equation.[7] We decided to exclude domestic variables from the equation since we would not expect a small open economy such as the UK to have any significant influence on oil prices. The resultant oil price equation, estimated over the period 1965q1–1999q4, is given by:

$$\Delta p_t^o = \underset{(0.0352)}{-0.0039} + \underset{(0.1070)}{0.04787}\Delta p_{t-1}^o + \underset{(2.6818)}{2.7731}\Delta y_{t-1}^* + \underset{(1.8572)}{0.4199}\Delta p_{t-1}^* + \underset{(11.635)}{2.4855}\Delta r_{t-1}^* + \hat{\varepsilon}_{ot}, \tag{10.9}$$

$$\sqrt{\widehat{\omega}_{oo}} = 0.1661, \ \chi_{SC}^2[4] = 1.86, \ \chi_N^2[2] = 6558.9.$$

where standard errors are in brackets, $\omega_{oo} = \text{var}(\varepsilon_{ot})$ and χ_{SC}^2 and χ_N^2 are chi-squared statistics for serial correlation and normality, respectively. None of the coefficients are statistically significant at the conventional levels, although there is some evidence of a positive effect from past changes in foreign output. The hypothesis that the residuals are serially uncorrelated cannot be rejected either. But, not surprisingly, there is a clear evidence of non-normal errors, primarily reflecting the three major oil price shocks experienced during the period under consideration. These results are in line with the widely held view that oil prices follow a geometric random walk, possibly with a drift. Therefore, we base our computations of impulse responses on the following simple model:

$$\Delta p_t^o = \underset{(0.0139)}{0.0173} + \hat{\varepsilon}_{ot}, \tag{10.10}$$

$$\sqrt{\widehat{\omega}_{oo}} = 0.16485, \ \chi_{SC}^2[4] = 2.19, \ \chi_N^2[2] = 6399.$$

10.2.1 *Effects of an oil price shock*

Over the past three decades, oil price changes have had a significant impact on the conduct of monetary policy in the UK and elsewhere. Increases in oil prices have often been associated with rising prices, falling output and

[7] This relates to the discussion surrounding (4.44) in Chapter 4.

a tightening of monetary policy which has in turn contributed to further output falls. It is important that special care is taken to separate the output and inflation effects of an oil price shock from those of a monetary shock as they are likely to be positively correlated. In our framework, this is achieved by treating oil prices as long-run forcing, and by explicitly modelling the contemporaneous dependence of monetary policy shocks on the oil price shocks, as well as on shocks to exchange rates and foreign interest rates.[8]

Figure 10.1 provides the persistence profiles of the effects of a one standard error increase in oil prices (around 16.5% per quarter) on the five long-run relationships. Figure 10.2 gives the impulse responses of the oil price shock on the levels of all the eight endogenous variables in the model. Both figures also provide bootstrapped 95% confidence error bands (see Section 6.5 for more details).[9] All the persistence profiles converge towards zero, thus confirming the cointegrating properties of the long-run relations. In addition, the persistence profiles provide useful information on the speed with which the different relations in the model, once shocked, will return to their long-run equilibria. The results are generally in line with those found in the literature, with *PPP* and output gap relations showing much slower rates of adjustments to shocks. The effect of the oil price shock on the output gap takes some ten years to complete. This is rather slow, but is comparable to those implied by Barro and Sala-i-Martin's (1995) analyses of international output series.[10] Similarly, deviations from *PPP* are relatively long lived, but the slow speed of convergence towards equilibrium in this relationship is again consistent with existing results which put the half life of deviations from *PPP* at about four years for the major industrialised countries.[11] Convergence to the *FIP*, *IRP* and *MME* relationships is much more rapid, reflecting the standard view that arbitrage in asset markets functions much faster than in the goods markets in restoring equilibria.

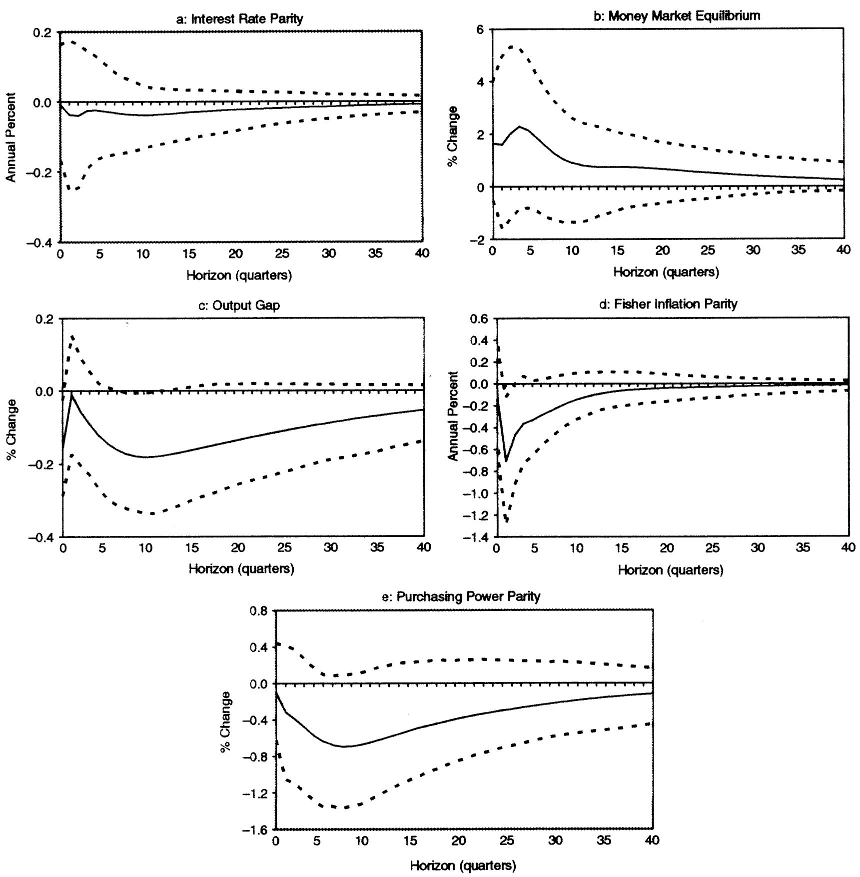
[8] For an alternative identification scheme applied to the US economy, see Bernanke *et al.* (1997).

[9] Point estimates and 95% confidence intervals are plotted in Figures 10.1–10.10. We also calculated the empirical means and medians of the bootstrap estimates and generally found them to be close to the point estimates.

The calculations were performed using *GAUSS* and the programs are described in Appendix D. A forthcoming version of Microfit, *Microfit 5.0,* may also be used to calculate the impulse responses and persistence profiles reported here. See Pesaran and Pesaran (2006).

[10] However, Barro and Sala-i-Martin (1995) assume that output series are trend stationary and study convergence to a common trend growth rate. The present study assumes the output series are difference stationary and tests for cointegration between UK and OECD output series. For further discussion, see Lee *et al.* (1997, 1998).

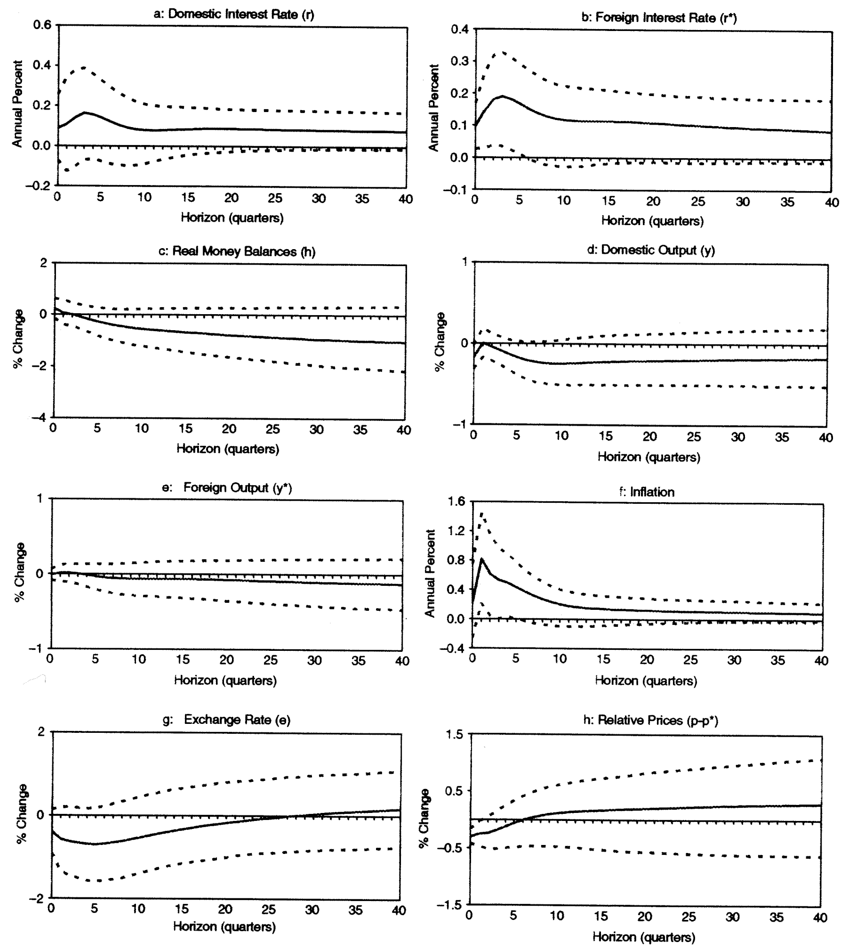
[11] See, for example, Johansen and Juselius (1992), Pesaran and Shin (1996), or Rogoff (1996).



Note: The graphs define the long-run relationships as follows: Interest Rate Parity: $r_t - r_t^*$, Money Market Equilibrium Condition: $h_t - y_t + 56.1r_t + 0.0073t$, the Output Gap: $y_t - y_t^*$, Fisher Inflation Parity: $r_t - \Delta p_t$ and the *PPP* (real exchange rate): $e_t + p_t^* - p_t$. The size of the shock is equal to the standard deviation of the selected equation error. The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.1 Persistence profiles of the long-run relations of a positive unit shock to the oil price.

