4

An economic theory of the long run

As was noted in Chapter 2, there are various theoretical approaches to the derivation of the long-run, steady-state relations of a macroeconometric model. However, we have argued that many of the approaches yield very similar results as far as the long-run relations are concerned and that there is a degree of consensus on these long-run properties across macroeconomic models. In this chapter, we outline the theoretical basis of the long-run relations to be considered for the modelling of a small open economy such as the UK. The analysis emphasises stock-flow equilibria and arbitrage conditions, appropriately modified to allow for the risks associated with market uncertainties. The arbitrage conditions provide intertemporal links between prices, interest rates and asset returns in the economy as a whole. The approach is distinct from the intertemporal optimisation approach underlying the DSGE models, but it is closely related and yields similar results on the long-run relations.

The minimal structure required of any model of the macroeconomy must accommodate a description of the production technology, the role of market forces and a characterisation of the institutional set-up (including financial institutions and the role of money, for example). The model described in this chapter is based on three sectors (namely the private, government and foreign sectors) and, to provide the required minimal economic structure, in what follows we begin with a description of output determination and the technological diffusion process, comment on the important arbitrage conditions arising from market forces, and note the implications of institutional solvency requirements for the long-run relationships in the macroeconomy.

4.1 Production technology and output determination

We assume that, in the long run, aggregate output is determined according to the following constant returns to scale production function in labour (denoted by N_t) and capital stock (denoted by K_t):

$$\frac{\tilde{Y}_t}{P_t} = F(K_t, A_t N_t) = A_t N_t F\left(\frac{K_t}{A_t N_t}, 1\right), \tag{4.1}$$

where $\tilde{Y}_t$ is gross domestic product measured in pounds sterling (with nominal magnitudes denoted with a '$\sim$' throughout), P_t, is a general price index, $\tilde{Y}_t/P_t$ is real aggregate output, and A_t stands for an index of labour-augmenting technological progress, assumed to be composed of a deterministic component, $a_0 + g\,t$, and a stochastic mean-zero component, u_{at} :

$$\ln(A_t) = a_0 + g\,t + u_{at}. \tag{4.2}$$

The process generating u_{at} is likely to be quite complex and there is little direct evidence on its evolution. But a few studies that have used patent data or R&D expenditures to directly analyse the behaviour of u_{at} over the course of the business cycle generally find highly persistent effects of technological disturbances on output (discussed in Fabiani (1996) and the references cited therein). The indirect evidence on u_{at}, obtained from empirical analysis of aggregate output, also corroborates this finding and generally speaking does not reject the hypothesis that u_{at} contains a unit root. (See, for example, Nelson and Plosser (1982) and, for the UK, Mills (1991).)

We further assume that the fraction of the population which is employed at time t, $\lambda_t = N_t/POP_t$, is a stationary process such that

$$N_t = \lambda POP_t \exp(\eta_{nt}), \tag{4.3}$$

where POP_t is population at the end of period t and η_{nt} represents a stationary, mean-zero process capturing the cyclical fluctuations of the unemployment rate around its steady-state value, $1 - \lambda$. The presence of, for example, real and nominal wage and price rigidities could generate deviations from the equilibrium, and these might be large and prolonged. However, these influences are captured by the presence of the η_{nt}, and ultimately it is assumed that the long-run equilibrium unemployment rate is re-established. The assumption that the steady-state unemployment rate is constant is by no means innocuous: it requires labour supply to be inelastic

with respect to the real wage in the long run, and abstracts from the possibility that there exist factors which might cause permanent changes in labour supply decisions.[1] The first requirement might not be too unrealistic in the very long run, given the absence of a long-term trend increase in unemployment corresponding to the unremitting rise in real wages (when measured over decades rather than years). However, many commentators would point to changes in labour market conditions which persist and which might reasonably be expected to influence the equilibrium unemployment rate; these might include shocks to incentives due to changes in the incidence of direct and indirect taxation; changes in the size and coverage of benefit payments; changes in the extent of union influence and other institutional changes in wage-bargaining arrangements; and so on. Our approach can be justified on the grounds that these institutional changes are by their nature constrained not to change continually and without bounds (so that they can be subsumed into the η_{nt}) or that their effects are small compared to the consequences of technical progress which dominates output determination.[2]

Under the above assumptions and using the relations (4.1), (4.2) and (4.3) it now readily follows that

$$y_t = a_0 + \ln(\lambda) + g\,t + \ln(f(\kappa_t)) + u_{at} + \eta_{nt}, \tag{4.4}$$

where $y_t = \ln[\tilde{Y}_t/(POP_t \times P_t)]$ is the logarithm of real per capita output, $\kappa_t = K_t/A_t N_t$ is the capital stock per effective labour unit, and $f(\kappa_t) = F(\kappa_t, 1)$ is a well-behaved function in the sense that it satisfies the Inada conditions. See, for example, Barro and Sala-i-Martin (1995, p. 16). Assuming the aggregate saving rate is monotonic in κ_t and that certain other mild regularity conditions hold, Binder and Pesaran (1999) show that, irrespective of whether the process generating u_{at} is stationary or contains a unit root, κ_t converges to a steady-state probability distribution with $\kappa_t \to \kappa_\infty$, where κ_∞ is a time-invariant random variable with a non-degenerate probability distribution function. Hence, in the long run the evolution of per capita output will be largely determined by technological process, with $E[\Delta \ln(y_t)] = g$. Also whether y_t contains a unit root crucially depends on whether there is a unit root in the process generating technological progress.[3]

1 See Nickell (1990) for a review of the literature on unemployment determination.

2 Notice that the assumption that the unemployment rate is stationary in effect rules out long-run hysteresis effects in the unemployment process.

3 See Lee *et al.* (1997, 1998) and Pesaran (2004a) for further discussion of the time series properties of output series derived under the stochastic Solow model framework.

