

■ 1 Position Effects

Perceived difficulty of an item does not only depend on its difficulty, but also on the position at which it was administered.

$$\underbrace{\beta_{ip}}_{\text{difficulty of item } i \text{ on position } p} = \underbrace{\eta_i}_{\text{'real' difficulty of item } i} + \underbrace{\omega_i}_{\text{difficulty of position } p} \quad (1)$$

η_i is also called the 'item root'

item	pos.
A	1
B	2
C	3
D	4
A	4
B	3
C	2
D	1
A	2
B	4
C	1
D	3
A	3
B	1
C	4
D	2

These are four items administered in four sequences.

One way of dealing with this would be to assume a linear increase of difficulty or easiness with each position.

item	pos.	η_A	η_B	η_C	η_D	ω	β_{ip}
A	1	1				1	$\eta_A + \omega_1 \omega$
B	2		1			2	$\eta_B + \omega_2 \omega$
C	3			1		3	$\eta_C + \omega_3 \omega$
D	4				1	4	$\eta_D + \omega_4 \omega$
A	4	1				4	$\eta_A + \omega_4 \omega$
B	3		1			3	$\eta_B + \omega_3 \omega$
C	2			1		2	$\eta_C + \omega_2 \omega$
D	1				1	1	$\eta_D + \omega_1 \omega$
A	2	1				2	$\eta_A + \omega_2 \omega$
B	4		1			4	$\eta_B + \omega_4 \omega$
C	1			1		1	$\eta_C + \omega_1 \omega$
D	3				1	3	$\eta_D + \omega_3 \omega$
A	3	1				3	$\eta_A + \omega_3 \omega$
B	1		1			1	$\eta_B + \omega_1 \omega$
C	4			1		4	$\eta_C + \omega_4 \omega$
D	2				1	2	$\eta_D + \omega_2 \omega$

Problem: X needs a special shape for this (*virtual items*).

Of course there are only 4 items, but to account for the 4 sequences, a total of 16 items must be present in X .

Pos.	1	2	3	4	4	3	2	1	2	4	1	3	3	1	4	2
Item	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
Block 1	x	x	x	x												
Block 1	x	x	x	x												
Block 2					x	x	x	x								
Block 2					x	x	x	x								
Block 3									x	x	x	x				
Block 3									x	x	x	x				
Block 4													x	x	x	x
Block 4													x	x	x	x

Now the data can be analyzed using an LLTM.

Caution: because of the data structure we have to used LPCM (ill-conditioned data).

If we want to model single positions' effects, W has to look like this:

$$\begin{array}{cc|cccc|cccc|c}
 \text{item} & \text{pos.} & \eta_A & \eta_B & \eta_C & \eta_D & \omega_1 & \omega_2 & \omega_3 & \omega_4 & \beta_{ip} \\
 \hline
 A & 1 & 1 & & & & 1 & & & & \eta_A + \omega_1 \\
 B & 2 & & 1 & & & & 1 & & & \eta_B + \omega_2 \\
 C & 3 & & & 1 & & & & 1 & & \eta_C + \omega_3 \\
 D & 4 & & & & 1 & & & & 1 & \eta_D + \omega_4 \\
 \hline
 A & 4 & 1 & & & & & & & 1 & \eta_A + \omega_4 \\
 B & 3 & & 1 & & & & & 1 & & \eta_B + \omega_3 \\
 C & 2 & & & 1 & & & 1 & & & \eta_C + \omega_2 \\
 D & 1 & & & & 1 & 1 & & & & \eta_D + \omega_1 \\
 \hline
 A & 2 & 1 & & & & & 1 & & & \eta_A + \omega_2 \\
 B & 4 & & 1 & & & & & & 1 & \eta_B + \omega_4 \\
 C & 1 & & & 1 & & 1 & & & & \eta_C + \omega_1 \\
 D & 3 & & & & 1 & & & 1 & & \eta_D + \omega_3 \\
 \hline
 A & 3 & 1 & & & & & & 1 & & \eta_A + \omega_3 \\
 B & 1 & & 1 & & & 1 & & & & \eta_B + \omega_1 \\
 C & 4 & & & 1 & & & & & 1 & \eta_C + \omega_4 \\
 D & 2 & & & & 1 & & 1 & & & \eta_D + \omega_2
 \end{array}$$

In addition to each item difficulty i , each position p has its own difficulty ω_p

■ 2 Effects of Response Formats

We administer the same 5 items using three different response formats:

- free format
- multiple choice with 4 distractors
- multiple choice with 3 distractors

Now we have item difficulty plus an effect of the response format.

item	resp.	η_A	η_B	η_C	η_D	η_E	ω_f	ω_4	ω_3	β_{ir}
A	free	1					1			$\eta_A + \omega_f$
B	free		1				1			$\eta_B + \omega_f$
C	free			1			1			$\eta_C + \omega_f$
D	free				1		1			$\eta_D + \omega_f$
E	free					1	1			$\eta_E + \omega_f$
A	4	1							1	$\eta_A + \omega_4$
B	4		1						1	$\eta_B + \omega_4$
C	4			1					1	$\eta_C + \omega_4$
D	4				1				1	$\eta_D + \omega_4$
E	4					1			1	$\eta_E + \omega_4$
A	3	1							1	$\eta_A + \omega_3$
B	3		1						1	$\eta_B + \omega_3$
C	3			1					1	$\eta_C + \omega_3$
D	3				1				1	$\eta_D + \omega_3$
E	3					1			1	$\eta_E + \omega_3$

X is partitioned in 3 Blocks containing 5 items each.