Turning to the impulse response functions in Figure 10.2, the oil price shock has a permanent effect on the level of the individual series, reflecting their unit root properties. Its effect on output has the expected negative sign, reducing domestic output by approximately 0.24% below its base after 2.5 years. Foreign output also declines to the same long-run value but at a much slower speed. On impact, the oil price shock raises the



Note: The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.2 Generalised impulse responses of a positive unit shock to the oil price.

domestic rate of inflation by 0.20%, and by 0.82% after one quarter, before gradually falling back close to zero after approximately three years. Despite the higher domestic prices, the oil price shock generates a small appreciation of the nominal exchange rate, as can be seen from Figure 10.2g. This initial movement is then followed by further appreciations, although the process starts to reverse after approximately one year. In the long run, the nominal exchange rate fully adjusts to the change in relative prices with

PPP restored but, as noted above, the speed of adjustment is relatively slow. The oil price shock is accompanied by increases in both domestic and foreign interest rates, suggesting a possible tightening of the monetary policy in response to the rise in oil prices. Domestic interest rates increase by some nine basis points on impact, rising to 16 basis points after approximately three quarters, and then falling to a long-run values of eight basis points above its pre-shock level. The oil price shock affects real money balances both directly and indirectly through its impact on interest rates. The overall outcome is to reduce real money balances by around 1% in the long run. This is indicative of the presence of a strong liquidity effect in our model. The oil price shock also causes the real rate of interest to fall, initially by 0.1% and then by 0.7%, before gradually returning to its equilibrium value of zero.

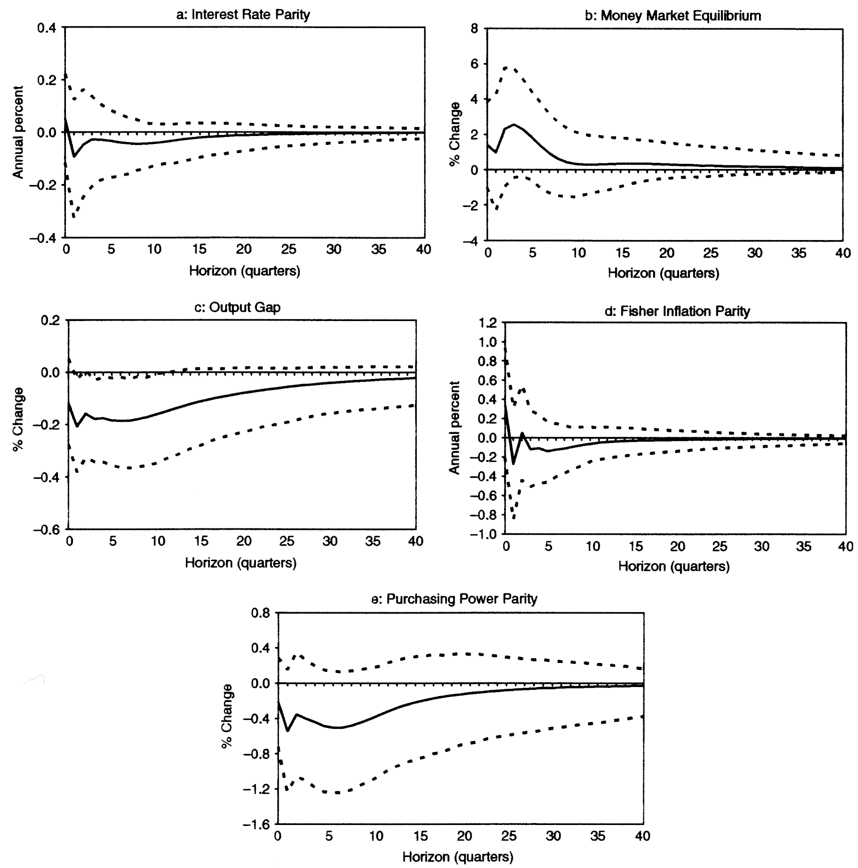
10.2.2 *Effects of a foreign output equation shock*

The Generalised Impulse Responses (GIR), outlined in Chapter 6, describe the time profile of the effect of a unit shock to a *particular equation* on all the model's endogenous variables. The dynamics which result from the shock will embody the contemporaneous interactions of all the endogenous variables of the system. These are captured by the elements of the estimated covariance matrix of the shocks to the endogenous variables which reflects the historical patterns of correlations across the shocks in the sample period under consideration. There are many issues that could be analysed through the GIR analysis and here we focus on the effects of shocks to the foreign output equation. As was noted earlier, unlike the orthogonalised impulse responses, the GIRs are invariant to the ordering of the variables in the VAR, and only require that the particular shock under consideration does not significantly alter the parameters of the model (see Section 6.1.3).[12]

Figure 10.3 plots the persistence profiles of the effects of a unit shock to the foreign output equation for the five long-run relations. The size of the deviations from equilibrium are much smaller than those compared to the oil price shock but the pattern is similar. Hence, the *PPP* and output gap relations show much slower rates of adjustments to shocks, whilst the convergence to the *FIP*, *IRP* and *MME* relationships is much more rapid.

Figure 10.4 gives the impulse responses of the foreign output shock on the levels of all the eight endogenous variables in the model. Given

[12] Here we mean policy changes that do not result in significant changes in the covariance structure of the shocks and/or the coefficients of the underlying VAR model.

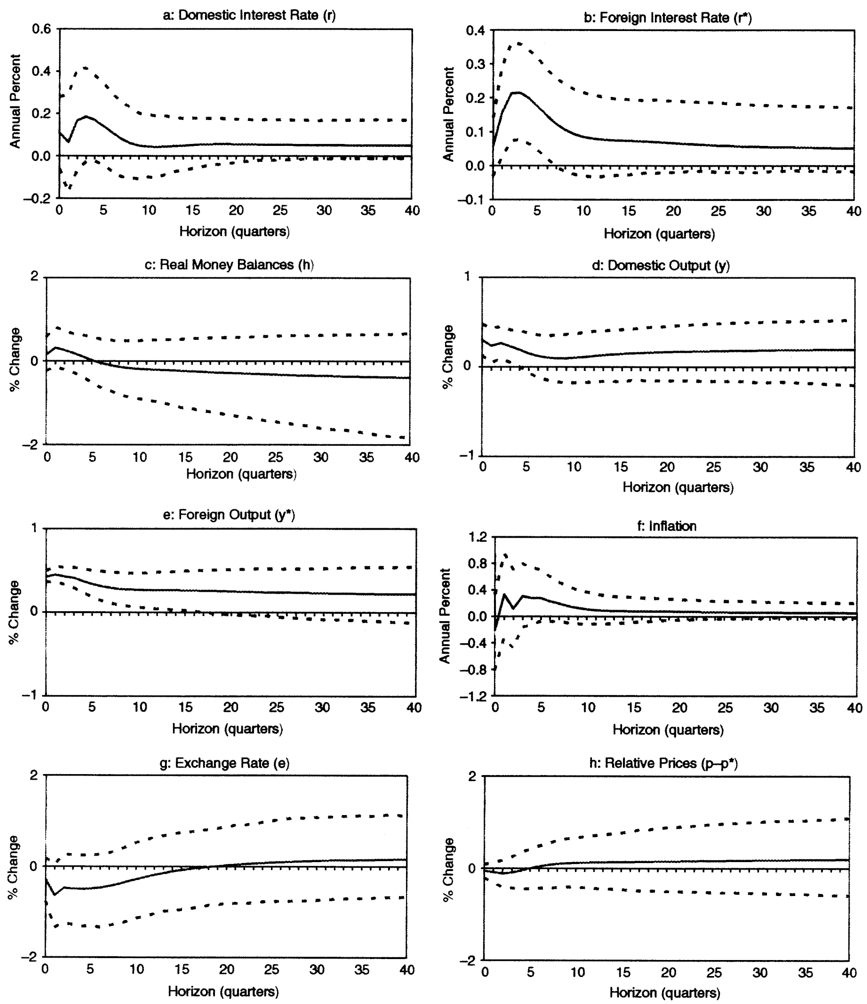


Note: The graphs define the long-run relationships as follows: Interest Rate Parity: $r_t - r_t^*$, Money Market Equilibrium Condition: $h_t - y_t + 56.1r_t + 0.0073t$, the Output Gap: $y_t - y_t^*$, Fisher Inflation Parity: $r_t - \Delta p_t$ and the *PPP* (real exchange rate): $e_t + p_t^* - p_t$. The size of the shock is equal to the standard deviation of the selected equation error. The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.3 Persistence profiles of the long-run relations of a positive unit shock to the foreign output equation.

the strong positive correlation that exists between foreign and domestic output innovations, the effect of the foreign output shock on impact is to cause domestic output to increase by approximately 0.3% (see Figure 10.4d).[13] These effects continue to persist over the subsequent quarters. In the long run, the effect of a unit shock to the foreign output

[13] All percentage changes quoted in this section are computed at annual rates.



Note: The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.4 Generalised impulse responses of a positive unit shock to the foreign output equation.

equation is to increase both domestic and foreign output by 0.2% above their baseline values. However, it is important to note that the gap between domestic and foreign output growths persists even after 20 quarters, with the foreign output level remaining considerably higher than domestic output through this time.

The GIRs for the foreign output shock on the domestic inflation and the nominal exchange rates are displayed in Figures 10.4f and 10.4g. The shock initially reduces domestic prices by 0.23% and appreciates the nominal exchange rate on impact by 0.27%. The fall in inflation is reversed in the following quarter, though, returning to near its baseline value after about 12 quarters. In the long run, the effect of the foreign output shock on the domestic inflation rate is zero, so that the effects are purely temporary.