Given the small and open nature of the UK economy, it might be reasonable to assume that, in the long run, A_t is determined by the level of technological progress in the rest of the world; namely

$$A_t = \gamma A_t^* \exp(\eta_{at}), \tag{4.5}$$

where A_t^* represents the level of foreign technological progress, γ captures productivity differentials based on fixed, initial technological endowments, and η_{at} represents stationary, mean zero disturbances capturing the effects of information lags or (transitory) legal impediments to technology flows across different countries, for example. Assuming that per capita output in the rest of the world is also determined according to a neoclassical growth model, and using a similar line of reasoning as above, we have

$$y_t - y_t^* = \ln(\gamma) + \ln(\lambda/\lambda^*) + \ln[f(\kappa_t)/f^*(\kappa_t^*)] + \eta_{at} + (\eta_{nt} - \eta_{nt}^*), \tag{4.6}$$

where foreign variables are shown with a 'star'. Similarly to κ_t the foreign capital stock per effective labour unit, κ_t^*, also tends to a time-invariant probability distribution function, and hence under the assumption that A_t^* (or A_t) contain a unit root, (y_t, y_t^*) will be cointegrated with a cointegrating vector equal to $(1, -1)$. (See Lee (1998) and Pesaran (2004a) for further discussion.)

The above stochastic formulation of the neoclassical growth model also has important implications for the determination of the real rate of return, which we denote by ρ_t. Profit maximisation on the part of firms ensures that, in the steady state, ρ_t will be equal to the marginal product of capital, so that

$$\rho_t = f'(\kappa_t), \tag{4.7}$$

where $f'(\kappa_t)$ is the derivative of $f(\kappa_t)$ with respect to κ_t. Since $\kappa_t \to \kappa_\infty$, it therefore follows that $\rho_t \to f'(\kappa_\infty)$; thus establishing that the steady-state distribution of the real rate of return will also be ergodic and stationary. This result allows us to write

$$1 + \rho_{t+1} = (1 + \rho)\exp(\eta_{\rho,t+1}), \tag{4.8}$$

where $\eta_{\rho,t+1}$ is a stationary process normalised so that $E[\exp(\eta_{\rho,t+1}) \mid I_t] = 1$, and where I_t is the publicly available information set at time t. This normalisation ensures that ρ is in fact the mean of the steady-state distribution of real returns, ρ_t, given by $E[f'(\kappa_\infty)]$.

4.2 Arbitrage conditions

Market forces in the model motivate a set of arbitrage conditions that are included in many macroeconomic models in one form or another. They are the (relative) Purchasing Power Parity (*PPP*), the Fisher Inflation Parity (*FIP*), and the Uncovered Interest Parity (*UIP*) relationships. We consider each of these in turn.

Purchasing Power Parity is based on the presence of goods market arbitrage, and captures the idea that the price of a common basket of goods will be equal in different countries when measured in a common currency. Information disparities, transportation costs or the effects of tariff and non-tariff barriers are likely to create considerable deviations from (absolute) *PPP* in the short run and, with the likely exception of information disparities, these might persist indefinitely. However, if the size of these influences has a constant mean over time, then the common currency price of the basket of goods in the different countries will rise one-for-one over the longer term, and this is captured by the (weaker) concept of 'relative *PPP*'. The primary explanation of long-run deviations from relative *PPP* is the 'Harrod–Balassa–Samuelson (H–B–S) effect' in which the price of a basket of traded and non-traded goods rises more rapidly in countries with relatively rapid productivity growth in the traded goods sector.[4] Following these arguments, we express relative *PPP* as

$$P_{t+1} = E_{t+1}P_{t+1}^* \exp(\eta_{ppp,t+1}), \tag{4.9}$$

where E_t is the effective exchange rate, defined as the domestic price of a unit of foreign currency at the beginning of period t (so that an increase in the exchange rate represents a depreciation of the home country currency), P_t^* is the foreign price index and the term in brackets captures the deviations from *PPP*. Here, $\eta_{ppp,t+1}$ is assumed to follow a stationary (or possibly trend-stationary) process capturing short-run variations in transport costs, information disparities, and the effects of tariff and non-tariff barriers. The errors $\eta_{ppp,t+1}$ could be conditionally heteroscedastic, although this particular source of variability is unlikely to be very important in quarterly macromodels. The effects of differential productivity growth rates in the traded and non-traded goods sectors at home and abroad, accommodating the H–B–S effect, can be captured by assuming that $\eta_{ppp,t+1}$ contains a trend.

[4] See Obstfeld and Rogoff (1996, Chapter 4) and Rogoff (1996) for further discussion of this effect and alternative modifications to *PPP*.

Deviations from *PPP* might be observed because real exchange rates are measured using price indices which involve different baskets of commodities across countries. In this case, real shocks which cause changes in the relative price of particular commodities will have differential impacts on countries' prices, and deviations from *PPP* remain consistent with goods market arbitrage. In the case of the UK, which is an oil producer, the (potential) direct effect of changes in the relative price of oil on the UK's real exchange rate could be accommodated in the model by including a multiplicative term in the relative oil price variable, say $(P^o_{t+1}/P^*_{t+1})^\theta$ on the right-hand side of (4.9).[5] Of course, one might doubt that changes in relative oil prices would have a permanent effect on real exchange rates over long horizons, in which case $\theta = 0$. However, even in this case, the relative oil price variable could still affect real exchange rates over prolonged periods, given the size of the oil price changes in recent years, because of differential speeds of adjustment to the productivity shock in different economies. Ultimately, this is a matter to be investigated empirically.[6]

