5

An economic theory of the short run

The modelling strategy described in the previous chapter has the twofold advantage that it is capable of accommodating the relationships that exist between variables in the long run as suggested by an explicit macroeconomic theory, and that the estimated model reflects parsimoniously the complex dynamic relationships that exist between variables at shorter horizons. In this chapter, we consider the problem of identification of contemporaneous relationships, and in particular discuss the identification of the monetary policy shocks and the associated impulse responses.

Our view is that the economic theory of the long run described in the previous chapter can be held with some degree of confidence (and, as we shall see, we can judge whether this confidence is justifiable through a formal statistical test of the over-identifying restrictions suggested by the theory). We are less confident in the economic theory of the short run that we shall present. The theory that is described is 'tentative' in the sense described in Section 3.2.3 of Chapter 3, relying to a large extent on *a priori* views on the sequencing of decisions and the institutional detail of the decision-making process. As we noted in that discussion, there is no consensus on the appropriate form of short-run restrictions in macroeconometric models, and we recognise the frailty of the theory elaborated here. On the other hand, some identifying restrictions are essential if we are to investigate specific types of shock (*e.g.* monetary policy shocks) and we present a theory which seems the most reasonable to us and which involves the imposition of the minimum possible identifying structure.

Before describing the proposed theory of the short run, it is worth briefly summarising the specification of the macromodel derived in the previous chapter. Combining the oil price equation (4.44) and the conditional model explaining the remaining eight variables of interest (4.46), we obtain the reduced form specification

$$\Delta z_t = \begin{pmatrix} \Delta p_t^o \\ \Delta y_t \end{pmatrix} = a - \alpha\left[\beta' z_{t-1} - b_1(t-1)\right] + \sum_{i=1}^{p-1} \Gamma_i \Delta z_{t-i} + v_t, \quad (5.1)$$

where

$$z_t = (p_t^o, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*)'.$$

and

$$a = \begin{pmatrix} \delta_o \\ \psi_{yo}\delta_o + a_y - \alpha_y b_0 \end{pmatrix}, \alpha = \begin{pmatrix} 0 \\ \alpha_y \end{pmatrix}, \Gamma_i = \begin{pmatrix} \delta_{oi} \\ \psi_{yo}\delta_{oi} + \Gamma_{yi} \end{pmatrix},$$

$$v_t = \begin{pmatrix} 1 & 0 \\ \psi_{yo} & I_8 \end{pmatrix}\begin{pmatrix} u_{ot} \\ u_{yt} \end{pmatrix} = \begin{pmatrix} u_{ot} \\ \psi_{yo}u_{ot} + u_{yt} \end{pmatrix}.$$

Under standard assumptions, all the reduced form coefficients can be consistently estimated from the ML estimates of the parameters of the conditional model (4.46), and the OLS regression of the oil price equation (4.44). In particular, the vector of the reduced form errors v_t, and its covariance matrix, Σ, can be estimated consistently from the reduced form parameters. The 'structural' VECM associated with the *long-run structural* macroeconometric model defined by (4.44) and (4.46) can be written as:

$$A\ \Delta z_t = \tilde{a} - \tilde{\alpha}\left[\beta' z_{t-1} - b_1(t-1)\right] + \sum_{i=1}^{p-1} \tilde{\Gamma}_i \Delta z_{t-i} + \varepsilon_t, \quad (5.2)$$

where A represents the 9×9 matrix of contemporaneous structural coefficients, $\tilde{a} = Aa$, $\tilde{\alpha} = A\alpha$, $\tilde{\Gamma}_i = A\Gamma_i$, and $\varepsilon_t = Av_t$ are the associated structural shocks which are serially uncorrelated and have zero means and the positive definite variance covariance matrix, $\Omega = A\Sigma A'$. So far, the only restrictions relating to the contemporaneous dependencies in (5.2) are those imposed on the first row of A through the assumption that oil prices are long-run forcing.

This is precisely the set-up described in Chapter 3. For exact identification of *all* the structural coefficients in our model, $9^2 = 81$ restrictions are required to be imposed on A, Ω, or, more unusually, on $\tilde{\Pi}$ or the $\tilde{\Gamma}_i$, $i = 1, \ldots, p-1$. Alternatively, it might be possible to identify a specific subset of the structural parameters and/or shocks imposing fewer than

81 restrictions. The restrictions might be motivated by a fully articulated DSGE model of the entire economy; or they might be motivated via a more limited linear–quadratic optimisation model involving adjustment costs; or they might be motivated by a less formal theory.

In our macroeconometric modelling, we do not address the problem of the identification of all the structural shocks. This is because such an exercise would require a detailed specification of all the structural relations and the underlying decisions as well as the mechanisms by which expectations are formed. Rather, we focus on the identification of the effects of oil and monetary policy shocks only, concentrating on the decisions made by monetary authorities and agents in the financial sector. These decisions are discussed with reference to the optimisation problem faced by the monetary authorities (modelled as an LQ model with adjustment costs) combined with 'tentative' theory on the sequencing of decisions and the institutional context. The decisions of the monetary authorities and agents in the financial sector are made taking as given the decision rules of the private agents as embodied in the structural relations bearing on output and inflation in the structural VECM, (5.2). Whilst it might be desirable to consider the identification of other shocks, such as technology or demand shocks, this does not seem to be necessary for the analysis of monetary shocks. As we shall show, the identification of other shocks in the economy can be dealt with using the generalised impulse response approach developed in Pesaran and Shin (1998).