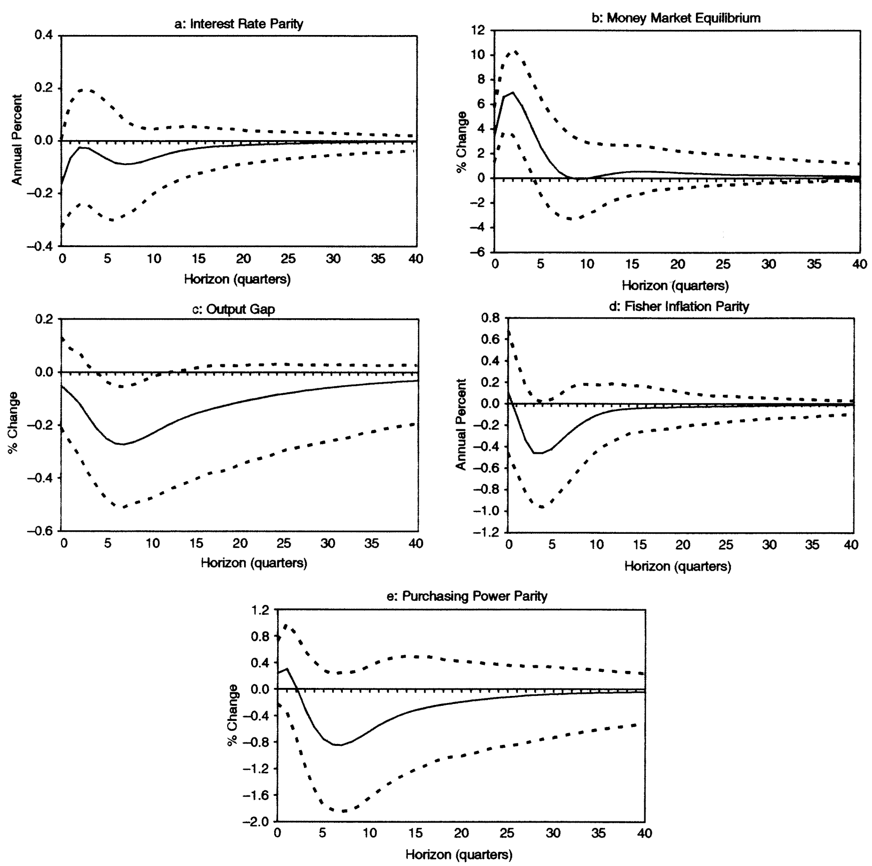
The effects of the shock on domestic and foreign interest rates are displayed in Figures 10.4a and 10.4b. The initial response to the shock is to increase domestic and foreign interest rates by 11 and six basis points, respectively. Subsequently the foreign interest rate rises above the domestic interest rate, but eventually this gap disappears, as predicted by the long-run interest parity relation embodied in the core model.

10.2.3 *Effects of a foreign interest rate equation shock*

In Figures 10.5 and 10.6, we report the GIRs for a shock to the foreign interest rate equation, where the size of the shock is scaled to ensure that the foreign interest rate rises by one standard deviation of the error variance on impact. Figure 10.5 again confirms the varying speeds of adjustments of the long-run relationships as before.

Figure 10.6 shows the impact effect of the shock to the foreign interest rate equation is to increase the domestic interest rate by 23 basis points whilst domestic output is unchanged. Domestic output falls thereafter, down by 0.37% after four quarters and reaching 0.5% below its baseline value after approximately 16 quarters. This suggests a complicated relationship between interest rate and output changes over the course of the business cycle. The shock to the foreign interest rate equation depreciates the nominal exchange rate on impact by 0.14% and by approximately 0.5% in the long run.

The effects of the shock on domestic interest rates, foreign interest rates, and domestic inflation are displayed in Figures 10.6a–b and 10.6f. The fact that the impulse response function for the foreign interest initially slopes upwards reflects the highly persistent nature of the interest rate movements in the short run. Perhaps not surprisingly, the domestic interest rate is much less affected by the shock with the result that, during the first three years following the shock, the foreign interest rate tends to rise above the domestic interest rate. This interest rate gap (relative to its baseline value)

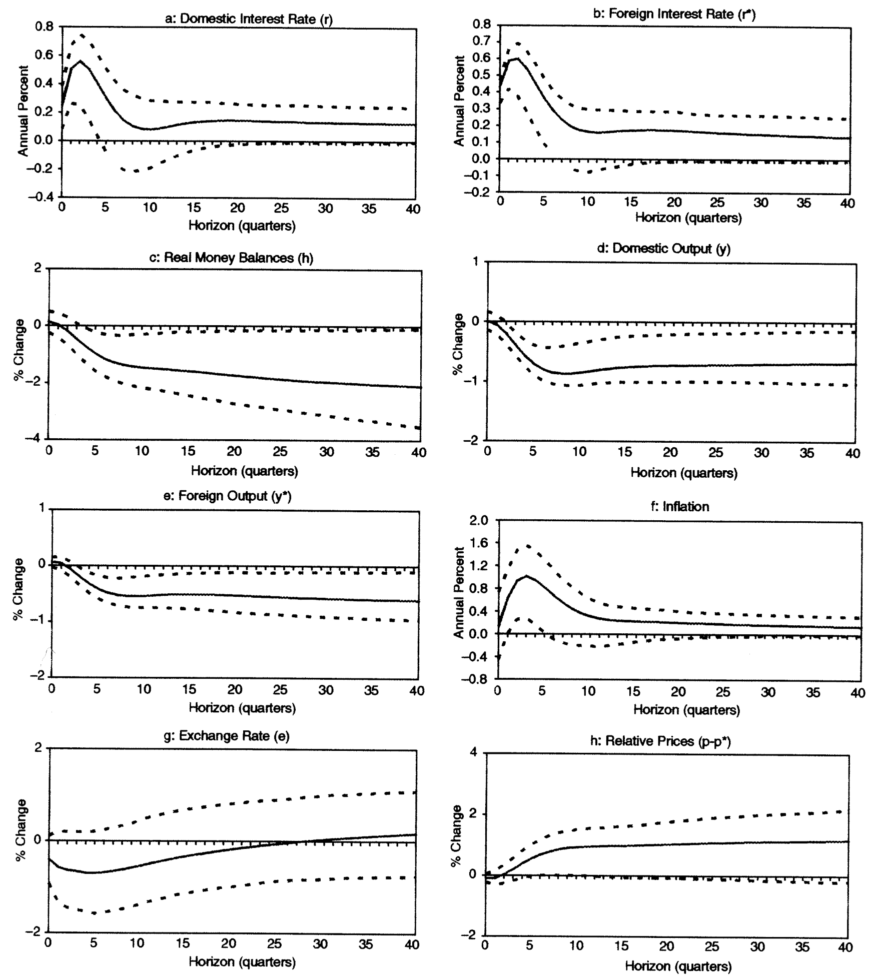


Note: The graphs define the long-run relationships as follows: Interest Rate Parity: $r_t - r_t^*$, Money Market Equilibrium Condition: $h_t - y_t + 56.1r_t + 0.0073t$, the Output Gap: $y_t - y_t^*$, Fisher Inflation Parity: $r_t - \Delta p_t$ and the *PPP* (real exchange rate): $e_t + p_t^* - p_t$. The size of the shock is equal to the standard deviation of the selected equation error. The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.5 Persistence profiles of the long-run relations of a positive unit shock to the foreign interest rate equation.

will eventually disappear, however, as predicted by the long-run interest parity relation embodied in the core model.

The initial effect of the interest rate shock on domestic inflation is to increase the rate of inflation by 0.13% followed by 0.57% after one quarter, and 0.88% after four quarters. This effect is reversed from this point onwards, with the inflation rate falling to be approximately 0.1% above its baseline value after about 14 quarters. In the long run, the effect of



Note: The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.6 Generalised impulse responses of a positive unit shock to the foreign interest rate equation.

the interest rate shock on the domestic inflation rate is zero. Throughout, the effects of the shock to the foreign interest rate equation on real money balances are negative, which is in line with the strong negative effect of interest rates on real money balances obtained at the estimation stage.

10.2.4 *Effects of a monetary policy shock*

We turn now to the more economically meaningful monetary policy shocks or, more precisely under our identification scheme set out in Chapter 5, the non-systematic (or unanticipated) component of the policy. Recall that the shock is defined by ε_{rt}, the shock in the structural equation for the market interest rate, and allows oil prices, exchange rates and foreign interest rates to have contemporaneous effects on r_t. The algorithms necessary for the computation of the associated impulse responses are set out above in Section 10.1.

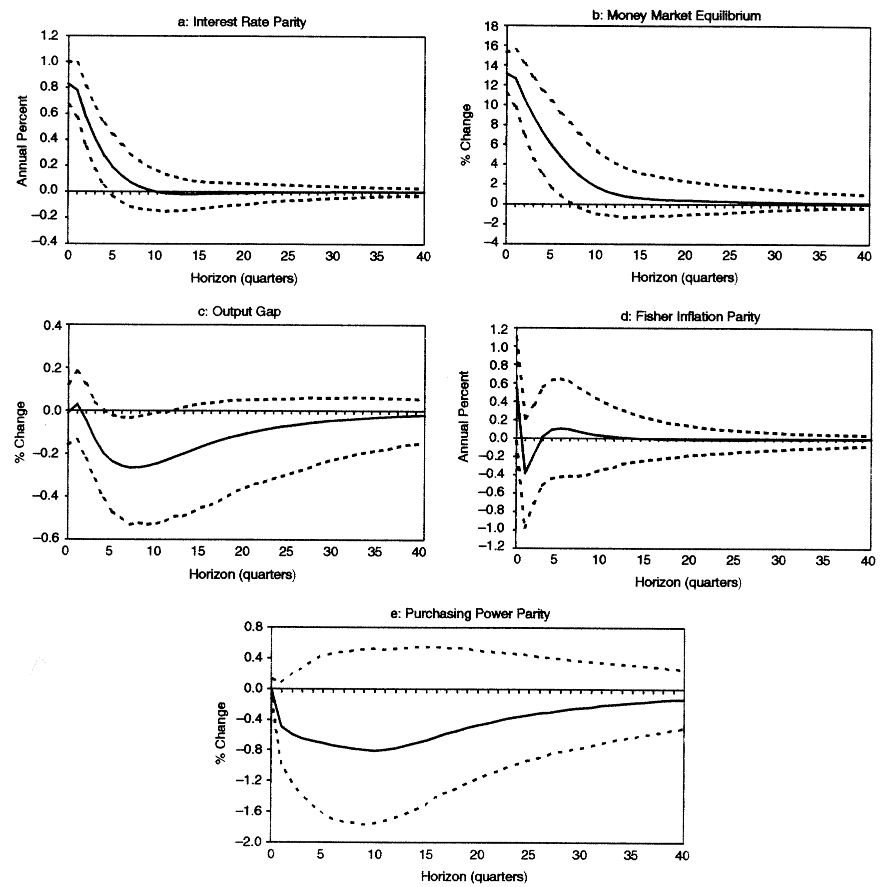
Figure 10.7 presents the persistence profiles of the effects of one standard error unexpected increase in the interest rate (*i.e.* a rise of 91 basis points) on the five long-run relations of the model.