The *FIP* relationship captures the equilibrium outcome of the arbitrage process between holding bonds and investing in physical assets. Denoting the expected real rate of return on physical assets over the period t to $t+1$ by ρ^e_{t+1}, and denoting inflation expectations over the same period by $(P^e_{t+1} - P_t)/P_t$, we have

$$\begin{aligned}(1+R_t) &= (1+\rho^e_{t+1})\left(1+\frac{P^e_{t+1}-P_t}{P_t}\right)\exp(\eta_{fip,t+1})\\ &= (1+\rho^e_{t+1})\left(\frac{P^e_{t+1}}{P_{t+1}}\right)\left(1+\frac{\Delta P_{t+1}}{P_t}\right)\exp(\eta_{fip,t+1}),\end{aligned} \quad (4.10)$$

where R_t is the nominal interest rate on domestic assets held from the beginning to the end of period t and $\eta_{fip,t+1}$ is the risk premium, capturing the effects of money and goods market uncertainties on risk-averse agents. We assume that $\eta_{fip,t+1}$ follows a stationary process with a finite mean and variance. Also recall that in the context of the neoclassical growth model

[5] This approach is advocated in Chauduri and Daniel (1998), for example. The inclusion of the relative oil price term in (4.9) can also be justified with reference to the H–B–S effect. Certainly, (relative) oil price changes have a pervasive effect on productivity, and these might have a differential effect in the traded and non-traded sectors of different economies. See Bruno and Sachs (1984) or Perron (1989), among others, for discussion of the role of the 1973 oil price shock in the worldwide slowdown in productivity.

[6] Distinguishing whether these effects are permanent or transitory is likely to be difficult using available datasets. However, the importance of explicitly taking into account the effects of oil price changes on the dynamics of real exchange rates has been widely acknowledged in applied work; see, for example, Johansen and Juselius (1992).

the real rate of interest (which we take to be the same as the real rate of return on capital) follows a stationary process; see (4.7) and (4.8).

The third arbitrage condition is based on the *UIP* relationship, which captures the equilibrium outcome of the arbitrage process between holding domestic and foreign bonds. In this, any differential in interest rates across countries must be offset by expected exchange rate changes to eliminate the scope for arbitrage. The presence of transactions costs, risk premia and speculative effects provide for the possibility of short-run deviations from *UIP*, and we therefore define the *Interest Rate Parity* (*IRP*) relationship as follows:

$$\begin{aligned}(1+R_t) &= (1+R^*_t)\left(1+\frac{E^e_{t+1}-E_t}{E_t}\right)\exp(\eta_{uip,t+1})\\ &= (1+R^*_t)\left(\frac{E^e_{t+1}}{E_{t+1}}\right)\left(1+\frac{\Delta E_{t+1}}{E_t}\right)\exp(\eta_{uip,t+1}),\end{aligned} \quad (4.11)$$

where R^*_t is the nominal interest rate paid on foreign assets during period t and $\eta_{uip,t+1}$ is the risk premium associated with the effects of bond and foreign exchange uncertainties on risk-averse agents. As before, we shall assume that $\eta_{uip,t+1}$ is stationary and ergodic.[7]

For the purpose of long-run modelling, we assume that the expectations errors $\eta^e_{i,t+1}$, $i = p, e, \rho$, defined by

$$\begin{aligned}P^e_{t+1} &= P_{t+1}\exp(\eta^e_{p,t+1}),\\ E^e_{t+1} &= E_{t+1}\exp(\eta^e_{e,t+1}),\\ \text{and } (1+\rho^e_{t+1}) &= (1+\rho_{t+1})\exp(\eta^e_{\rho,t+1})\end{aligned} \quad (4.12)$$

follow stationary processes. The assumption that the expectation errors are stationary seems quite plausible and is consistent with a wide variety of hypotheses concerning the expectations formation process.[8] In this case, the three arbitrage relationships discussed above can be written in terms of the observables using the expressions in (4.8) and (4.12). Specifically, the *FIP* relation can be written as:

$$r_t = \ln(1+\rho) + \Delta p_t + \eta_{fip,t+1} + \eta_{\rho,t+1} + \eta_{\Delta\Delta p,t+1} + \eta^e_{p,t+1} + \eta^e_{\rho,t+1}, \quad (4.13)$$

[7] As noted earlier, the relationships in (4.10) and (4.11) can also be derived from Euler equations obtained from consumer and producer optimisation in an intertemporal model of an economy with well-behaved preferences and technologies.

[8] This assumption is consistent with the Rational Expectations Hypothesis (REH), for example. However, it is much less restrictive than the REH, and can accommodate the possibility of systematic expectational errors in the short run, possibly due to incomplete learning.

where lower cases denote the logarithm of a variable, so that $r_t = \ln(1+R_t)$ and $p_t = \ln(P_t)$,

$$\Delta p_t = \ln\left(1 + \frac{\Delta P_t}{P_{t-1}}\right)$$

and

$$\eta_{\Delta\Delta p,t+1} = \ln\left(\frac{P_{t+1}}{P_t} \bigg/ \frac{P_t}{P_{t-1}}\right).$$

Similarly, the *IRP* relation can be written as

$$r_t = r_t^* + \eta_{\Delta e,t+1} + \eta_{uip,t+1} + \eta_{e,t+1}^e, \tag{4.14}$$

where $r_t^* = \ln(1+R_t^*)$ and $\eta_{\Delta e,t+1} = \Delta \ln(E_{t+1})$. And the log-linear version of the *PPP* relationship in (4.9) is given by

$$p_{t+1} = p_{t+1}^* + e_{t+1} + \eta_{ppp,t+1}, \tag{4.15}$$

where $p_{t+1}^* = \ln(P_{t+1}^*)$ and $e_{t+1} = \ln(E_{t+1})$.