5.1 Modelling monetary policy

5.1.1 *The monetary authority's decision problem*

For identification of the monetary policy shocks, we first need to formally articulate the decision problem of the monetary authorities. We assume that, at the start of each period, the monetary authorities try to influence the market interest rate, r_t, by setting the base rate, r_t^b that they have under their direct control.[1] We then impose a structure on the sequencing of decisions, assuming that the difference between the market rate and the base rate, the term premium, is determined by unanticipated factors such as oil

[1] In the case of the UK, it is reasonable to assume that the Bank of England determines r_t^b, the price of liquidity. This is a characteristic of the institutional framework in the UK and contrasts with the US (as described in Gordon and Leeper (1994), for example).

price shocks, unexpected changes in foreign interest rates and exchange rates. This is justified on the grounds that these four variables, *i.e.* p_t^o, e_t, r_t^*, and $r_t - r_t^b$, are likely to be contemporaneously determined in the market place on a daily (even intra-daily) basis. The remaining variables, y_t, y_t^*, Δp_t, $p_t - p_t^*$, and $h_t - p_t$, are much less frequently observed, often with relatively long delays, and their contemporaneous values can be reasonably excluded from the determination of the term premium, $r_t - r_t^b$. However, as we shall see below, lagged values of these variables can still affect the term premium.

To help explain the structure imposed by the sequencing of decisions described above, and to aid the subsequent exposition, it is worth introducing some notation to explicitly denote the structural parameters of interest. So, let us distinguish between three sets of variables: the first set consists of the four variables determined contemporaneously with r_t^b, namely, p_t^o, e_t, r_t^*, and r_t; the second set, denoted $\mathbf{w}_t$, contains output and inflation, which we shall assume are the variables of direct concern to the monetary authorities; and the third set, denoted $\mathbf{q}_t$, consists of the remaining variable in the model. Hence, we have the following partitioning of the variables:

$$\mathbf{z}_t = \begin{pmatrix} p_t^o \\ \mathbf{y}_t \end{pmatrix}, \quad \mathbf{y}_t = \begin{pmatrix} e_t \\ r_t^* \\ r_t \\ \mathbf{w}_t \\ \mathbf{q}_t \end{pmatrix}, \quad \mathbf{w}_t = \begin{pmatrix} y_t \\ \Delta p_t \end{pmatrix}, \quad \mathbf{q}_t = \begin{pmatrix} p_t - p_t^* \\ h_t - y_t \\ y_t^* \end{pmatrix}.$$

The assumption that oil prices are determined as in (4.44) and that e_t, r_t^*, and r_t are determined prior to the variables in $\mathbf{w}_t$ and $\mathbf{q}_t$ imposes a structure on the parameter matrices of (5.2) as follows:[2]

$$\tilde{\mathbf{a}} = \begin{pmatrix} \delta_0 \\ \tilde{a}_e \\ \tilde{a}_{r^*} \\ \tilde{a}_r \\ \tilde{\mathbf{a}}_w \\ \tilde{\mathbf{a}}_q \end{pmatrix}, \quad \tilde{\boldsymbol{\alpha}} = \begin{pmatrix} 0 \\ \tilde{\alpha}_e \\ \tilde{\alpha}_{r^*} \\ \tilde{\alpha}_r \\ \tilde{\boldsymbol{\alpha}}_w \\ \tilde{\boldsymbol{\alpha}}_q \end{pmatrix}, \quad \tilde{\boldsymbol{\Gamma}}_i = \begin{pmatrix} \delta_i \\ \tilde{\boldsymbol{\Gamma}}_{e,i} \\ \tilde{\boldsymbol{\Gamma}}_{r^*,i} \\ \tilde{\boldsymbol{\Gamma}}_{r,i} \\ \tilde{\boldsymbol{\Gamma}}_{w,i} \\ \tilde{\boldsymbol{\Gamma}}_{q,i} \end{pmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{o,t} \\ \varepsilon_{e,t} \\ \varepsilon_{r^*,t} \\ \varepsilon_{r,t} \\ \boldsymbol{\varepsilon}_{w,t} \\ \boldsymbol{\varepsilon}_{q,t} \end{pmatrix},$$

[2] In fact, for expositional purposes, we make the further assumption that exchange rates are determined prior to foreign interest rates in what follows.

and

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -\tilde{\psi}_e & 1 & 0 & 0 & 0 & 0 \\ -\tilde{\psi}_{r^*} & a_{r^*e} & 1 & 0 & 0 & 0 \\ -\tilde{\psi}_r & a_{re} & a_{rr^*} & 1 & 0 & 0 \\ -\tilde{\boldsymbol{\psi}}_w & \mathbf{A}_{we} & \mathbf{A}_{wr^*} & \mathbf{A}_{wr} & \mathbf{A}_{ww} & \mathbf{A}_{wq} \\ -\tilde{\boldsymbol{\psi}}_q & \mathbf{A}_{qe} & \mathbf{A}_{qr^*} & \mathbf{A}_{qr} & \mathbf{A}_{qw} & \mathbf{A}_{qq} \end{pmatrix}.$$