As with the previous shocks, the effects of the monetary policy shock on these relations disappear eventually, but the speed with which this occurs varies considerably across the different arbitrage conditions. The interest parity condition is the quickest to adjust followed by the Fisher inflation parity, the monetary equilibrium condition, purchasing power parity and the output gap. It is worth emphasising that, in our model, the long-run equilibrium condition for interest rate parity rules out the phenomena observed in Eichenbaum and Evans (1995), where a contractionary monetary policy shock could result in a permanent shift in the interest rate differential.

On impact, the effect of the monetary policy shock is most pronounced on the money market equilibrium condition, resulting in a 12.7% unexpected excess supply of money. With foreign interest rates unchanged on impact (by construction), the shock raises the domestic interest rate above the foreign interest rate by 91 basis points, but it also raises the real interest rate by 59 basis points while leaving the real exchange rate unchanged. The output gap is initially left intact, reflecting a lagged response of real output to interest rate changes. However, the contractionary impact of the shock on domestic output (relative to the foreign output) begins to be seen after the second quarter, with domestic output falling below foreign output by 0.29% after two years.

The impulse response functions for the effects of the monetary shock on the various endogenous variables in the model are given in Figure 10.8.

Most of these plots exhibit familiar patterns. After the initial impact, the domestic interest rate declines at a steady rate settling down after approximately four years at an equilibrium value of five basis points above the

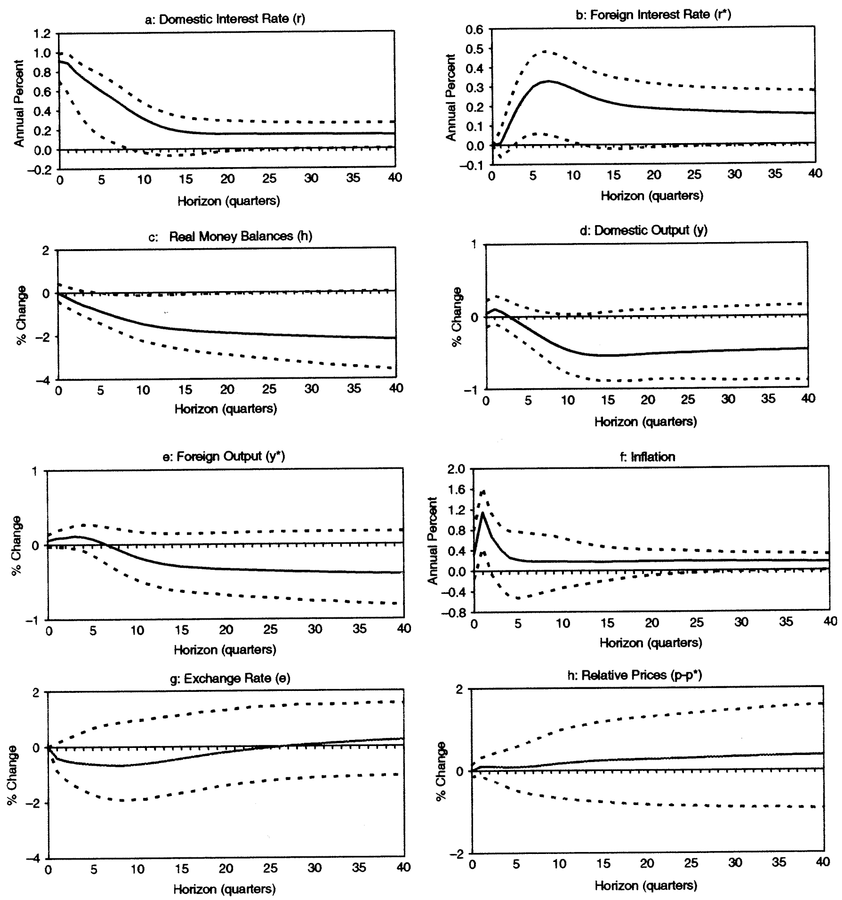


Note: The graphs define the long-run relationships as follows: Interest Rate Parity: $r_t - r_t^*$, Money Market Equilibrium Condition: $h_t - y_t + 56.1r_t + 0.0073t$, the Output Gap: $y_t - y_t^*$, Fisher Inflation Parity: $r_t - \Delta p_t$ and the *PPP* (real exchange rate): $e_t + p_t^* - p_t$. The size of the shock is equal to the standard deviation of the selected equation error. The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.7 Persistence profiles of the long-run relations of a positive unit shock to monetary policy.

baseline value. In tandem with the fall in the interest rate, the excess supply of money declines to approximately 8.6% after one year, then to 5.0% after two years, reaching its equilibrium after approximately five years. These results clearly show the presence of a sizeable 'liquidity effect' in our model following the unexpected tightening of the monetary policy.[14]

[14] See, for example, the analysis of liquidity effects in Pagan and Robertson (1998).



Note: The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.8 Generalised impulse responses of a positive unit shock to monetary policy.

The monetary policy shock has little immediate effects on the real side of the economy. The contractionary effects of the policy begin to be felt on output and real money balances after one quarter. The impulse responses of domestic and foreign output are given in Figures 10.8d and 10.8e, each showing a relatively smooth decline to around 0.46% and 0.17%, respectively, below base after two and half years. The speed of adjustments

of the two series differ, however, as was seen clearly from the persistence profile of the output gap presented in Figure 10.7c. Figure 10.8f provides evidence of the well-known 'price puzzle', as inflation increases in immediate response to the contractionary monetary shock, falling back to close to zero after three years. Note, however, that with the exception of the first few quarters the inflation responses are insignificantly different from zero so that, insofar as the puzzle is apparent, the underlying long-run relations ensure that the anomaly are observed in the short run only.

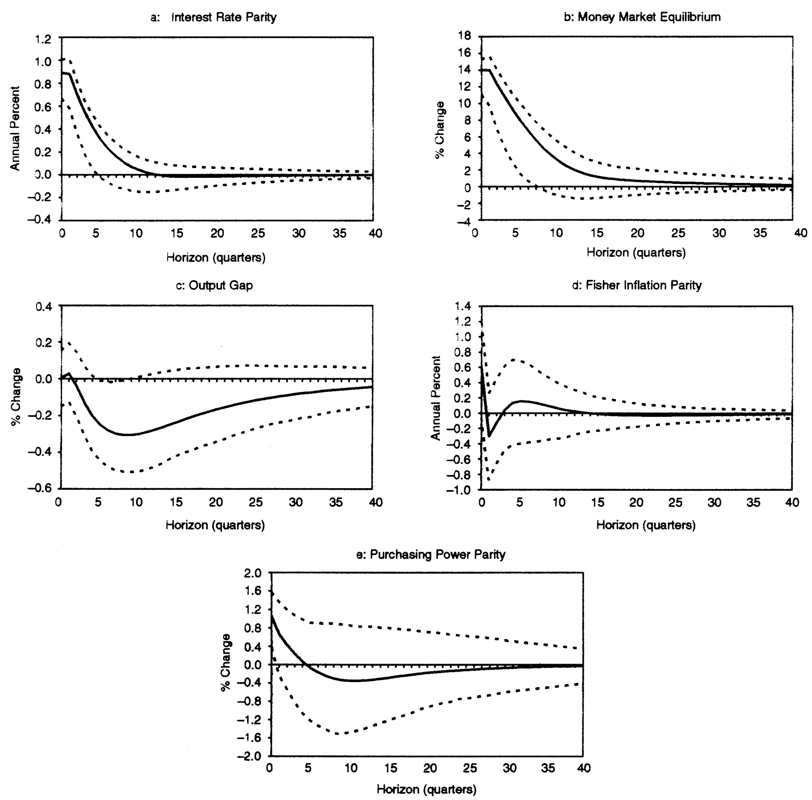
The impact effect of the monetary policy shock on the nominal exchange rate is zero by construction but, as can be seen from Figure10.8g, the shock causes the exchange rate to appreciate by around 0.5% in the following period. The exchange rate remains roughly constant for the subsequent year and then depreciates back to close to its original level after 20 quarters. This pattern is reasonably consistent with the Dornbusch (1976) overshooting model which would predict a large initial appreciation in the exchange rate in response to a monetary contraction, followed by subsequent depreciation to its long-run level. Certainly it matches well the broader view of overshooting discussed in Eichenbaum and Evans (1995) in which there might be a *sequence of periods* of appreciation followed by depreciation because of secondary effects of the shock on risk premia, speculative behaviour and information imperfections relating to the permanence of the shock. Moreover, this time profile for the exchange rate is observed in a set of responses in which interest rate parity is re-established relatively quickly and in which a positive differential of domestic over foreign interest rates is associated with a constant or depreciating exchange rate as suggested by UIP. This accords well with theory, therefore, and is in contrast to the ' exchange rate puzzle' observed by Eichenbaum and Evans (1995) in which the interest rate differential is maintained indefinitely and is associated with a persistently appreciating exchange rate.[15]

By way of comparison, we also provide here the time profiles of the effects of shocks to a unit (one standard error) increase in the domestic interest rate equation. Figure 10.9 provides the persistence profiles of the effects of a unit shock to domestic interest rates (the size of the shock is scaled to ensure domestic interest rates rise by one standard error on impact) on the five long-run relationships. Figure 10.10 gives the

[15] See Gali and Monacelli (2005) for a small open economy model which shows the key difference between alternative rule-based policy regimes as being one of the relative amount of exchange rate volatility.