4.3 Accounting identities and stock-flow relations

The institutional set-up of the model is captured through the use of the relevant accounting identities and stock-flow relations. We use the following stock identities:

$$\tilde{D}_t = \tilde{H}_t + \tilde{B}_t, \tag{4.16}$$

$$\tilde{F}_t = E_t\tilde{B}_t^* - (\tilde{B}_t - \tilde{B}_t^d), \tag{4.17}$$

$$\tilde{L}_t = \tilde{H}_t + \tilde{B}_t^d + E_t\tilde{B}_t^*, \tag{4.18}$$

where $\tilde{D}_t$ is net government debt, $\tilde{H}_t$ is the stock of high-powered money, $\tilde{B}_t$ is the stock of domestic bonds issued by the government, $\tilde{F}_t$ is the net foreign asset position of the economy, $\tilde{B}_t^*$ is the stock of foreign assets held by domestic residents, $\tilde{B}_t^d$ is the stock of domestic assets held by domestic residents, and $\tilde{L}_t$ $(= \tilde{D}_t + \tilde{F}_t)$ is the stock of financial assets held by the private sector.[9] All the stocks are measured at the beginning of period t. Recall that nominal magnitudes are denoted with a '~', and these are expressed

[9] It is assumed that foreign asset holdings of domestic residents and domestic holdings of foreign residents are composed of government bonds only.

in pounds sterling, except $\tilde{B}_t^*$ which is expressed in foreign currency. It is assumed that the government holds no foreign assets of its own.

We also have the output–expenditure flow identity:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t + (\tilde{X}_t - \tilde{M}_t), \tag{4.19}$$

where $\tilde{C}_t$ is consumption expenditures, $\tilde{I}_t$ investment expenditures, $\tilde{G}_t$ government expenditures, $\tilde{X}_t$ expenditures on exports and $\tilde{M}_t$ expenditures on imports, all are in current market prices and expressed in pounds sterling. The private sector disposable income is defined by

$$\tilde{Y}_t^d = \tilde{Y}_t - \tilde{T}_t + R_t\tilde{B}_t^d + E_tR_t^*\tilde{B}_t^*, \tag{4.20}$$

where $\tilde{T}_t$ represents taxes net of transfers to the private sector.

The model economy's stock-flow relationships are:

$$\Delta\tilde{D}_{t+1} = \tilde{G}_t + R_t\tilde{B}_t - \tilde{T}_t, \tag{4.21}$$

$$\Delta\tilde{L}_{t+1} = \tilde{Y}_t^d - \tilde{C}_t - \tilde{I}_t + (E_{t+1}^e - E_t)\tilde{B}_t^*, \tag{4.22}$$

$$\Delta\tilde{F}_{t+1} = \tilde{X}_t - \tilde{M}_t + \widetilde{NFA}_t + (E_{t+1}^e - E_t)\tilde{B}_t^*, \tag{4.23}$$

where $\widetilde{NFA}_t = E_tR_t^*\tilde{B}_t^* - R_t(\tilde{B}_t - \tilde{B}_t^d)$ is net factor income from abroad, and E_{t+1}^e stands for exchange rate expectations formed on the basis of publicly available information at time t. Hence, the term $(E_{t+1}^e - E_t)\tilde{B}_t^*$ is the (expected) revaluation of foreign assets held by domestic residents accruing through exchange rate appreciation in period t.[10] Note that, since $\tilde{L}_t = \tilde{D}_t + \tilde{F}_t$, any two of (4.21)–(4.23) implies the third.

4.4 Long-run solvency requirements

The assumption that the private sector remains solvent, taken with the stock-flow relationships given by (4.21)–(4.23), provides the motivation for further long-run relationships between macroeconomic variables. In order to ensure the long-run solvency of the private sector asset/liability position, we assume

$$\tilde{L}_{t+1}/\tilde{Y}_t = \mu \exp(\eta_{ly,t+1}), \tag{4.24}$$

[10] In most formulations of stock-flow relationships the asset revaluation term is either ignored or is approximated by an *ex post* counterpart such as $(E_t - E_{t-1})\tilde{B}_t^*$. But for consistency with the arbitrage (equilibrium) conditions to be set out below, we prefer to work with the *ex ante* asset revaluation term.

where $\eta_{ly,t+1}$ is a stationary process, so that the ratio of total financial assets to the nominal income level is stationary and ergodic. Expression (4.24) captures the idea that domestic residents are neither willing nor able to accumulate claims on, or liabilities to, the government and the rest of the world which are out of line with their current and expected future income. This condition, in conjunction with assumptions on the determinants of the equilibrium portfolio balance of the private sector assets, provides additional long-run relations.

In modelling the equilibrium portfolio balance of private sector assets, we follow Branson's (1977) Portfolio Balance Approach. From (4.18), we note that the stock of financial assets held by the private sector consists of the stock of high-powered money plus the stock of domestic and foreign bonds held by domestic residents. Given this adding-up constraint, we specify two independent equilibrium relationships relating to asset demand; namely, those relating to the demand for high-powered money and for foreign assets. These relationships are characterised in our model by the following:

$$\frac{\tilde{H}_{t+1}}{\tilde{L}_t} = F_h\left(\frac{Y_t}{P_t}, \rho^e_{b,t+1}, \rho^{*e}_{b,t+1}, \frac{\Delta P^e_{t+1}}{P_t}, t\right)\exp(\eta_{h,t+1}), \tag{4.25}$$

and

$$\frac{\tilde{F}_{t+1}}{\tilde{L}_t} = F_f\left(\frac{Y_t}{P_t}, \rho^e_{b,t+1}, \rho^{*e}_{b,t+1}, \frac{\Delta P^e_{t+1}}{P_t}, t\right)\exp(\eta_{f,t+1}), \tag{4.26}$$

where $F_{h1} \geq 0, F_{h2} \leq 0, F_{h3} \leq 0, F_{h4} \leq 0$, and $F_{f1} \leq 0, F_{f2} \leq 0, F_{f3} \geq 0, F_{f4} \geq 0$, and where

$$Y_t = \frac{\tilde{Y}_t}{POP_{t-1}}, \qquad \rho^e_{b,t+1} = \frac{(1+R_t)}{\left(1+\frac{P^e_{t+1}-P_t}{P_t}\right)} - 1,$$

and

$$\rho^{*e}_{b,t+1} = \frac{(1+R^*_t)\left(1+\frac{E^e_{t+1}-E_t}{E_t}\right)}{\left(1+\frac{P^e_{t+1}-P_t}{P_t}\right)} - 1.$$