Note that $\mathbf{A}$ has a lower triangular structure only in the case of the variables, p_{ot}, e_t, r_t^* and r_t that are primarily market determined on a daily basis. However, $\mathbf{A}$ taken as a whole is not triangular. The parameters of the corresponding reduced form equations given in (5.1) can be similarly defined, using

$$\mathbf{a} = \begin{pmatrix} \delta_0 \\ a_e \\ a_{r^*} \\ a_r \\ \mathbf{a}_w \\ \mathbf{a}_q \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_e \\ \alpha_{r^*} \\ \alpha_r \\ \boldsymbol{\alpha}_w \\ \boldsymbol{\alpha}_q \end{pmatrix}, \quad \boldsymbol{\Gamma}_i = \begin{pmatrix} \delta_i \\ \boldsymbol{\Gamma}_{e,i} \\ \boldsymbol{\Gamma}_{r^*,i} \\ \boldsymbol{\Gamma}_{r,i} \\ \boldsymbol{\Gamma}_{w,i} \\ \boldsymbol{\Gamma}_{q,i} \end{pmatrix}, \quad \mathbf{v}_t = \begin{pmatrix} v_{o,t} \\ v_{e,t} \\ v_{r^*,t} \\ v_{r,t} \\ \mathbf{v}_{w,t} \\ \mathbf{v}_{q,t} \end{pmatrix}.$$

Given the above relationships between the structural and reduced form parameters, we can return to the monetary authorities' decision problem. The sequencing of decisions assumes that the term premium equation has the following form:

$$\begin{aligned} r_t - r_t^b = {} & \rho_{b,t-1} + a_{rr^*}\left[r_t^* - E\left(r_t^* \mid \Im_{t-1}\right)\right] \\ & + a_{re}\left[e_t - E\left(e_t \mid \Im_{t-1}\right)\right] \\ & + \tilde{\psi}_r\left[p_t^o - E\left(p_t^o \mid \Im_{t-1}\right)\right] + \varepsilon_{rt}, \end{aligned} \tag{5.3}$$

where $\Im_{t-1}$ is the information set of the monetary authorities at the end of $t-1$, $\rho_{b,t-1}$ is the predictable component of the term premium, which could be a general function of one or more elements in the information set $\Im_{t-1}$, r_t^b is the systematic component of monetary policy, and ε_{rt} is the monetary policy shock.[3] Hence, in addition to ε_{rt}, the unexpected component of the term premium is assumed to vary linearly with the

[3] In the absence of a fully specified model of the markets which link the monetary authorities with other financial markets, we associate monetary policy shocks with innovations to short-term interest rates. It is worth noting that several researchers, including Sims (1992) and Bernanke *et al.* (1997), have argued that, within this context, innovations to short-term interest rates are preferable to using innovations in monetary aggregates.

unanticipated changes in oil prices, the exchange rate, and the foreign interest rate. We shall assume that the monetary policy shocks, ε_{rt}, satisfy the following standard orthogonality condition:

$$E\left(\varepsilon_{rt} \mid \Im_{t-1}\right) = 0,$$

and the associated time-varying expected term premium is given by

$$E\left(r_t - r_t^b \mid \Im_{t-1}\right) = \rho_{b,t-1}.$$

The term premium equation of (5.3) can be written equivalently as

$$\Delta r_t = r_t^b - r_{t-1} + \rho_{b,t-1} + a_{rr^*}\left[\Delta r_t^* - E\left(\Delta r_t^* \mid \Im_{t-1}\right)\right] + a_{re}\left[\Delta e_t - E\left(\Delta e_t \mid \Im_{t-1}\right)\right] + \widetilde{\psi}_r\left[\Delta p_t^o - E\left(\Delta p_t^o \mid \Im_{t-1}\right)\right] + \varepsilon_{rt}. \quad (5.4)$$

Under expectations formation mechanisms consistent with the reduced form VECM (5.1), the expectational variables $E\left(\Delta r_t^* \mid \Im_{t-1}\right)$, $E\left(\Delta e_t \mid \Im_{t-1}\right)$, and $E\left(\Delta p_t^o \mid \Im_{t-1}\right)$ can be solved in terms of the error correction terms, $\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)$, and $\Delta \mathbf{z}_{t-i}$, $i = 1, 2, \ldots, p-1$, to yield:

$$\Delta r_t - a_{rr^*}\Delta r_t^* - a_{re}\Delta e_t - \widetilde{\psi}_r \Delta p_t^o = r_t^b - r_{t-1} + \rho_{b,t-1} + \boldsymbol{\phi}_r^*\left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\phi}_{zi}^* \Delta \mathbf{z}_{t-i} + \varepsilon_{rt}, \quad (5.5)$$

where the parameters $\boldsymbol{\phi}_r^*$ and $\boldsymbol{\phi}_{zi}^*$ are functions of a_{rr^*}, a_{re}, a_{ro} and the coefficients in the rows of (5.1) associated with the equations for Δr_t^*, Δe_t and Δp_t^o. This relates the change in the market rate to the base rate, to changes in the contemporaneously determined variables r_t^*, e_t and p_t^o, and to past information. We turn now to the determination of the base rate.