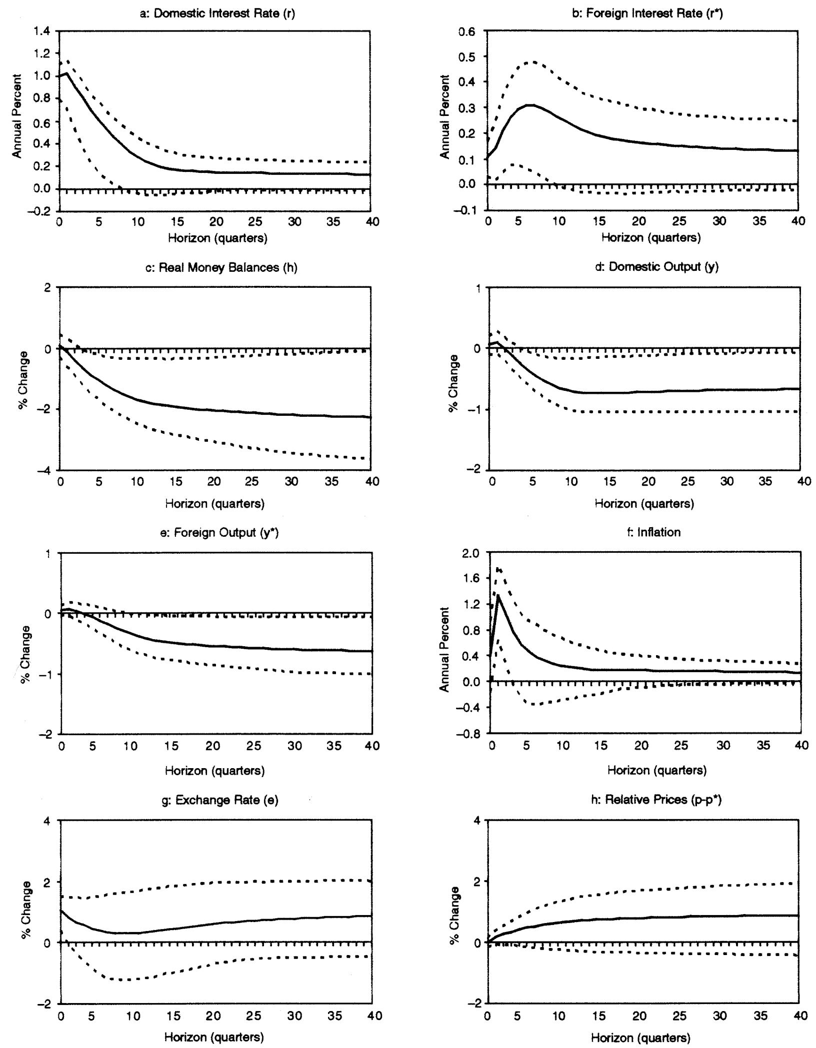
generalised impulse responses of a unit shock to the interest rate on the levels of all the eight endogenous variables in the model.

As is immediately apparent, the time profiles of the impulse responses of Figures 10.9 and 10.10 are very similar, in both size and shape, to those plotted in Figures 10.7 and 10.8 resulting from the identified monetary policy shock. This may not be too surprising as the impulses in the two experiments are clearly related and the long-run properties of the systems are the same. However, there are differences between the two which are



Note: The graphs define the long-run relationships as follows: Interest Rate Parity: $r_t - r_t^*$, Money Market Equilibrium Condition: $h_t - y_t + 56.1r_t + 0.0073t$, the Output Gap: $y_t - y_t^*$, Fisher Inflation Parity: $r_t - \Delta p_t$ and the *PPP* (real exchange rate): $e_t + p_t^* - p_t$. The size of the shock is equal to the standard deviation of the selected equation error. The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.9 Persistence profiles of the long-run relations of a positive unit shock to the UK interest rate equation.



Note: The solid and dashed lines plot the point estimates and 95% confidence intervals, respectively, of the impulse responses. The confidence intervals are generated from a bootstrap procedure using 2000 replications.

Figure 10.10 Generalised impulse responses of a positive unit shock to the UK interest rate equation.

important in terms of interpretation of the responses. In particular, the response of the exchange rate in Figure 10.10g shows important differences to those in Figure 10.8g, indicating a *depreciation* of the exchange rate on impact in response to the positive shock to interest rates. Moreover, the positive differential of domestic over foreign interest rates observed over the first ten quarters is associated in Figure 10.10g with an *appreciating* exchange rate. It is worth emphasising that the GIRs obtained for the shock to the interest rate equation in Figure 10.10 take into account the contemporaneous innovations in the other variables typically observed when the interest rate is shocked. It does not have the interpretation of a monetary policy shock and one should not expect to be able to relate the profile of responses to economically motivated dynamics as we could those for Figure 10.8. But the comparison with the responses of Figure 10.8, which do have this reasonable match with an economically motivated interpretation, illustrates well both the strengths of the GIR analysis of reduced form shocks and its limitations.

10.3 Trend/cycle decomposition in cointegrating VARs

In this section, we consider a decomposition of the variables in the UK model into trends and cycles, with the former further decomposed into deterministic and stochastic components, following Garratt, Robertson and Wright (2005, GRW). As we shall see, the stochastic components will be present only if the underlying VAR contains a unit root. The decomposition can be viewed as a multivariate version of the well-known Beveridge–Nelson (BN) permanent/transitory decomposition, but has the advantage that it is characterised fully in terms of the observables.[16] We illustrate the analysis with an empirical example, highlighting the permanent components of selected variables of the core VEC model of the UK economy developed in the earlier chapters.

It is worth noting that the choice of a permanent trend/transitory cycle decomposition relies on *a priori* assumptions on the extent of the correlation between permanent and transitory innovations. In the literature, views have ranged from the assumption that the innovations arise from the same sources (so that the correlation is perfect) to the assumption that they are entirely unrelated (so that the correlation is zero). Decompositions in the spirit of BN assume that shocks to the transitory component and to the stochastic permanent component have a correlation of one. This is in contrast to the unobserved component's approach to permanent and

[16] Beveridge and Nelson (1981) describe the decomposition in the case of a univariate specification. For a multivariate version of the BN decomposition, see Stock and Watson (1988) and Evans and Reichlin (1994). Other decompositions are provided by Gonzalo and Granger (1995), Proietti (1997), Hecq, Palm and Urbain (2000), and Gonzalo and Ng (2001).

transitory decomposition, for example, which assumes the correlation is zero.[17]

To explain our proposed decomposition scheme, suppose we take any arbitrary partitioning of $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{x}_t')'$ into permanent trend, $\mathbf{z}_t^P$, and transitory cycle, $\mathbf{z}_t^C$ components of the form:

$$\mathbf{z}_t = \mathbf{z}_t^P + \mathbf{z}_t^C, \tag{10.11}$$

where the permanent component may be further subdivided into deterministic and stochastic components

$$\mathbf{z}_t^P = \mathbf{z}_{dt}^P + \mathbf{z}_{st}^P.$$

Following GRW, we define the deterministic and the stochastic trend components of $\mathbf{z}_t$, respectively, by

$$\mathbf{z}_{dt}^P = \mathbf{g}_0 + \mathbf{g}t,$$

$$\mathbf{z}_{st}^P = \lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h} - \mathbf{z}_{d,t+h}^P\right) = \lim_{h\to\infty} E_t\left[\mathbf{z}_{t+h} - \mathbf{g}_0 - \mathbf{g}(t+h)\right], \tag{10.12}$$

where $\mathbf{g}_0$ is an $m \times 1$ vector of fixed intercepts, $\mathbf{g}$ is an $m \times 1$ vector of (restricted) trend growth rates, t is a deterministic trend term, and $E_t(\cdot)$ denotes the expectations operator conditional on the information at time t, taken to be $\{\mathbf{z}_t, \mathbf{z}_{t-1}, \ldots, \mathbf{z}_0\}$. Then we have

$$\mathbf{z}_t^P = \lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h} - \mathbf{g}h\right). \tag{10.13}$$

This definition of permanent trend has a number of important features that are worth emphasising

Remark 1 *Even if we are interested in the permanent/transitory decomposition of the endogenous variables, $\mathbf{y}_t$, we would still need to work with the VECM in $\mathbf{z}_t$ since this allows for long-run restrictions as well as for the short-term interactions that might exist between $\mathbf{y}_t$ and $\mathbf{x}_t$, under which the permanent/transitory properties of $\mathbf{x}_t$ would have a direct bearing on those of $\mathbf{y}_t$. This point reaffirms the desirability of multivariate approaches to trend/cycle decomposition over the univariate such as the Hodrick and Prescott (1997) and the original Beveridge–Nelson decompositions.*

[17] For a description of this alternative approach, see Harvey (1985), Watson (1986), Clarke (1987) and Harvey and Jaeger (1993). A review is provided in Canova (1998). Recently, Morley *et al.* (2003) have shown in a univariate context that when the (identifying) restriction in the unobserved components model that trend and cycle innovations are uncorrelated is relaxed, both decompositions will be identical.

Remark 2 *The stochastic permanent component of* $\mathbf{z}_t^P$, *namely* $\mathbf{z}_{st}^P$, *satisfies the property:*[18]

$$\lim_{h\to\infty} E_t\left(\mathbf{z}_{s,t+h}^P\right) = \mathbf{z}_{st}^P, \qquad (10.14)$$

which is a limiting martingale property shared by the random walk models. Recall that a process X_t *is said to follow a martingale process if* $E_t\left(X_{t+h}\right) = X_t$ *for all h. In the context of cointegrating VAR models,* $\mathbf{z}_{st}^P$ *satisfy the martingale property in the limit, whilst the permanent component of the BN decomposition is a martingale process and satisfy the property for all h. To establish the limit martingale property, (10.14), we first note that*

$$E_t\left(\mathbf{z}_{t+h} - \mathbf{z}_{d,t+h}^P\right) = E_t\left(\mathbf{z}_{s,t+h}^P\right) + E_t\left(\mathbf{z}_{t+h}^C\right).$$

Since $\mathbf{z}_{t+h}^C$ *is transitory,* $\lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h}^C\right) = 0$, *and therefore*

$$\lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h} - \mathbf{z}_{d,t+h}^P\right) = \lim_{h\to\infty} E_t\left(\mathbf{z}_{s,t+h}^P\right).$$

Then, the result in (10.14) follows using (10.12).

Remark 3 *As pointed out earlier the definition of the permanent component given by (10.13) has the advantage that it is defined directly in terms of the observables* $\{\mathbf{z}_t, \mathbf{z}_{t-1}, \ldots, \mathbf{z}_0\}$. *But this does not render it unique. For example, suppose that* $\mathbf{z}_t$ *is cointegrated and co-trended such that* $\boldsymbol{\beta}'\mathbf{z}_t + \mathbf{c}_0$ *is a stationary process with zero mean, and set* $\mathbf{z}_{st}^{PP} = \mathbf{z}_{st}^P + \boldsymbol{\beta}'\mathbf{z}_t + \mathbf{c}_0$. *Then it readily follows that*

$$\lim_{h\to\infty} E_t\left(\mathbf{z}_{s,t+h}^{PP}\right) = \lim_{h\to\infty} E_t\left(\mathbf{z}_{s,t+h}^P\right) = \mathbf{z}_{st}^P.$$

Therefore, $\mathbf{z}_{st}^{PP}$ *is also a stochastic permanent component with the same limiting martingale property as* $\mathbf{z}_{st}^P$.