The last two terms are the expected real rates of return on domestic and foreign bonds, respectively (both measured in domestic currency), η_{ht} is a stationary process which captures the effects of various factors that contribute to the short-run deviations of the ratio of money balances to total financial assets from its long-run determinants, and η_{ft} is the corresponding stationary process capturing the effects of short-run deviations

of the ratio of foreign assets to total financial assets from its long-run position. The determinants of the ratio of money to total financial assets in (4.25) include the real output level, to capture the influence of the transactions demand for money, and the expected real rates of return on the three alternative forms of holding financial assets; namely domestic bonds, foreign bonds and high-powered money. We have also specified a deterministic trend in $F_h(\cdot)$ to allow for the possible effect of the changing nature of financial intermediation, and the increasing use of credit cards in settlement of transactions on the convenience value of money. One would expect a downward trend in H/L, reflecting a trend reduction in the proportion of financial assets held in the form of non-interest bearing high-powered money over time. The determinants of the ratio of foreign assets to total financial assets in (4.26) are the same, with the decision to hold assets in the form of bonds mirroring that relating to holding assets in the form of money.

In view of the *IRP* relationship of (4.11), it is clear that, in the steady state, domestic and foreign bonds become perfect substitutes, and their expected rates of return are equal. Similarly, given the *FIP* relationship of (4.10) the real rates of return on (both) domestic and foreign bonds are equal to the (stationary) real rate of return on physical assets in the steady state. Hence, the asset demand relationships of (4.25) and (4.26) can be written equally as:

$$\frac{\tilde{H}_{t+1}}{\tilde{L}_t} = F_{hl}\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{hl,t+1}), \quad F_{hl1} \geq 0, \; F_{hl2} \leq 0, \tag{4.27}$$

and

$$\frac{\tilde{F}_{t+1}}{\tilde{L}_t} = F_{fl}\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{fl,t+1}), \quad F_{fl1} \leq 0, \; F_{fl2} \geq 0, \tag{4.28}$$

where the effects of the short-run deviations from *IRP* and *FIP* are now subsumed into the more general stationary processes $\eta_{hl,t+1}$ and $\eta_{fl,t+1}$ and where the effects of the expected real rate of return on non-interest bearing money holdings (*i.e.* minus the expected inflation rate) are captured by the domestic nominal interest rate (again making use of (4.10)). Note that this final effect implies that different rates of inflation, and hence different levels of nominal interest rates, could change the equilibrium portfolio composition, depending on the responsiveness of the asset demands to the relative returns on the three assets, so that changes in nominal rates of interest can potentially have lasting real effects.[11]

4.4.1 *Liquidity (real money balances)*

The solvency condition in (4.24) combined with the asset demand relationship of equation (4.27) now yields

$$\frac{\tilde{H}_{t+1}}{\tilde{Y}_t} = \frac{H_{t+1}}{Y_t} = \mu F_h\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{ly,t+1} + \eta_{hl,t+1}), \tag{4.29}$$

where $H_t = \tilde{H}_t/POP_{t-1}$ or, in its approximate log-linear form,

$$(h_t - y_t) = \ln(\mu) + \mu_1 t + \mu_2 r_t + \mu_3 y_t + \eta_{ly,t+1} + \eta_{hl,t+1},$$

where $h_t - y_t = \ln(H_{t+1}/P_t) - \ln(Y_t/P_t) = \ln(H_{t+1}/Y_t)$ and with unknown parameters μ_i, $i = 1, 2, 3$.[12] The equivalent relationship, based on (4.24) and (4.28), yields the following expression for the ratio of net foreign assets (measured in domestic currency) to the nominal output level:

$$\frac{\tilde{F}_{t+1}}{\tilde{Y}_t} = \mu F_f\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{fl,t+1} + \eta_{ly,t+1}), \tag{4.30}$$

although foreign asset levels are less frequently the focus of attention in macroeconometric models.[13] Equation (4.29) therefore provides the final long-run relationship to be considered in our model of the UK macroeconomy, along with the other four relationships described in (4.6), (4.13), (4.14) and (4.15).

4.4.2 *Imports and exports*

Before moving on to consider how the five steady-state relationships given in (4.6), (4.13), (4.14), (4.15), and (4.29) can be incorporated into an empirical model, it is worth briefly elaborating on the potential role that might be played by the demand for foreign assets in the domestic economy.

[11] The possibility of the 'super-non-neutrality' of monetary policy arising through this route is discussed in Buiter (1980), for example.

[12] For expositional simplicity, we have chosen to denote $\ln(H_{t+1}/P_t)$ by h_t, rather than h_{t+1}. Recall that H_{t+1} relates to the stock of high-powered money at the beginning of period $t+1$.

[13] The stock-flow relationship of (4.23) can be used in conjunction with (4.17) to motivate a relationship between net foreign assets, net exports and domestic and foreign interest rates. Assuming that net exports depend on domestic and foreign output and the terms of trade, substitution of these relationships into (4.28) provides the justification for a further possible long-run relationship between Y_t, Y_t^*, R_t, R_t^*, and $E_tP_t^*/P_t$.

Specifically, we can show that the conditions (4.24) and (4.28), when taken with assumptions on import and export determination, provide a further equilibrium condition between the real exchange rate, domestic and foreign outputs and the interest rate. Given the stationarity of the real exchange rate expressed by (4.15) and given the relationship between domestic and foreign outputs in (4.6), it is reasonable to believe that this extra equilibrium relationship will provide little additional explanatory power in a model that incorporates the effects of (4.15) and (4.6) already. Indeed, in the empirical model of the later chapters, we do not include this additional equilibrium relationship. But it is worth elaborating the relationship here both to clarify the potential role of foreign asset demand and to note that, in practice, the equilibrating pressures assigned to deviations from *PPP* and the 'output gap' relationship may in fact confound these effects and those arising from balance of payments outcomes.

