On the Welfare Cost of Inflation and the Recent Behavior of Money Demand

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1 Phillips-Perron Unit Root Test

The Phillips-Perron (1988) unit root test is also described by Hamilton (1994, Ch.17). Let $\hat{\rho}$ denote the OLS estimate and $\hat{\sigma}_\rho$ its associated standard error from the regression

$$y_t = \alpha + \rho y_{t-1} + u_t.$$

Let $t = (\hat{\rho} - 1)/\hat{\sigma}_\rho$ denote the usual t-statistic associated with the null hypothesis of a unit root.

In general, u_t will be serially correlated, so that its long-run variance, denoted λ^2, must be computed as suggested by Newey and West (1987). Let

$$\gamma_0 = T^{-1} \sum_{t=1}^{T} u_t^2$$

and, similarly, for $j = 1, 2, ..., q$, let

$$\gamma_j = T^{-1} \sum_{t=j+1}^{T} u_t u_{t-j}.$$

Then

$$\lambda^2 = \gamma_0 + 2 \sum_{j=1}^{q} [1 - j/(q+1)] \gamma_j.$$

Finally, let

$$s^2 = (T-2)^{-1} \sum_{t=1}^{T} u_t^2$$

denote the usual OLS estimate for the variance of u_t. Then the Phillips-Perron statistic

$$Z_t = (\gamma_0/\lambda^2)^{1/2} t - (1/2)[(\lambda^2 - \gamma_0)/\lambda](T\hat{\sigma}_\rho/s)$$

has critical values reported under the heading "case 2" in Hamilton's table B.6 (p.763).

2 Phillips-Ouliaris Cointegration Test

The Phillips-Ouliaris (1990) test for cointegration is also described by Hamilton (1994, Ch.19). The approach starts by estimating the regression equation

$$y_t = \alpha + \beta' x_t + u_t$$

by OLS where, as above, y_t is a scalar, but here x_t is $g \times 1$ so that β is also $g \times 1$. Next, the residual u_t is regressed on its own lagged value:

$$u_t = \rho u_{t-1} + \varepsilon_t.$$

Let $\hat{\rho}$ denote the OLS estimate of ρ, let $\hat{\sigma}_\rho$ denote the associated OLS standard error, and let $t = (\hat{\rho} - 1)/\hat{\sigma}_\rho$ denote the usual t-statistic associated with the null hypothesis of a unit root in the process for the residuals u_t.

In general, ε_t will be serially correlated, so that its long-run variance, denoted λ^2, must be computed as suggested by Newey and West (1987). Let

$$\gamma_0 = (T-1)^{-1} \sum_{t=2}^{T} \varepsilon_t^2$$

and, similarly, for $j = 1, 2, ..., q$, let

$$\gamma_j = (T-1)^{-1} \sum_{t=j+2}^{T} \varepsilon_t \varepsilon_{t-j}.$$

Then

$$\lambda^2 = \gamma_0 + 2 \sum_{j=1}^{q} [1 - j/(q+1)]\gamma_j.$$

Finally, let

$$s^2 = (T-2)^{-1} \sum_{t=1}^{T} \varepsilon_t^2$$

denote the usual OLS estimate of the variance of ε_t. Then the Phillips-Ouliaris statistic

$$Z_t = (\gamma_0/\lambda^2)^{1/2} t - (1/2)[(\lambda^2 - \gamma_0)/\lambda][(T-1)\hat{\sigma}_\rho/s]$$

has critical values reported under the heading "case 2" in Hamilton's table B.9 (p.766) when all of series in the $n \times 1$ vector $(n = g+1)$ $y_t^* = [y_t, x_t']'$ are driftless and critical values reported under the heading "case 3" in Hamilton's table B.9 (p.766) when at least one of the series in x_t has nonzero drift.

3 Stock and Watson's Dynamic OLS Estimator

Hamilton (1994, Ch.19) notes that while consistent estimates of α and β can be obtained by applying "static" OLS to the regression

$$y_t = \alpha + \beta' x_t + u_t,$$

these estimates have nonstandard distributions in the general case in which x_t and u_t are correlated. Hamilton goes on to explain how Stock and Watson (1993) and others correct for this correlation by including leads and lags of $\Delta x_t = x_t - x_{t-1}$ in the expanded regression

$$y_t = \alpha + \beta' x_t + \sum_{s=-p}^{p} \beta_s' \Delta x_{t-s} + u_t,$$

assuming that the correlation between x_t and u_{t-s} is zero for all $|s| > p$. The "dynamic" OLS estimates of β are asymptotically efficient and asymptotically equivalent to Johansen's (1988) maximum likelihood estimates. In addition, once they are corrected for the serial correlation that remains in u_t, Wald statistics for testing linear restrictions $R\beta = r$ formed using the DOLS estimates have standard asymptotic distributions.

Let $z_t = [1, x_t', \Delta x_{t-p}', \Delta x_{t-p+1}', ..., \Delta x_t', ..., \Delta x_{t+p-1}', \Delta x_{t+p}']'$ and, once again following Newey and West (1987), let

$$\gamma_0 = (T-2p-1)^{-1} \sum_{t=p+2}^{T-p} u_t^2,$$

$$\gamma_j = (T-2p-1)^{-1} \sum_{t=j+p+2}^{T-p} u_t u_{t-j},$$

for $j = 1, 2, ..., q$, and

$$\lambda^2 = \gamma_0 + 2\sum_{j=1}^{q}[1 - j/(q+1)]\gamma_j.$$

Then

$$W = (R\beta - r)' \left\{ \lambda^2 \left[\begin{array}{ccc} 0_{m\times 1} & R & 0_{m\times(2p+1)g} \end{array} \right] \left[\sum_{t=p+2}^{T-p} z_t z_t' \right]^{-1} \left[\begin{array}{c} 0_{1\times m} \\ R' \\ 0_{(2p+1)g\times m} \end{array} \right] \right\}^{-1} (R\beta - r)$$

is asympotically distributed as a $\chi^2(m)$ random variable.