10.3.1 *Relationship of GRW and BN Decompositions*

For a comparison of the BN and GRW decompositions, it is instructive to consider the UK model, which is given by the following vector error correction specification with restricted (deterministic) trend coefficients:[19]

$$\Delta\mathbf{z}_t = \mathbf{a} - \boldsymbol{\alpha}\boldsymbol{\beta}'\left[\mathbf{z}_{t-1} - \boldsymbol{\gamma}(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{z}_{t-i} + \mathbf{u}_t. \qquad (10.15)$$

[18] The deterministic permanent components are the same for GRW and BN.

[19] See, for example, equation (10.5) and note that under case IV, $\mathbf{b}_1 = \boldsymbol{\beta}'\boldsymbol{\gamma}$, where $\boldsymbol{\gamma}$ is an $m \times 1$ vector of restricted trend coefficients.

Denote the deviation of the variables in $\mathbf{z}_t$ from their deterministic components as $\tilde{\mathbf{z}}_t$, namely

$$\tilde{\mathbf{z}}_t = \mathbf{z}_t - \mathbf{g}_0 - \mathbf{g}t.$$

Then in terms of $\tilde{\mathbf{z}}_t$ we have

$$\Delta\tilde{\mathbf{z}}_t = \mathbf{a} - \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{g}_0 - \left(\mathbf{I}_m - \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\right)\mathbf{g} - \boldsymbol{\alpha}\beta'\left(\mathbf{g} - \gamma\right)(t-1)$$
$$- \boldsymbol{\alpha}\boldsymbol{\beta}'\tilde{\mathbf{z}}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\tilde{\mathbf{z}}_{t-i} + \mathbf{u}_t.$$

Since $\tilde{\mathbf{z}}_t$ has no deterministic components by construction, it must be that

$$\mathbf{a} = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{g}_0 + \left(\mathbf{I}_m - \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\right)\mathbf{g}, \qquad (10.16)$$

and

$$\boldsymbol{\beta}'\mathbf{g} = \boldsymbol{\beta}'\boldsymbol{\gamma}. \qquad (10.17)$$

Hence

$$\Delta\tilde{\mathbf{z}}_t = -\boldsymbol{\alpha}\boldsymbol{\beta}'\tilde{\mathbf{z}}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\tilde{\mathbf{z}}_{t-i} + \mathbf{u}_t, \qquad (10.18)$$

or, equivalently,

$$\tilde{\mathbf{z}}_t = \sum_{i=1}^{p}\boldsymbol{\Phi}_i\tilde{\mathbf{z}}_{t-i} + \mathbf{u}_t, \qquad (10.19)$$

where

$$\boldsymbol{\Phi}_1 = \mathbf{I}_m + \boldsymbol{\Gamma}_1 - \boldsymbol{\alpha}\boldsymbol{\beta}', \;\; \boldsymbol{\Phi}_i = \boldsymbol{\Gamma}_i - \boldsymbol{\Gamma}_{i-1}, \; i = 2, \ldots, p-1, \;\; \boldsymbol{\Phi}_p = -\boldsymbol{\Gamma}_{p-1}.$$

The BN decomposition of $\mathbf{z}_t$ can now be written as[20]

$$\mathbf{z}_t = \mathbf{z}_0 + \mathbf{g}t + \mathbf{C}(1)\,\mathbf{s}_{ut} + \mathbf{C}^*(L)\left(\mathbf{u}_t - \mathbf{u}_0\right), \qquad (10.20)$$

[20] See also Stock and Watson (1988) and Evans and Reichlin (1994), and the discussion in Section 6.2.1.

where

$$s_{ut} = \sum_{i=1}^{t} \mathbf{u}_i, \; \mathbf{C}^*(L) = \sum_{i=0}^{\infty} \mathbf{C}_i^* L^i,$$

$$\mathbf{C}_i = \mathbf{C}_{i-1}\mathbf{\Phi}_1 + \mathbf{C}_{i-2}\mathbf{\Phi}_2 + \cdots + \mathbf{C}_{i-p}\mathbf{\Phi}_p, \quad \text{for } i = 1, 2, \ldots,$$

with $\mathbf{C}_0 = \mathbf{I}_m$, $\mathbf{C}_1 = -(\mathbf{I}_m - \mathbf{\Phi}_1)$, and $\mathbf{C}_i = \mathbf{0}$ for $i < 0$; $\mathbf{C}_i^* = \mathbf{C}_{i-1}^* + \mathbf{C}_i$, for $i = 1, 2, \ldots$, with $\mathbf{C}_0^* = \mathbf{C}_0 - \mathbf{C}(1)$, and $\mathbf{C}(1) = \sum_{i=0}^{\infty} \mathbf{C}_i$. Hence, the stochastic trend in this approach is defined by

$$\mathbf{z}_{st}^{BN} = \mathbf{C}(1) \sum_{i=1}^{t} \mathbf{u}_i, \tag{10.21}$$

and satisfies the martingale property

$$E_t\left(\mathbf{z}_{s,t+h}^{BN}\right) = \mathbf{z}_{st}^{BN}, \quad \text{for all } h.$$

The two decompositions differ in the way the permanent stochastic components are defined and yield identical results only in the case where $\mathbf{z}_t$ follows a random walk model, possibly with a drift. This arises in the case of univariate models, or in the case of multivariate models without cointegration.[21] For example, considering univariate models, Morley et al. (2003, p. 3) also define z_{st}^{BN} by

$$z_{st}^{BN} = \lim_{h\to\infty} E_t\left(z_{t+h} - gh\right),$$

and show that it reduces to $z_{st}^{BN} = c(1)\sum_{i=1}^{t} u_i$. Therefore, at first it appears that the two definitions, $\mathbf{z}_{st}^{P} = \lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h} - \mathbf{g}h\right)$ and $\mathbf{z}_{st}^{BN} = \mathbf{C}(1)\sum_{i=1}^{t} \mathbf{u}_i$ are the same in general. But as noted above and the applications below illustrate this is not true in the multivariate case where $\mathbf{z}_t$ is cointegrated.

10.3.2 *Computation of the GRW decomposition*

As noted earlier, the GRW decomposition also has the advantage that it can be computed directly from the error correction or the VAR representation in terms of $\mathbf{z}_t, \mathbf{z}_{t-1}, \ldots, \mathbf{z}_{t-p+1}$. For computational purposes it is convenient to use the companion form of (10.19) given by

$$\widetilde{\mathbf{Z}}_t = \mathbf{F}\widetilde{\mathbf{Z}}_{t-1} + \mathbf{U}_t, \quad t = 1, \ldots, T,$$

[21] Note, however, that the two decompositions are based on the same deterministic trend specifications, with the restrictions on $\mathbf{g}$, defined by (10.17), applicable to both.

where

$$\underset{mp\times 1}{\widetilde{\mathbf{Z}}_t} = \begin{bmatrix} \widetilde{\mathbf{z}}_t \\ \widetilde{\mathbf{z}}_{t-1} \\ \vdots \\ \widetilde{\mathbf{z}}_{t-p+1} \end{bmatrix}, \; \underset{mp\times 1}{\widetilde{\mathbf{Z}}_t} = \begin{bmatrix} \widetilde{\mathbf{z}}_{t-1} \\ \widetilde{\mathbf{z}}_{t-2} \\ \vdots \\ \widetilde{\mathbf{z}}_{t-p} \end{bmatrix}, \; \underset{mp\times 1}{\mathbf{U}_t} = \begin{bmatrix} \mathbf{u}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix},$$

$$\underset{mp\times mp}{\mathbf{F}} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 & \mathbf{\Phi}_3 & \cdots & \mathbf{\Phi}_{p-1} & \mathbf{\Phi}_p \\ \mathbf{I}_m & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_m & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_m & \mathbf{0} \end{bmatrix}.$$

It is easily seen that

$$\widetilde{\mathbf{Z}}_{t+h} = \mathbf{F}^h\widetilde{\mathbf{Z}}_t + \sum_{j=0}^{h-1} \mathbf{F}^j\mathbf{U}_{t+h-j}.$$

Therefore, we have

$$\widetilde{\mathbf{z}}_{t+h} = \mathbf{J}\mathbf{F}^h\widetilde{\mathbf{Z}}_t + \sum_{j=0}^{h-1} \mathbf{J}\mathbf{F}^j\mathbf{U}_{t+h-j} = \mathbf{J}\mathbf{F}^h\widetilde{\mathbf{Z}}_t + \sum_{j=0}^{h-1}\left(\mathbf{J}\mathbf{F}^j\mathbf{J}'\right)\mathbf{u}_{t+h-j},$$

and

$$E_t\left(\widetilde{\mathbf{z}}_{t+h}\right) = \mathbf{J}\mathbf{F}^h\widetilde{\mathbf{Z}}_t,$$

where $\underset{m\times mp}{\mathbf{J}} = (\mathbf{I}_m, \mathbf{0}, \ldots, \mathbf{0})$ is a selection matrix.

In the case of the cointegrating VAR system with $I(1)$ variables, the eigenvalues of the underlying VAR model are either on or inside of the unit circle, and thus we have

$$\mathbf{z}_{st}^{P} = \lim_{h\to\infty} E_t\left(\widetilde{\mathbf{z}}_{t+h}\right) = \mathbf{J}\mathbf{F}^{\infty}\widetilde{\mathbf{Z}}_t, \tag{10.22}$$

where $\mathbf{F}^{\infty}$ is the limit of $\mathbf{F}^h$ as $h \to \infty$. In the case where all the variables in the VAR are stationary, $\mathbf{F}^{\infty} = \mathbf{0}$ and $\mathbf{z}_t$ will have no stochastic trend components. But in the general case where $\mathbf{z}_t$ contains $I(1)$ variables (and possibly cointegrated), $\mathbf{F}^{\infty}$ tends to a finite non-zero matrix and the overall trend component of $\mathbf{z}_t$ will be given by

$$\mathbf{z}_t^{P} = \lim_{h\to\infty} E_t\left(\mathbf{z}_{t+h} - \mathbf{g}h\right) = \mathbf{g}_0 + \mathbf{g}t + \mathbf{J}\mathbf{F}^{\infty}\widetilde{\mathbf{Z}}_t. \tag{10.23}$$

The cycle or the transitory component of z_t is then defined simply as

$$z_t^C = z_t - z_t^P.$$

In the applications below we set the vector of intercepts, g_0, so that the cyclical components have mean zero.