5.1.2 *The derivation of the base rate*

For the derivation of r_t^b, we follow the literature on inflation targeting and assume that it is derived as the solution to the following optimisation problem:[4]

$$\min_{r_t^b} \left\{E\left[C(\mathbf{w}_t, r_t) \mid \Im_{t-1}\right]\right\}, \quad (5.6)$$

[4] For recent accounts, see Blinder (1998), Bernanke *et al.* (1999) and Svensson (1999), for example.

where $C(\mathbf{w}_t, r_t)$ is the loss function of the monetary authorities, assumed to be quadratic so that

$$C(\mathbf{w}_t, r_t) = \frac{1}{2}(\mathbf{w}_t - \mathbf{w}_t^{\dagger})'\mathbf{Q}(\mathbf{w}_t - \mathbf{w}_t^{\dagger}) + \frac{1}{2}\theta(r_t - r_{t-1})^2, \quad (5.7)$$

where $\mathbf{w}_t = (y_t, \Delta p_t)'$ and $\mathbf{w}_t^{\dagger} = (y_t^{\dagger}, \pi_t^{\dagger})'$ are the target variables and their desired values, respectively. Since the target variables are both assumed to be $I(1)$ in our model, the desired target values, $\mathbf{w}_t^{\dagger}$ also need to be $I(1)$ and cointegrated with $\mathbf{w}_t$ for the optimisation problem to be controllable; otherwise the solution to the control problem will not be consistent with the assumed structural model. The 2×2 matrix $\mathbf{Q}$ characterises the authorities' short term trade-off between output growth and a reduction in the rate of inflation. The final term in (5.7) is intended to capture the institutional and political costs of changes to the interest rate.

The solution to the above optimisation problem requires the specification of a model linking the target variables, $\mathbf{w}_t$, to the policy instrument, r_t^b. Within our framework, such a model can be derived as a sub-system of the general long-run structural model specified in (5.2), with (5.5) as its structural interest rate equation. Subject to this sub-model, we can derive the first-order condition for the minimisation of (5.6), which is easily seen to be given by[5]

$$E\left[\left(\frac{\partial \mathbf{w}_t}{\partial r_t^b}\right)' \mathbf{Q}(\mathbf{w}_t - \mathbf{w}_t^{\dagger}) + \theta\left(\frac{\partial r_t}{\partial r_t^b}\right)\Delta r_t \mid \Im_{t-1}\right] = 0. \quad (5.8)$$

The outcome of this optimisation is a feedback rule (or reaction function) explaining the determination of r_t^b given the information available to the authorities. The parameters of the feedback rule depend on the preference parameters and the parameters of the econometric model. A complete description of the optimisation and the relationship between the parameters of the feedback rule is given in Appendix A. Stated simply, though, the feedback rule is of the following form:

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \boldsymbol{\Upsilon}'\left[E(\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0) - \mathbf{w}_t^{\dagger}\right] \quad (5.9)$$

where $\boldsymbol{\Upsilon}'$ is a function of the parameters of the econometric model and of the preference parameters of the monetary authorities, $\mathbf{Q}$ and θ. Here,

[5] The problem of the credibility of the monetary policy, discussed in the literature by Barro and Gordon (1983), Rogoff (1985) and, more recently, by Svensson (1997), for example, is resolved in our application by the common knowledge assumption and the information symmetry. It is also worth noting that the extension of the decision problem to an intertemporal setting will complicate the analysis but does not materially alter our main conclusions.

$E(\mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0)$ indicates the value of the target variables that would occur in time t in the absence of any interest rate adjustment and in the absence of any structural innovations to the economic system. The expression $E(\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0)$ is a function of information available to the monetary authorities and, as demonstrated in Appendix A, (5.9) can be written in more detail as:

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \boldsymbol{\phi}^{\diamond} - \boldsymbol{\Upsilon}' \left(\mathbf{w}_t^{\dagger} - \mathbf{w}_{t-1} \right) + \boldsymbol{\phi}_r^{\diamond} \left[\boldsymbol{\beta}' \mathbf{z}_{t-1} - \mathbf{b}_1 (t-1) \right] + \sum_{i=1}^{p-1} \boldsymbol{\phi}_{zi}^{\diamond} \Delta \mathbf{z}_{t-i}, \tag{5.10}$$

where $\boldsymbol{\phi}^{\diamond}$, $\boldsymbol{\phi}_r^{\diamond}$, and $\boldsymbol{\phi}_{zi}^{\diamond}$ are again functions of the parameters of the econometric model and of the preference parameters of the monetary authorities.

INFLATION TARGETING AND THE BASE RATE REACTION FUNCTION

The term $E(\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0)$ expresses the monetary authorities' conditional forecast of the target variables in time t assuming that they leave the base rate unchanged. The term $\left[E(\mathbf{w}_t | \Im_{t-1}, \Delta r_t^b = 0) - \mathbf{w}_t^{\dagger} \right]$ therefore has a natural interpretation as the *gap* between the desired level of the target variables and the authorities' forecast of the target variables in the absence of policy intervention. Written in this way, the reaction function of (5.9) clarifies the link between 'instrument rules' and 'target rules' in guiding monetary policy, as discussed recently in Svensson (2001, 2002), for example.

