# Economic Valuation of Product Features and Patent Applications 

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## Economic Valuation of Product Features

If we enhance a product with a new or improved feature, this creates additional consumer surplus.

Consumer surplus can be monetized via a WTP computation.
WTP is defined as the compensating variation for this enhanced set of choices, i.e. what additional income do I have to pay consumers to be indifferent between set of products without feature enhancement and with?

This is not the value of the feature to the firm, however.

## Economic Valuation of Product Features

The extent to which a firm whose product has the enhancement can capture the surplus is determined by the competitive structure of the industry.
Are there many closely substitutable products?
What happens to firm profits in the world with and without the feature?

Economic value to the firm is determined by incremental profits.

## Measuring Incremental Profits

Requirements:

1. A valid, characteristics-based demand system to measure consumer surplus.
2. Cost assumptions
3. Equilibrium calculations: we must calculate the industry equilibrium with and without feature enhancement

## Product Features and Patent Litigation

Many patents are designed to enable certain product attributes (infamous example of the rounded edge icons on the iPhone screen).
Patent Litigation has three parts: 1) validity 2 ) infringement 3) damages.
How are damages calculated?

1. If patent holder has products in the market that practice the patent, lost profits are appropriate
2. If not, patent holder may seek a "reasonable" royalty, the result of a hypothetical negotiation

## The "But-for" World

To calculate damages, we need to think about the "but-for" world. That is the world in which the infringing products are not in the marketplace.
The but-for world is a counter-factual claim involving two issues:

1. Are there non-infringing alternatives that would provide the feature without infringing on the patent? If so, how substitutable are these?
2. How would the industry adjust to the removal of the feature in terms of prices, shares and total industry demand?

## Toward a Better Paradigm for Damages

A naïve model of damages: so-called the "whole" market rule - that is, attributing the entire source of the product value to the feature.
We need to decompose the total "value" of the product (iPhone) into that which can be attributed to rounded-edge icon and that which is not.

For this reason, patent litigators have been very attracted to conjoint analysis.

A partial list of apps: 1. smartphones 2. tablets 3. anti-virus software, 4. gaming consoles, 5. medical devices. Damages $>\$ 100$ million!

## A Better Paradigm for Damages

Compare the industry equilibrium with and without feature on the alleged infringing product.

Use conjoint to estimate demand system for industry.

Turn on and off features for the patent holder and possibly those who are alleged to infringe.

Compute changes in industry shares, total industry sales, and prices using cost assumptions and a model of competition
Damages calculated as the change in profits for the patent-holder (Apple) when there is an infringer (Samsung).

## Constructing a Valid Demand System

We must estimate a characteristics-based demand system. Requires data in which characteristics and prices have exogenous variation.

For many products there is little price variation of any kind (think consumer electronics).

We don't see the world without the feature present except via time series data.
Concerns regarding omitted characteristics and price.

Experimentally induced variation would be ideal!

## What is Conjoint?

| rsion: 2; Task: 1 | Survey Support Progress |  |  |  | 15\% Complete |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Scenario 1 of 16 | Camera 1 | Camera 2 | Camera 3 | Camera 4 | None of these |
| Brand | Canon Powershot | Panasonic Lumix | Sony Cyber-shot | Nikon COOLPIX |  |
| Megapixels | 16 | 16 | 16 | 16 |  |
| Optical Zoom | 10x | 4x | 10x | 4x |  |
| Video | Full HD Video (1080p) with Stereo Microphone | HD Video (720p) | HD Video (720p) | Full HD Video (1080p) with Stereo Microphone |  |
| Swivel Screen | No | Yes | No | Yes |  |
| Wifi | Yes | Yes | No | Yes |  |
| Price | \$79 | \$279 | \$379 | \$179 |  |
| Which of these digital cameras do you prefer? |  | 0 | 0 | 0 | 0 |
| As a reminder, you can hover over the features to view the definitions. |  |  |  |  |  |
| CONTINUE $>$ |  |  |  |  |  |

## Choice-based Conjoint Design

- Select attributes
- Select the levels for each attribute (e.g $8 \mathrm{mp}, 10$ mp, Wi-Fi enabled (yes or no)
- Create choice tasks (10-16 per respondent).

Orthogonal or fractional factorial designs to determine combinations of attributes/levels

Each task is created by drawing from this design matrix and finding tasks that have good balance (high and low levels are evenly distributed)

## Conjoint Design

We observe choice among hypothetical products which are specified by an attribute design matrix. Each row of the matrix specifies an attribute level combination.
$A=\left[\begin{array}{c}a_{1} \\ a_{2} \\ \vdots \\ a_{J}\end{array}\right]$ J choice alternatives

## Conjoint as Experimentally Generated Demand Data

Some advantages:
true exogenous variation. As much as you want?
Problems:
stated vs revealed preferences?
Incentives?
Some believe people display less price sensitivity than in the marketplace.

Respondents who exhibit low betas in general - i.e. behave as though choices are made at random.

## How should you analyze conjoint data?

We could analyze this using a standard Multinomial Logit choice model.

$$
\operatorname{Pr}(j \mid A, \beta)=\frac{\exp \left(a_{j}^{\prime} \beta\right)}{\sum_{j=1}^{J} \exp \left(a_{j}^{\prime} \beta\right)}
$$

Of course, every respondent is different (different preferences or beta vectors).

## A Hierarchical MNL Model

We can use a hierarchical model of the form

$$
\begin{array}{ll}
y_{i} \mid A_{i} ; \beta_{i} & \mathrm{MNL} \\
\beta_{i} \mid \tau & \text { Heterogeneity } \\
\tau \mid h & \text { Prior }
\end{array}
$$

The data for respondent $i$ consists of the design facing that respondent and the a vector of choices made.