To derive the extra equilibrium relationship, we note that the stock-flow relationship (4.23) can be used in conjunction with the definition of the country's net foreign asset position in (4.17) to write

$$\begin{aligned}\tilde{F}_{t+1} &= \tilde{X}_t - \tilde{M}_t + \tilde{F}_t + E_tR_t^*\tilde{B}_t^* - R_t(\tilde{B}_t - \tilde{B}_t^d) + (E_{t+1}^e - E_t)\tilde{B}_t^* \\ &= \tilde{X}_t - \tilde{M}_t + (1 + R_t)\tilde{F}_t - E_t\tilde{B}_t^*\left(R_t - R_t^* - \frac{\Delta E_{t+1}^e}{E_t}\right).\end{aligned}$$

Dividing through by nominal income, and writing the various ratios in per capita terms, we obtain

$$\frac{F_{t+1}}{Y_t} = \frac{X_t - M_t}{Y_t} + (1 + R_t)\left(\frac{F_t}{Y_{t-1}}\right)\left(\frac{Y_{t-1}}{Y_t}\right) - \frac{E_tB_t^*}{Y_t}\left(R_t - R_t^* - \frac{\Delta E_{t+1}^e}{E_t}\right),$$

where $Y_t = \tilde{Y}_t/POP_{t-1}$, $X_t = \tilde{X}_t/POP_{t-1}$ and $F_t = \tilde{F}_t/POP_{t-1}$. Let g_t denote the growth of per capita output and note that $Y_t/Y_{t-1} = (1 + g_t)(1 + \Delta P_t/P_{t-1})$. Hence

$$\begin{aligned}\frac{F_{t+1}}{Y_t} = \frac{X_t - M_t}{Y_t} &+ \frac{(1 + R_t)}{(1 + g_t)(1 + \Delta P_t/P_{t-1})}\left(\frac{F_t}{Y_{t-1}}\right) \\ &- \frac{E_tB_t^*}{Y_t}\left(R_t - R_t^* - \frac{\Delta E_{t+1}^e}{E_t}\right).\end{aligned} \tag{4.31}$$

Now, under our assumptions, $(1+g_t)$ and $(1+R_t)/(1+\Delta P_t/P_{t-1})$ both tend to stationary processes with constant means, $1+g$ and $1+\rho$, respectively, and the term in $R_t - R_t^* - (\Delta E_{t+1}^e/E_t)$ itself tends to a stationary process. Recalling from (4.30) that the value of $\tilde{F}_{t+1}/\tilde{Y}_t$ depends on $Y_t/P_t, R_t$ and t, the solvency condition and the relationships describing the determinants of the ratio of foreign to total financial assets provides, through (4.31), a long-run relationship between $(X_t - M_t)/Y_t$, Y_t/P_t and R_t. We represent this relationship by the following:

$$\frac{X_t - M_t}{Y_t} = F_b\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{b,t+1}), \quad F_{b1} \leq 0, \; F_{b2} \geq 0, \tag{4.32}$$

where $\eta_{b,t+1}$ is a stationary process.

To complete our derivations, we further assume that real per capita imports (M_t/P_t) and exports (X_t/P_t) are determined according to the following relations:

$$\frac{X_t}{P_t} = F_x\left(\frac{Y_t^*}{P_t^*}, \frac{E_tP_t^*}{P_t}\right)\exp(\eta_{xt}), \qquad F_{x1} > 0, \; F_{x2} > 0, \tag{4.33}$$

$$\frac{M_t}{E_tP_t^*} = F_m\left(\frac{Y_t}{P_t}, \frac{E_tP_t^*}{P_t}\right)\exp(\eta_{mt}), \qquad F_{m1} > 0, \; F_{m2} < 0,$$

where η_{xt} and η_{mt} are stationary processes with zero means. In the long run, real per capita exports are assumed to depend on real activity levels abroad, Y_t^*/P_t^*, and on the relative price of goods abroad compared to those at home, while real per capita imports depend on domestic real per capita output and relative prices. The stationary processes η_{xt} and η_{mt} characterise the short-run departure of exports and imports from their long-term determinants. Using (4.32) and (4.33), we obtain

$$F_x\left(\frac{Y_t^*}{P_t^*}, \frac{E_tP_t^*}{P_t}, \frac{P_t^o}{P_t^*}\right)\exp(\eta_{xt}) - \frac{E_tP_t^*}{P_t}F_m\left(\frac{Y_t}{P_t}, \frac{E_tP_t^*}{P_t}, \frac{P_t^o}{P_t^*}\right)\exp(\eta_{mt})$$
$$= \frac{Y_t}{P_t}F_b\left(\frac{Y_t}{P_t}, R_t, t\right)\exp(\eta_{b,t+1}), \tag{4.34}$$

or, in its approximate log-linear form,

$$(e_t + p_t^* - p_t) = \mu_4 + \mu_5 t + \mu_6 y_t + \mu_7 y_t^* + \mu_8 r_t + \eta_{xt} + \eta_{mt} + \eta_{b,t+1},$$

with unknown parameters μ_i, $i = 4, \ldots, 8$. In summary, then, the interplay between the stock-flow equilibria, the demand for foreign assets, the solvency condition, and simple assumptions on the determinants of import and export demand generates a further long-run relationship between the real exchange rate, domestic and foreign outputs and the interest rate. In principle, this 'trade balance' relationship could be investigated alongside the five steady-state relationships in (4.6), (4.13), (4.14), (4.15) and (4.29). However, comparison of the log-linear version of (4.34) with those of the *PPP* relationship of (4.15) and the output relationship of (4.6) shows that it is likely to be difficult to distinguish the separate contributions of the trade balance relationship empirically. For these reasons, we do not pursue the effects of the trade balance relationship in what follows.