Svensson defines a 'target rule' as a commitment to set a policy instrument so as to achieve specific criteria for target variables. He contrasts this with an 'instrument rule' in which the central bank mechanically sets its instrument as a simple function of a small subset of the information available to the central bank, via a reaction function. He notes that, while most of the literature discusses central banks' behaviour in terms of reaction functions and instrument rules, few central banks have committed to mechanical instrument rate rules in practice. Rather, *inflation targeting* has been adopted as a strategy for implementing monetary policy by various countries in recent years, meaning that: (i) there is a numerical inflation target announced (either as a point target or as an acceptable range); (ii) monetary instruments are set such that the inflation forecasts of the authorities, conditional on the instrument setting, are

consistent with the target (so that the decision process can be described as 'inflation-forecast targeting'); and (iii) the authorities provide transparent and explicit policy reports presenting forecasts and are held accountable for achieving the target. Svensson argues that inflation targeting is better described and prescribed as a commitment to a targeting rule than a commitment to an instrument rule.

As it is written in (5.10), the base rate reaction function derived above could clearly form the basis of an instrument rule, linking the authorities' base interest rate decision to known information on lagged variables in a complex but mechanical formula. However, the form in (5.9) shows that, despite the complexity, the reaction function has a relatively straightforward form based on the authorities' view of conditional forecasts of the target variables expressed relative to their desired levels. Moreover, as is demonstrated in detail in Appendix A, the reaction function can readily provide the motivation for a target rule.

To demonstrate this idea in Appendix A, we substitute the base rate reaction function into the structural relationships explaining the determination of the target variables in (5.2). This shows that, when the monetary authorities are following the reaction function derived above, the target variable outcome can be written as

$$\mathbf{w}_t = (\mathbf{I}_2 - \boldsymbol{\Lambda})\, E(\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0) + \boldsymbol{\Lambda} \mathbf{w}_t^{\dagger} + \mathbf{v}_{ww,t}^{\diamond}$$

where $\boldsymbol{\Lambda}$ is a 2×2 matrix of fixed parameters and $\mathbf{v}_{ww,t}^{\diamond}$ is a composite shock generated by the structural shocks impacting on the p_t^o, e_t, r_t^* and target variables in time t. Hence, the target variable outcomes are a weighted average of the expected levels that would be achieved if the base rate is left unchanged and their desired levels, plus a random component, $\mathbf{v}_{ww,t}^{\diamond}$. The weights on the expected target variables and the desired target variables, $(\mathbf{I}_2 - \boldsymbol{\Lambda})$ and $\boldsymbol{\Lambda}$ respectively, depend on the preference parameters and the parameters of the econometric model. In particular, they reflect the relative importance of deviations of the target variables from their desired levels and the costs of changing the base rate in the monetary authorities' objective function in (5.6). Hence, in the simple case where there is only one target variable (say inflation), so that $\mathbf{A}_{wr}'$, $\mathbf{A}_{ww}'$ and $\mathbf{Q}$ in (5.2) and (5.7) are scalars, and equal to a_{wr}, 1, and q, respectively, say; then the weight is simply given by

$$\Lambda = \frac{a_{wr}^2 q}{a_{wr}^2 q + \theta}.$$

As $q/\theta \rightarrow \infty$, so that the cost of deviations of inflation from its desired level rises relative to the cost of changing the base rate, we have $\frac{a_{wr}^2 q}{a_{wr}^2 q+\theta} \rightarrow 1$ and

$$\mathbf{w}_t = \mathbf{w}_t^{\dagger} + \mathbf{v}_{ww,t}^{\diamond}. \tag{5.11}$$

Abstracting from the unpredictable structural shocks, the target variable would track the desired level precisely therefore. Hence, in the case where the costs of changing the base rate are small,[6] a commitment to an instrument rule of the form in (5.9), based on the gap between the inflation rate expected in the absence of a policy response and the desired inflation rate, is operationally equivalent to a commitment to a target rule in which policy is undertaken so as to achieve a specific desired inflation target.[7]

5.1.3 *The structural interest rate equation*

The final stage in deriving the structural interest rate equation is made in two further steps. First, we require a specification for $\mathbf{w}_t^{\dagger}$. Given that the desired target values need to be $I(1)$ and cointegrated with $\mathbf{w}_t$, a simple specification for $\mathbf{w}_t^{\dagger}$ that satisfies these requirements is given by

$$\mathbf{w}_t^{\dagger} = \begin{pmatrix} y_{t-1} + g_y^{\dagger} \\ \Delta p_{t-1} - g_\pi^{\dagger} \end{pmatrix} = \mathbf{w}_{t-1} + \mathbf{g}^{\dagger}, \tag{5.12}$$

where $\mathbf{g}^{\dagger} = (g_y^{\dagger}, -g_\pi^{\dagger})'$, $g_y^{\dagger} > 0$ is the fixed target level for output growth, and $g_\pi^{\dagger} > 0$ is the desired reduction in the rate of inflation. This specification is realistic and provides a reasonable working hypothesis with which to complete an empirical model. A discussion of alternative specifications for $\mathbf{w}_t^{\dagger}$ is considered below.