10.3.3 *An application to the UK model*

To compute the decomposition described above, all the required parameters can be estimated from the maximum likelihood estimates of the underlying VEC model, except for $\mathbf{g}$. Note that under Case IV, the estimation of the cointegrating VAR yields an estimate of $\boldsymbol{\beta}'\boldsymbol{\gamma}$, and $\boldsymbol{\gamma}$ cannot be separately identified from $\boldsymbol{\beta}$ in the presence of cointegration. But, noting that Δz_t is stationary with mean $\mathbf{g}$, we can estimate $\mathbf{g}$ by estimating

$$\Delta z_t = \mathbf{g} + \boldsymbol{\vartheta}_t, \qquad (10.24)$$

subject to the restrictions $\boldsymbol{\beta}'\mathbf{g} = \boldsymbol{\beta}'\boldsymbol{\gamma}$ with $\boldsymbol{\beta}'\boldsymbol{\gamma}$ given by the maximum likelihood estimates, say $\widehat{\boldsymbol{\beta}'\boldsymbol{\gamma}}$. A consistent estimate of $\mathbf{g}$ can be obtained by application of the SURE procedure to (10.24) subject to the restrictions, $\boldsymbol{\beta}'\mathbf{g} = \widehat{\boldsymbol{\beta}'\boldsymbol{\gamma}}$. A more efficient estimator can be obtained by exploiting the serial correlation properties of $\boldsymbol{\vartheta}_t$ as well.

In the case of the UK model where

$$z_t = \left(p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*\right)',$$

$$\mathbf{g} = (g_o, g_e, g_{r^*}, g_r, g_{\Delta p}, g_y, g_{p-p^*}, g_{h-y}, g_{y^*})'$$

and

$$\hat{\boldsymbol{\beta}}'\mathbf{g} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -56.098 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g_o \\ g_e \\ g_{r^*} \\ g_r \\ g_{\Delta p} \\ g_y \\ g_{p-p^*} \\ g_{h-y} \\ g_{y^*} \end{pmatrix}$$

$$= \begin{pmatrix} g_e - g_{p-p^*} \\ g_{r^*} - g_r \\ g_y - g_{y^*} \\ -56.098 g_r \\ g_r - g_{\Delta p} \end{pmatrix}.$$

Also, the estimated version of the model yields

$$\widehat{\boldsymbol{\beta}'\boldsymbol{\gamma}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -0.007300 \\ 0 \end{pmatrix}.$$

This yields the following restrictions

$$g_e = g_{p-p^*}, \; g_{r^*} = g_r, \; g_y = g_{y^*}, \; g_r = 0.007300/56.098, \text{ and } g_r = g_{\Delta p}.$$

The presence of a linear trend in the money demand equation implies a very small but a non-zero value for g_r (= 0.00013). We decided to set $g_r = 0$ as it is unlikely that the trend in the money demand equation could prevail in the very long run. Therefore, we estimate $\mathbf{g}$ subject to the following restrictions:

$$g_e = g_{p-p^*}, \; g_y = g_{y^*} \quad \text{and} \quad g_{\Delta p} = g_r = g_{r^*} = 0,$$

and obtained the estimate,

$$\hat{\mathbf{g}} = (0.018557, 0.002179, 0, 0, 0, 0.005256, 0.002179, -0.007287, 0.005256)'.$$

Using these estimates together with the estimated parameters from equation (10.15), we can construct a permanent/transitory or trend/cycle decomposition.

To illustrate the decomposition, Figures 10.11–10.18 plot a range of transitory and permanent components for some selected endogenous variables of the model. Figure 10.11 plots the actual series and the GRW permanent component of domestic output y_t. The GRW permanent component of UK GDP is not as smooth as other trend estimates and it is also subject to some fairly significant downward, as well as upward, shifts at various points in the sample. However, the important point here is that, by construction, the permanent component of y_t is perfectly correlated with the permanent component of y_t^*. This interesting feature, that the permanent stochastic components of the variables that cointegrate and co-trend should be perfectly correlated, also applies to the pairs r_t and r_t^*; and r_t and Δp_t (once the long-run theory restrictions are imposed).

In Figure 10.12, we plot the transitory component of the UK output alongside the transitory component of inflation so that we might look at the cyclical movements in the inflation–output trade-off.[22] Given our

[22] To provide a clearer picture of the relationships we have normalised all the transitory components so that they have mean zero over the sample period under consideration.

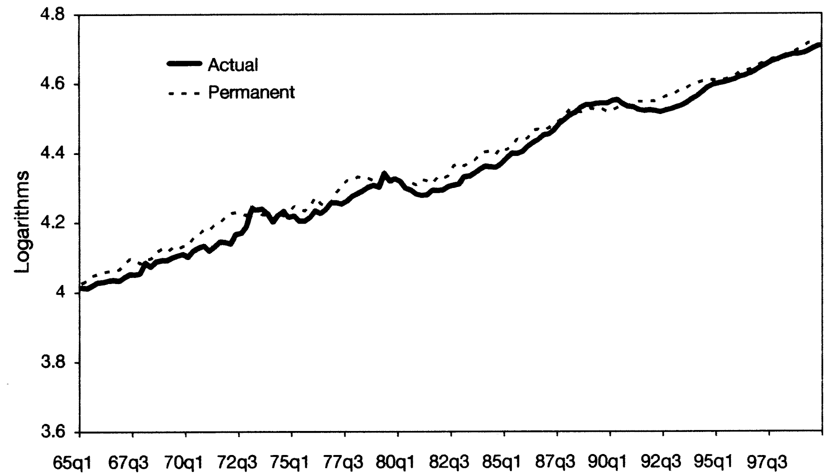


Figure 10.11 Actual UK output (y_t) and the GRW permanent component.

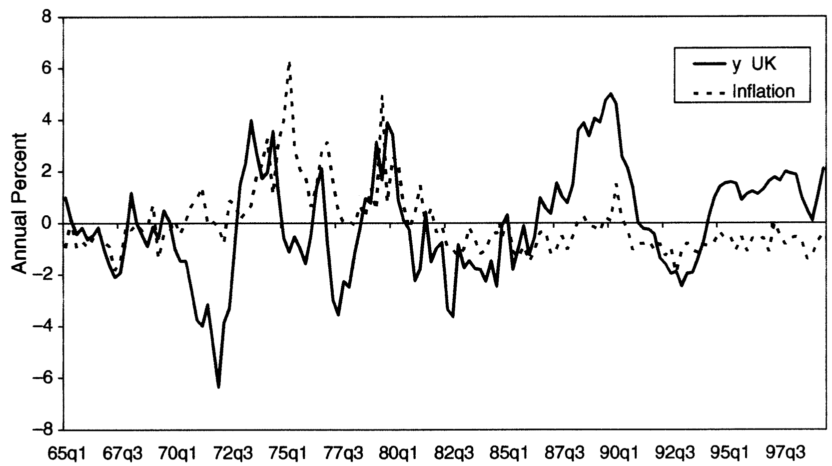


Figure 10.12 GRW transitory components of UK output and inflation: y_t and Δp_t.

explicit multivariate approach to detrending, the figure automatically takes account of the interactions between the variables when analysing the nature of the relationship between output and inflation. As the figure shows, there is a limited degree of positive co-movement between inflation and output, with a correlation coefficient of just 0.14, which is

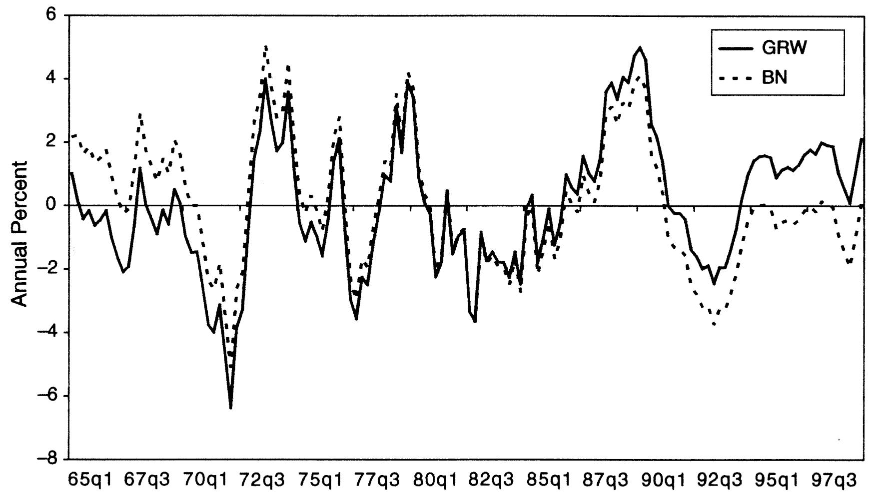


Figure 10.13 GRW and BN transitory components of UK output: y_t.

consistent with a demand-shock view of the business cycle, but the cyclical dependence seems rather weak. A sub-sample analysis could well be more revealing here.

Figure 10.13 plots the GRW and BN transitory components for y_t. There is a clear degree of consensus between the two series, reflected in a correlation coefficient of 0.827, although the BN transitory component suggests higher growth in the early 1960s but lower growth in the late 1990s. In fact, there is a high degree of co-movement between the GRW and BN transitory components for all the endogenous variables, with correlation coefficients of 1.00, 0.965, 0.980, 0.997, 0.999, 0.901 and 0.929 for $e_t, r_t^*, r_t, \Delta p_t, p_t - p_t^*, h_t - y_t$ and y_t^*, respectively. The two decompositions yield the same result for the exchange rate due to its random walk property.

Figure 10.14 plots the GRW transitory components of y_t and y_t^*.[23] The most noticeable feature is the limited degree of co-movement exhibited by the two series, with a correlation coefficient of 0.28, particularly in the late 1960s, early 1980s and late 1990s. For a comparison, Figures 10.15 and 10.16 plot the BN and Hodrick–Prescott (HP) transitory components of the same two series.[24] Both the GRW and BN decompositions show

[23] To make any meaningful comparison between the transitory components of variables with different levels we require that each variable be mean zero and hence we first de-mean (using the sample mean) the transitory components.

[24] The HP filter uses a smoothing parameter value of 1600.