4.5 Econometric formulation of the model

In this section, we adopt the modelling strategy elaborated in Section 3.1.3 to derive an econometric formulation for our model based on the economic theory of the long run elaborated above. For empirical purposes, we employ a log-linear approximation of the five long-run equilibrium relationships set out in the previous section in (4.15), (4.14), (4.6), (4.29) and (4.13).[14] These constitute the theory-based long-run relationships of the model and take the following form:

$$p_t - p_t^* - e_t = b_{10} + b_{11}t + \xi_{1,t+1}, \tag{4.35}$$

$$r_t - r_t^* = b_{20} + \xi_{2,t+1}, \tag{4.36}$$

$$y_t - y_t^* = b_{30} + \xi_{3,t+1}, \tag{4.37}$$

$$h_t - y_t = b_{40} + b_{41}t + \beta_{44}r_t + \beta_{46}y_t + \xi_{4,t+1}, \tag{4.38}$$

$$r_t - \Delta p_t = b_{50} + \xi_{5,t+1}, \tag{4.39}$$

recalling that $p_t = \ln(P_t)$, $p_t^* = \ln(P_t^*)$, $e_t = \ln(E_t)$, $y_t = \ln(Y_t/P_t)$, $y_t^* = \ln(Y_t^*/P_t^*)$, $r_t = \ln(1+R_t)$, $r_t^* = \ln(1+R_t^*)$, $h_t - y_t = \ln(H_{t+1}/P_t) - \ln(Y_t/P_t) = \ln(H_{t+1}/Y_t)$ and $b_{50} = \ln(1+\rho)$. We have allowed for intercept and trend terms (when appropriate) in order to ensure that (long-run) reduced form disturbances, $\xi_{i,t+1}$, $i = 1, 2, \ldots., 5$, have zero means. These disturbances are

[14] We assume a trend term enters the log-linear *PPP* relationship, as mentioned earlier.

related to the long-run structural disturbances, the η_i's, in the following manner:[15]

$$\begin{aligned}
\xi_{1,t+1} &= \eta_{ppp,t} - b_{10} - b_{11}t, \\
\xi_{2,t+1} &= \eta_{uip,t+1} + \eta^e_{e,t+1} + \eta_{\Delta e,t+1} - b_{20}, \\
\xi_{3,t+1} &= \eta_{at} + (\eta_{nt} - \eta^*_{nt}) + (\eta_{kt} - \eta^*_{kt}), \\
\xi_{4,t+1} &= \eta_{ly,t} + \eta_{hl,t}, \\
\xi_{5,t+1} &= \eta_{fip,t+1} + \eta_{\rho,t+1} + \eta_{\Delta\Delta p,t+1} + \eta^e_{p,t+1} + \eta^e_{\rho,t+1}.
\end{aligned} \tag{4.40}$$

The above relationships between the long-run structural disturbances, η_i's, and the long-run reduced form disturbances, ξ_i's, clearly show the difficulties involved in identifying the effects of changes in particular structural disturbances on the dynamic behaviour of the macroeconomy. For example, $\xi_{5,t+1}$ is composed of the five structural disturbances, $\eta_{fip,t+1}$, $\eta_{\rho,t+1}$, $\eta_{\Delta\Delta p,t+1}$, $\eta^e_{p,t+1}$, $\eta_{\rho,t+1}$, representing the different factors that could be responsible for disequilibria between inflation and interest rates. In general, without further *a priori* restrictions, the effect of particular structural disturbances, η_i's, cannot be identified: firstly, there are many more long-run structural disturbances than there are long-run reduced form disturbances; and, secondly, there is no reason to believe that the η_i's are not themselves contemporaneously correlated. Empirical analysis at best enables us to identify the effect of changes in the long-run reduced form disturbances on the evolution of the macroeconomy towards its long-run equilibrium, although, as we discuss below, even identification of the effects of specific changes in these long-run reduced form disturbances will typically require further identifying restrictions based on an explicit model of short-run decision-making.

The five long-run relations of the model, (4.35)–(4.39), can be written more compactly as

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_0 - \mathbf{b}_1(t-1), \tag{4.41}$$

where

$$\mathbf{z}_t = \left(p^o_t, e_t, r^*_t, r_t, \Delta p_t, y_t, p_t - p^*_t, h_t - y_t, y^*_t\right)'. \tag{4.42}$$

$$\mathbf{b}_0 = (b_{10}, b_{20}, b_{30}, b_{40}, b_{50})', \ \mathbf{b}_1 = (b_{11}, 0, 0, b_{41}, 0)',$$

$$\boldsymbol{\xi}_t = (\xi_{1t}, \xi_{2t}, \xi_{3t}, \xi_{4t}, \xi_{5t})',$$

[15] In the case of $\xi_{2,t+1}$, we have taken account of the effect of exchange rate depreciation on the interest rate differential since, as we shall see below, the hypothesis that $\eta_{\Delta e,t+1}$ is stationary cannot be rejected.

and

$$\boldsymbol{\beta}' = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\beta_{44} & 0 & -\beta_{46} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.43}$$

The description of the long-run disturbances in (4.41) is, of course, of precisely the form of (3.4) introduced in the outline of our modelling strategy in Section 3.1.3.

For estimation purposes, we choose to partition $\mathbf{z}_t = (p^o_t, \mathbf{y}'_t)'$ where $\mathbf{y}_t = (e_t, r^*_t, r_t, \Delta p_t, y_t, p_t - p^*_t, h_t - y_t, y^*_t)'$. Here, p^o_t (the logarithm of oil prices) is considered to be a 'long-run forcing' variable for the determination of $\mathbf{y}_t$, in the sense that changes in p^o_t have a direct influence on $\mathbf{y}_t$, but changes in p^o_t are not affected by the presence of $\boldsymbol{\xi}_t$, which measure the extent of disequilibria in the UK economy. The treatment of oil prices as 'long-run forcing' represents a generalisation of the approach to modelling oil price effects in some previous applications of cointegrating VAR analyses (*e.g.* Johansen and Juselius, 1992, or Pesaran and Shin, 1996), where the change in the oil price is treated as a strictly exogenous $I(0)$ variable. The approach taken in the previous literature excludes the possibility that there might exist cointegrating relationships involving oil prices, while the approach taken here allows the validity of the hypothesised restriction to be tested and for the restriction to be imposed if it is not rejected.