[6] Svensson, for example, is sceptical about the importance of these issues; see Section 5.6 of Svensson (2002).

[7] This is a desired time-t inflation target, given that it is contemporaneous time-t inflation that enters into the objective function. We consider the case where future inflation enters into the objective function below.

Finally, substitution of (5.9) in (5.5) yields the following structural interest rate equation

$$\begin{aligned} &\Delta r_t - a_{rr^*}\Delta r_t^* - a_{re}\Delta e_t - \widetilde{\psi}_r \Delta p_t^o \\ &= \boldsymbol{\Upsilon}'\left[E(\mathbf{w}_t \mid \Im_{t-1}, \Delta r_t^b = 0)\right] - \mathbf{w}_t^{\dagger} \\ &+ \boldsymbol{\phi}_r^*\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\phi}_{z,i}^* \Delta \mathbf{z}_{t-i} + \varepsilon_{rt}, \end{aligned} \tag{5.13}$$

which illustrates clearly the need for $\mathbf{w}_t$ and $\mathbf{w}_t^{\dagger}$ to be cointegrated.[8] Further, using (5.12) and (5.10), we obtain

$$\begin{aligned} &\Delta r_t - a_{rr^*}\Delta r_t^* - a_{re}\Delta e_t - \widetilde{\psi}_r \Delta p_t^o \\ &= \left(\phi^{\diamond} - \boldsymbol{\Upsilon}'\mathbf{g}^{\dagger}\right) + (\boldsymbol{\phi}_r^{\diamond} + \boldsymbol{\phi}_r^*)\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_1(t-1)\right] \\ &+ \sum_{i=1}^{p-1} (\boldsymbol{\phi}_{zi}^{\diamond} + \boldsymbol{\phi}_{z,i}^*) \Delta \mathbf{z}_{t-i} + \varepsilon_{rt}, \end{aligned} \tag{5.14}$$

where ε_{rt} is the monetary policy shock, as discussed above. Note that this structural equation is consistent with the long-run properties of the general structural model specified in (5.2). In particular, although changes in the preference parameters of the monetary authorities affect the magnitude and the speed with which interest rates respond to economic disequilibria, such changes have no effect on the long-run coefficients, $\boldsymbol{\beta}$, that are determined by general arbitrage conditions. It is also easily shown that, while changes in the trade-off parameter matrix, $\mathbf{Q}$, affect all the short-run coefficients of the interest rate equation, changes to the desired target values, $g_y^{\dagger}$ and $g_\pi^{\dagger}$, affect only the intercept term, $\phi^{\diamond} - \boldsymbol{\Upsilon}'\mathbf{g}^{\dagger}$.[9]

[8] For example, the interest rate decision would not have been consistent with the assumed underlying structural model if a fixed inflation rate target were specified; *i.e.* if it was required that $\pi_t^{\dagger} = g_\pi^{\dagger}$ as opposed to our specification where $\pi_t^{\dagger} = \Delta p_{t-1} - g_\pi^{\dagger}$.

[9] These properties could form the basis of an empirical test of recent developments in the conduct of monetary policy in the UK. But such an analysis is beyond the scope of the present work.

5.2 Alternative model specifications

The model derived above provides a sufficiently complete description of the context within which the monetary authorities' make their decisions to explicitly identify monetary policy shocks. But, as has been noted already, the identifying structure is based on assumptions on the sequencing of decisions and on the information sets available to agents when making decisions which may be contentious. Identification also relies on an assumed structure for the authorities' objective function and assumptions on how desired values for target variables are formed. In this section, we briefly comment on alternative assumptions that could be made on these latter two issues and consider the impact these alternative assumptions would have on the identification of the monetary policy shocks.

5.2.1 *Forecast-inflation targeting*

The model above makes use of an objective function in which it is the *current* values of the variables in $\mathbf{w}_t$ that are assumed of importance to the monetary authorities. In contrast, King (1994) argues that *future* values of target variables should also be of interest to monetary authorities. Certainly in the UK, for example, the Bank of England overtly follows a targeting rule in which there is a commitment to set policy so that the inflation rate for about two years ahead is on target. The decision to focus attention on future values of target variables is reasonable if there is a significant delay between the implementation of the monetary policy decision and the time the effects are felt. In this case, the authorities' objective function might be written as

$$\min_{r_t^b} \left\{ E\left[\sum_{h=0}^{\infty} \delta^h C(\mathbf{w}_{t+h}, r_t) \mid \Im_{t-1} \right] \right\}, \qquad (5.15)$$

where $C(\mathbf{w}_{t+h}, r_t)$ might continue to have the quadratic form of (5.7) and δ is a discount rate. This complicates the authorities' decision-rule and focuses attention on forecasts of future target variables. However, the identification of monetary policy shocks is unaffected by this complexity and the impulse responses obtained on the basis of the identification scheme of the previous section remains unchanged here.