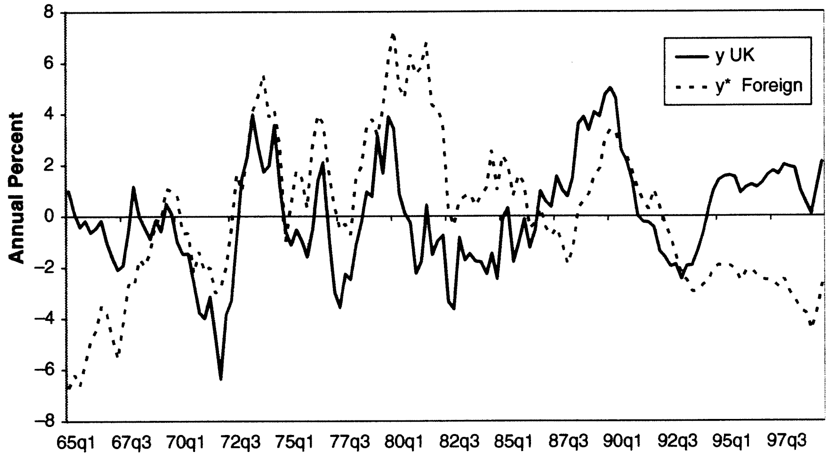


Figure 10.14 GRW transitory components of UK and foreign output: y_t and y_t^*.

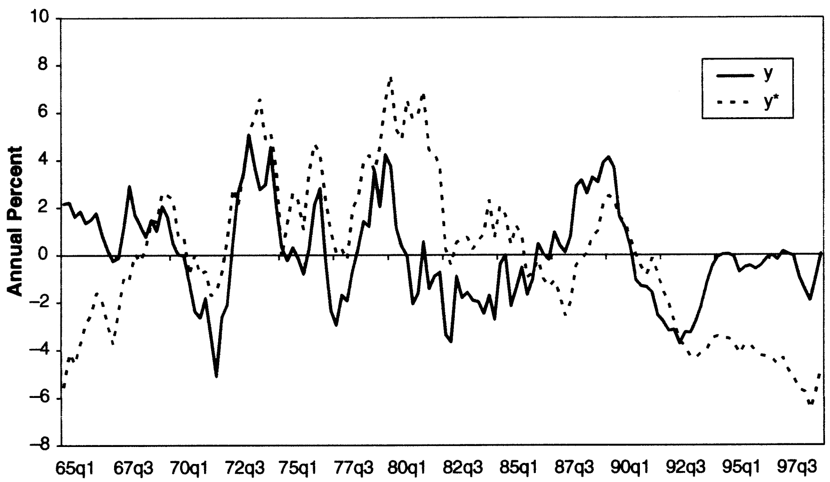


Figure 10.15 BN transitory components of UK and foreign output: y_t and y_t^*.

a low degree of co-movement between the transitory components of UK and foreign output, although the correlation coefficient is slightly higher at 0.38 (as compared to 0.28) in the case of the BN decomposition. The degree of co-movement between the HP transitory y_t and y_t^* components is noticeably higher, yielding a correlation coefficient of

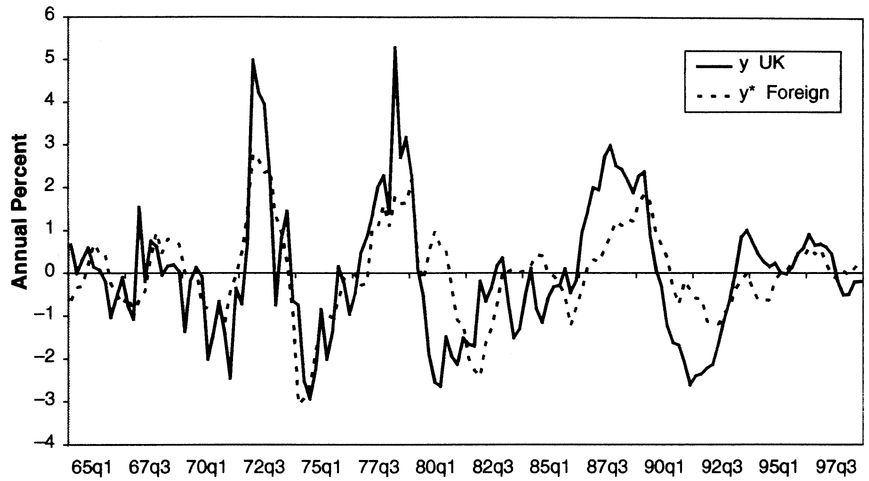


Figure 10.16 Hodrick–Prescott transitory components of UK and foreign output: y_t and y_t^*.

0.69. A tentative implication therefore is that the univariate HP filter overstates the degree of co-movement between the transitory components (*i.e.* induces highly synchronised business cycle for the UK relative to the rest of the world); the multivariate VECM, which imposes the long-run restrictions, does not support such a high degree of short-run synchronisations.

Figure 10.17 plots the transitory components of r_t and r_t^*. As in the case of y_t and y_t^*, the restriction that the permanent components are perfectly correlated is imposed. Here, our zero growth rate assumption on r_t and r_t^* implies that the change in the permanent component of these series is determined purely by the stochastic part (where the deterministic part is fixed at its initial value, see Figure 10.6). The co-movement between domestic and foreign interest rates is positive and reasonably strong, with a correlation coefficient of 0.5. The transitory component of domestic interest rates is more volatile but part of this difference reflects the fact that foreign interest rates are measured as the average of interest rates in a number of countries and hence is likely to be relatively smooth. Figure 10.18 plots the actual r_t series alongside its permanent component. We see here that a large part of movements in r_t could be defined as transitory, with the permanent component showing little variations.

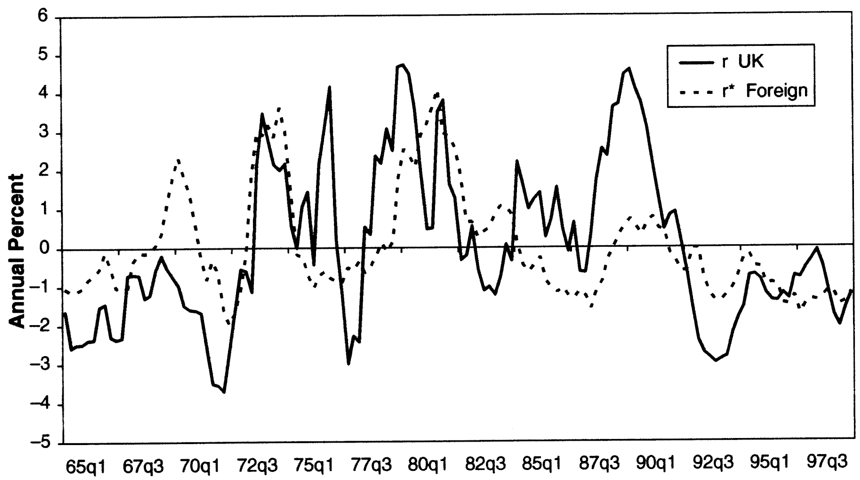


Figure 10.17 GRW transitory components of UK and foreign interest rates: r_t and r_t^*.

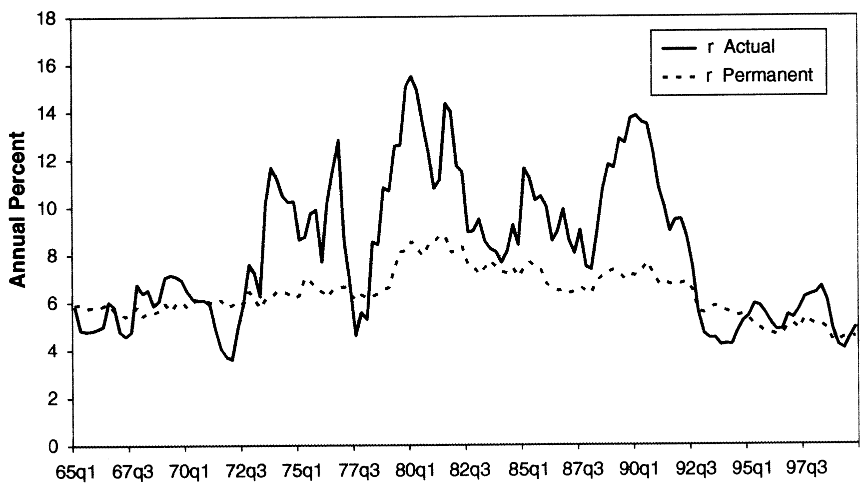


Figure 10.18 Actual and GRW permanent component of UK interest rates: r_t.

10.4 Concluding remarks

In this chapter, we have illustrated how we might use our modelling approach in the analysis of shocks through the use of both GIR and structural identified impulse responses. The cointegrating VAR model is not

only able to provide a reasonably flexible characterisation of the short-run dynamics of the macroeconomy but, by making explicit the link with the long-run relationships suggested by economic theory, it also enables us to consider explicitly the links between 'structural' and 'observable' shocks and provides an appropriate treatment of the analysis of the model's dynamic properties. Our use of Persistence Profiles show directly the dynamic effects of system-wide shocks to the equilibrium relations. Hence, for example, the estimated profiles illustrate clearly the differential speeds of response to the disequilibria involving financial variables compared to those involving real magnitudes. Our use of the Generalised Impulse Response functions allows us to investigate the effects of specific shocks to particular equations in the model, gaining insight on the dynamic response to particular events without the use of arbitrary orthogonalisation assumptions and without losing sight of the relationships that exist between the innovations and the underlying economic model.

There are, of course, a variety of other impulse response analyses that can be conducted using our model. Bernanke *et al.* (1997) and Cochrane (1998), for example, suggest counter-factual exercises aimed at distinguishing the effects of systematic changes to monetary policy rules from those that influence the economy's intrinsic propagation mechanisms. As a second example, one might consider impulse response functions associated with a once-and-for-all shift in the intercept of the interest rate equation.[25] This would help, for example, identify the time profile of the effects of shifts in the target variables for output growth or inflation reduction, *i.e.* $\Delta \mathbf{w}^{\dagger}$ in (5.12) of Chapter 5. The model presented in this book provides a potentially fruitful framework with which to investigate these and other counter-factual policy exercises.

Our discussion of the trend/cycle decomposition also highlights the importance of allowing for the long-run restrictions in identification and estimation of the transitory components. For example, by abstracting from the long-run relations that might exist in cross-country outputs, the use of univariate approaches such as the Hodrick–Prescott filter is likely to over state the degree of business cycle synchronisations that exist across countries.

[25] This is equivalent to a GIR function with certain zero restrictions on the error correction covariances.