We choose to treat foreign output and interest rates as endogenous for pragmatic reasons. As the discussion of Section 3.4 makes clear, the natural modelling choice for a small open economy like the UK would be to treat y^*_t and r^*_t as long-run forcing. However, we shall want to use our model for forecasting purposes and therefore require a world model with which to forecast future values of y^*_t and r^*_t. Rather than build a world model, we have implemented the model by treating these variables as endogenous (effectively supplementing simple autoregressive models of foreign output and interest rates with the lagged values of the UK variables as a substitute for the world model). It is worth noting that the endogenous treatment of foreign output and interest rates involves loss of efficiency in estimation if they were in fact long-run forcing or strictly exogenous, but this is clearly less serious than treating these variables as exogenous if this turned out to be false, for example.

Under the assumption that oil prices are long-run forcing for $\mathbf{y}_t$, the cointegrating properties of the model can be investigated without having to specify the oil price equation.[16] However, specification of an oil price equation is required in the analysis of the short-run dynamics and forecasting. For this purpose we shall adopt the following general specification for the evolution of oil prices:

$$\Delta p_t^o = \delta_o + \sum_{i=1}^{p-1} \delta_{oi} \Delta \mathbf{z}_{t-i} + u_{ot}, \tag{4.44}$$

where u_{ot} represents a serially uncorrelated oil price shock with a zero mean and a constant variance. The above specification ensures oil prices are long-run forcing for $\mathbf{y}_t$ since it allows lagged changes in the endogenous and exogenous variables of the model to influence current oil prices but rules out the possibility that the error correction terms, $\boldsymbol{\xi}_t$, have any effects on oil price changes. These assumptions are weaker than the requirement of 'Granger non-causality' often invoked in the literature.

Assuming that the variables in $\mathbf{z}_t$ are difference-stationary (as discussed in Chapter 8), our modelling strategy is now to embody $\boldsymbol{\xi}_t$ in an otherwise unrestricted VAR($p-1$) in $\mathbf{z}_t$. Under the assumption that oil prices are long-run forcing, it is efficient (for estimation purposes) to base our analysis on the following *conditional* error correction model

$$\Delta \mathbf{y}_t = \mathbf{a}_y - \boldsymbol{\alpha}_y \boldsymbol{\xi}_t + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{yi} \Delta \mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo} \Delta p_t^o + \mathbf{u}_{yt}, \tag{4.45}$$

where $\mathbf{a}_y$ is an 8×1 vector of fixed intercepts, $\boldsymbol{\alpha}_y$ is an 8×5 matrix of error correction coefficients (also known as the loading coefficient matrix), $\{\boldsymbol{\Gamma}_{yi}, i = 1, 2, ..., p-1\}$ are 8×9 matrices of short-run coefficients, $\boldsymbol{\psi}_{yo}$ is an 8×1 vector representing the impact effects of changes in oil prices on $\Delta \mathbf{y}_t$, and $\mathbf{u}_{yt}$ is an 8×1 vector of disturbances assumed to be $i.i.d.(0, \boldsymbol{\Sigma}_y)$, with $\boldsymbol{\Sigma}_y$ being a positive definite matrix, and by construction uncorrelated with u_{ot}. Using equation (4.41), we now have

$$\Delta \mathbf{y}_t = \mathbf{a}_y + \boldsymbol{\alpha}_y \mathbf{b}_0 - \boldsymbol{\alpha}_y \left[\boldsymbol{\beta}' z_{t-1} - b_1(t-1) \right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{yi} \Delta \mathbf{z}_{t-i} + \boldsymbol{\psi}_{yo} \Delta p_t^o + \mathbf{u}_{yt}, \tag{4.46}$$

where $\boldsymbol{\beta}' z_{t-1} - \mathbf{b}_1(t-1)$ is a 5×1 vector of error correction terms. The above specification embodies the economic theory's long-run predictions by construction.

[16] See, for example, Pesaran, Shin and Smith (2000).

Estimation of the parameters of the model, (4.46), can be carried out using the long-run structural modelling approach described in Pesaran and Shin (2002) and Pesaran, Shin and Smith (2000). It is based on a modified and generalised version of Johansen's (1991, 1995) maximum likelihood approach to the problem of estimation and hypothesis testing in the context of vector autoregressive error correction models. With this approach, having selected the order of the underlying VAR model (using model selection criteria such as the Akaike Information Criterion (AIC) or the Schwarz Bayesian Criterion (SBC)), we test for the number of cointegrating relations among the nine variables in $\mathbf{z}_t$. When performing this task, and in all the subsequent empirical analysis, we work in the context of a VAR model with unrestricted intercepts and restricted trend coefficients.[17] In terms of (4.46), we allow the intercepts to be freely estimated but restrict the trend coefficients so that $\boldsymbol{\alpha}_y \mathbf{b}_1 = \boldsymbol{\Pi}_y \boldsymbol{\gamma}$, where $\boldsymbol{\Pi}_y = \boldsymbol{\alpha}_y \boldsymbol{\beta}'$ and $\boldsymbol{\gamma}$ is a 9×1 vector of unknown coefficients. These restrictions ensure that the solution of the model in levels of $\mathbf{z}_t$ will not contain quadratic trends. We then compute Maximum Likelihood (ML) estimates of the model's parameters subject to exact and over-identifying restrictions on the long-run coefficients. Assuming that there is empirical support for the existence of five long-run relationships, as suggested by theory, exact identification in our model requires five restrictions on each of the five cointegrating vectors (each row of $\boldsymbol{\beta}$), or a total of 25 restrictions on $\boldsymbol{\beta}$. These represent only a subset of the restrictions suggested by economic theory as characterised in (4.43), however. Estimation of the model subject to all the (exact- and over-identifying) restrictions given in (4.43) enables a test of the validity of the over-identifying restrictions, and hence the long-run implications of the economic theory, to be carried out.

[17] This is referred to as Case IV in Pesaran, Shin and Smith (2000), see Subsection 6.2.1 below.