To see this, consider the simple case in which the monetary authorities care about just one particular period ahead, $t+h$ say, and face the

optimisation problem

$$\min_{r_t^b} \left\{ E\left[C(\mathbf{w}_{t+h}, r_t) \mid \Im_{t-1} \right] \right\}, \qquad (5.16)$$

with

$$C(\mathbf{w}_{t+h}, r_t) = \frac{1}{2}(\mathbf{w}_{t+h} - \mathbf{w}_{t+h}^{\dagger})' \mathbf{Q}(\mathbf{w}_{t+h} - \mathbf{w}_{t+h}^{\dagger}) + \frac{1}{2}\theta(r_t - r_{t-1})^2. \qquad (5.17)$$

Identification of the monetary policy shocks can be achieved following the steps described in the previous section. Hence, as explained in more detail in Appendix A, minimisation of (5.16) provides a reaction function of the form

$$r_t^b = r_{t-1} - \rho_{b,t-1} + \boldsymbol{\Upsilon}_h' \left[E(\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0) - \mathbf{w}_{t+h}^{\dagger} \right] \qquad (5.18)$$

where $\boldsymbol{\Upsilon}_h'$ is a function of the parameters of the econometric model and of the preference parameters of the monetary authorities (and $\mathbf{w}_{t+h}^{\dagger}$ is assumed known at time $t-1$). Assuming that the derived reaction function is followed, the target variable outcome, $\mathbf{w}_{t+h}$, will be a weighted average of $E(\mathbf{w}_{t+h} \mid \Im_{t-1}, \Delta r_t^b = 0)$ and $\mathbf{w}_{t+h}^{\dagger}$ plus the effects of structural shocks experienced between t and $t+h$. In the case where the costs of adjusting the base rate are small relative to the costs of deviating from the desired level, $\mathbf{w}_{t+h}$ will track $\mathbf{w}_{t+h}^{\dagger}$ closely (abstracting from the effects of the unknown structural innovations that occur between t and $t+h$) and the commitment to the instrument rule in (5.18) is equivalent to pursuing a target rule where policy attempts to achieve a specified forecast-inflation target. Moreover, having derived the base rate reaction function in (5.18), the structural interest rate equation is derived exactly as in Section 5.1.3 above. Hence, the form of the structural model for $\mathbf{z}_t$ is exactly as in (5.2) and monetary policy shocks are once more identified as changes in the interest rate not explained by unanticipated movements in oil prices, exchange rates and foreign interest rates.

5.2.2 *Choice of targets and their desired levels*

The form of the monetary authorities' objective function played a central role in the short-run economic theory that underlies the identification of monetary policy shocks in the previous section. There has been a lively debate in the literature in recent years over the terms that should enter into the authorities' objective function and over their desired levels. In this section, we briefly comment on this discussion and relate the arguments to our identification scheme.

The objective function described in the previous section is consistent with monetary authorities' statements about their objectives. The maintenance of low and stable inflation is an explicitly stated objective for the independent central banks that now implement monetary policy in many countries. And many policy-makers are also encouraged to consider the output consequences of monetary policy.[10] However, despite the general consensus among practitioners, there remains considerable controversy over certain aspects of the objectives of monetary policy in the academic community.

Among those who believe that inflation should be stabilised around a fixed target level, there is long-standing disagreement on whether this should be at zero or a small positive figure (see King (1999) for a review). Some argue that non-zero inflation is damaging because of the distortions to money demand that arise and because of the relative price distortions that are created in a world with less than instantaneous or asynchronised price adjustment.[11] Others argue that a low but positive inflation rate is optimal on the basis of the presence of downward nominal rigidities (*e.g.* Akerlof *et al.* (1996)) or on the basis of the dangers of a liquidity trap given a zero bound to nominal interest rates (*e.g.* Summers (1991)). Still others believe that monetary policy should aim at price *level* targeting rather than inflation targeting (*e.g.* Svensson (1999)).

The debate regarding the role of output in monetary authorities' objective functions is also unresolved. There is now a broad consensus within macroeconomics that there is relatively little scope for manipulating real magnitudes through monetary policy in the long run and attention focuses on the role of monetary policy in minimising short-run variation of output around its 'long-run level'. Hence, desired output levels found in the literature typically refer to 'potential output' and the 'output gap' between actual and potential output. The term is used in the sense introduced by Okun (1962) and refers to the level of output potentially available at full employment and given the level of technological knowledge, the capital stock, natural resources, skill of the labour force and so on. In practice, this desired level is frequently measured as a trend in actual output

[10] In the UK, for example, the Bank of England's remit is to achieve an annual rate of consumer price inflation of 2.0% (over an unspecified time horizon) and, insofar as it does not compromise the targeting of inflation, the Bank is to support the policy of the government including its objectives for growth and employment.

[11] See Feldstein (1998), Bakhshi *et al.* (1997) and Woodford (2002, 2003), for example.

data, obtained as a simple exponential trend or through the Hodrick–Prescott filter. These empirical measures are simple statistical constructs, however, and have little economic motivation. Indeed, if potential output is difference-stationary, as the evidence seems to suggest, these measures are unlikely to be satisfactory. Woodford (2002, 2003) provides a more detailed discussion of the appropriate measure of desired output based on an explicit model of private sector behaviour in which welfare can be considered explicitly. He demonstrates that a socially optimal desired level of output is the 'natural level' of output, defined as the equilibrium level of output that would obtain in the event of perfectly flexible prices. But again, an adequate measure of such a concept will require a relatively complete macroeconomic model to be developed.

In the model of the previous section, we assumed in expression (5.12) that there are fixed desired *growth rates* for output and inflation, so that desired levels of output and inflation in each period are a fixed markup over last period's level. The advantage of such a specification is that it ensures that the desired levels are $I(1)$ and this is necessary to generate $I(1)$ values for the target variable outcomes. The disadvantage is that this form for the desired target variables does not correspond to the economic concepts discussed above any better than the standard use of trends. However, while the form of the variables in (5.12) is the simplest that will ensure difference-stationarity, it is not the only assumption that will generate $I(1)$ variables. In particular, an alternative approach to motivating measures of desired target variables is suggested by the idea that it would be unreasonable to choose desired targets which are inconsistent with the long-run relationships outlined by economic theory. Hence, we might make use of the long-run relationships involving the target variables to suggest sensible measures of their desired levels which will be $I(1)$ by construction. In this, a balance is struck between the integration properties of the observed variables such as real output and inflation and their desired target values.

For example, following this approach in the case of output determination, we recall that the economic theory of the long run elaborated in Chapter 4 suggested a long-run relationship between domestic and foreign output. The relationship was based on a stochastic Solow growth model, a simple model of technological progress and an assumption that, subject to transitory impediments to information flows and legal impediments, technology flows across national boundaries. This view provided the long-run 'output gap' relationship in (4.37) that was embedded within our macroeconometric model, *i.e.* $y_t = y_t^* + b_{30} + \xi_{3,t+1}$, with stationary zero-mean long-run reduced form disturbance ξ_{3t}. Given this theory of the long run, and the acknowledgement that domestic and foreign outputs are being driven by an underlying (and unobserved) common measure of technological progress, a reasonable measure of potential output might be given by the value of foreign output at time t expected when policy is set. This also provides a natural alternative measure of desired UK output; *i.e.*

$$y_t^{\dagger} = E(y_t^* \mid \Im_{t-1}).$$

This alternative measure of target output has the required property that it is $I(1)$. Moreover, an expression for $E(y_t^* \mid \Im_{t-1})$ can be obtained, in terms of $\boldsymbol{\beta}' \mathbf{z}_{t-1}$ and $\Delta \mathbf{z}_{t-i}$, $i = 1, \ldots, p-1$, from the reduced form expression y_t^* in (5.1). Inclusion of this expression in (5.13) would provide a structural interest rate equation of precisely the same form as in (5.14). The coefficients of the associated reduced form model would have a different interpretation, but *the identified monetary policy shocks obtained on the basis of the estimated reduced form model would be the same and any impulse response analysis undertaken would be unchanged.*

In a similar vein, the description of the Fisher equation of Chapter 4 motivated a long-run relationship between nominal interest rates and inflation that might be used to motivate an alternative measure of the desired inflation rate. Specifically, the long-run theory provided a stationary real interest rate equation in (4.39) of the form $r_t - \Delta p_t = b_{50} + \xi_{5,t+1}$, with stationary zero-mean long-run reduced form disturbance ξ_{5t}. Simply rewriting the long-run relationship and abstracting from the long-run disturbances, we might consider $\Delta p_t^{\dagger} = E[r_t - b_{50} \mid \Im_{t-1}]$, so that the desired interest rate is that which would ensure the real interest rate is at its long-run level for any expected value of r_t. Further, given that the nominal interest rate is, via the base rate, the instrument under the control of the monetary authorities, and given that interest rate movements are intrinsically costly in the authorities' cost function in (5.7), the desired level might reasonably be defined under the assumption of no change in the base rate,[12] so that we might amend the above to give

$$\begin{aligned} \Delta p_t^{\dagger} &= E[r_t - b_{50} \mid \Im_{t-1}, \Delta r_t^b = 0] \\ &= r_{t-1} - b_{50}. \end{aligned}$$

[12] Even if the cost of adjusting the instrument rate is considered close to zero, the authorities might consider the desired level of inflation to be such that, if the economy is in equilibrium, no instrument rate adjustment is required. The definition of desired inflation would build in the assumption that $\Delta r_t^b = 0$ in this case.

Again, this alternative measure of the target inflation rate has the required property that it is $I(1)$.[13] And, once more, inclusion of this expression in (5.13) would provide a structural interest rate equation of precisely the same form as in (5.14), the identified monetary policy shocks would be the same, and any impulse response analysis undertaken on the basis of the model would be unchanged.

[13] Note that the Taylor (1993) rule is typically written as

$$r_t = (1-\lambda)\left[\rho + \Delta p_t + \gamma_1(\Delta p_t - \Delta p_t^{\dagger}) + \gamma_2(y_t - y_t^{\dagger})\right] + \lambda r_{t-1},$$

with ρ being the average real interest rate. Now, when Δp_t and y_t are at their desired levels in this expression, then

$$r_t - r_{t-1} = (1-\lambda)\left[\rho + \Delta p_t^{\dagger} - r_{t-1}\right]$$

and we again have $\Delta p_t^{\dagger} = r_{t-1} - \rho$ when $\Delta r_t = 0$.