Typically, we have 500 respondents, 16 choice tasks, and the beta vector is of dimension $\sim 10$. Depending how you count, about 5000 parameters.

## The Random Utility Model

We all know that the logit model comes from the stat literature, but economists have created a useful interpretation of the logit model as a Random Utility Model.

If

$$
u_{j}=a_{j}^{\prime} \beta+\varepsilon_{j}
$$

Then
Consumers observe utility and pick the alternative with maximum utility. We only observe $A$ and must calculate the probability of the observed choices.

## Interpreting the Logit Coefficients

The logit coefficients are often called part-worths. In the random utility model, then represent the incremental utility afforded an attribute level combination.

Suppose $\beta_{w_{i-F i}}=2$. This means we get 2 "utils" more utility from having Wi-Fi capability.

However, utility is measured on an interval scale. Each respondent has their own scale!!

## Interpreting the Logit Coefficients

How can I put the part-worths on a ratio scale (like $\$)$ ?

The price part-worth is how much utility changes for a one $\$$ increase in price. This is in the units of utility $/ \$$, sometimes called the marginal utility of income. If I divide the Wi-Fi coefficient by the absolute value of price coefficient, then I have a \$valued figure. Some people call this the Willingness To Pay for the feature. I call it pseudo-WTP.

$$
\mathrm{p}-\mathrm{WTP}=\frac{\beta_{\mathrm{w}_{i}-\mathrm{Fi}_{i}}}{\left|\beta_{\text {price }}\right|}=\frac{2}{.1}=\$ 20
$$

## Conjoint and Patent Valuation

Conjoint appears to be a very inexpensive solution to the thorny problem of ascribing total product value to each feature.
Conjoint studies can be fielded in a matter of days with internet panel providers and there is wellestablished turn-key software (SSI) to undertake the analysis.
However, if you want to have a credible basis for damages, the quality bar raises very substantially and the standard conjoint analysis tools must be supplemented.

## Sources of Economic Value

Feature valuation for either industry or litigation purposes must be based on the economic value of a feature to the firm - expected incremental profits.

Incremental profits come from:

1. A possible price premium for the product with the feature enhancement.
2. increased demand for the product (which can come at the expense of competitors or from expanded industry demand).

## How is Feature Valuation Done today?

There are two basic approaches. Both approaches are based entirely on demand and not on supply and, therefore, cannot be used to compute economic value.
"WTP": use part-worths and price coefficient to create some sort of "average" pseudo-WTP.
"WTB": hold prices constant and compute change in market share predicted by the conjoint model from switching feature on and off.

## Why WTP is not Economic Value

Properly designed and analyzed conjoint can be used to compute WTP.

We all know that HB methods show that there is a distribution of WTP. The distribution of WTP that IS the demand curve for the product. We can infer about WTP (albeit very imperfectly) at the respondent level. The question is how to summarize that distribution.

Market prices are determined by the WTP of the "marginal customer" whose WTP is close to marginal cost. Average WTP will frequently overstate economic value!

## Economic Problems with "Average" WTP

Consider example:

| WTP | Proportion |
| :--- | :--- |
| $\$ 5000$ | .01 |
| $\$ 100$ | .99 |

If cost=0, profit-maximizing price $=\$ 100$. Mean WTP is $\$ 149$.

WTP is determined by marginal not average consumer. Here the "marginal" person is the $\$ 100$ WTP respondent.
Equilibrium price depends on the entire distribution of WTP. Average, weighted average, median aren't the right summaries!

## WTP and Demand



## Why WTB is not Economic Value

In most situations (except the case of a flat supply curve), removing a product feature will result in lower equilibrium prices for the product and higher prices for competing substitute products.
This means that WTB also overstates the impact on market share from deletion of the product feature. The price adjustments will soften the effects on market share.

If the outside option is not included, then the WTB will overstate damages as the product feature is not allowed to increase total industry demand (zero sum game).


## Computing Equilibrium Outcomes

Economic Value is the incremental profits that accrue to the focal firm when the product feature is enhanced.

How do we properly compute the incremental profits?

We must allow other firms to compete in the market and we must allow for price competition.

We will use a standard Nash model of price competition.

## Assumptions used in Equilibrium Computations

## Product Characteristics are Fixed

That is, we don't consider the problem of reconfiguring products optimally. In the patent case, this is not interesting.

## No Exit

We assume that the product enhancement, while it will change the nature of competition, it will not cause firms to exit the market. That is we don't allow for a truly revolutionary feature that defines a new market. (no Segways, please).

## Nash Equilibrium

The standard price equilibrium in a market for differentiated products is a set of prices such that each firms product is profit-maximizing given the prices of all other firms.
The profit function for firm $j$ :

$$
\pi\left(p_{j} \mid p_{-j}\right)=E\left[\operatorname{Pr}\left(j \mid p_{1} A\right)\right]\left(p_{j}-c_{j}\right)
$$

Here the notation -j means all other indices than j. A is the set of features in the market.

## Nash Equilibrium

Expected market shares are obtained by integrating the choice probabilities (at fixed prices) over the distribution of respondent parameters.

$$
E[\operatorname{Pr}(j \mid p, A)]=\int \operatorname{Pr}(j \mid p, A, \beta) p(\beta) d \beta
$$

The First Order Conditions for the jth firm are

$$
\frac{\partial \pi}{\partial p_{j}}=E\left[\frac{\partial}{\partial p_{j}} \operatorname{Pr}(j \mid p, A)\right]\left(p_{j}-c\right)+E[\operatorname{Pr}(j \mid p, A)]
$$

## Nash Equilibrium and Computation

A Nash equilibrium price is a price vector which simultaneously satisfies the F.O.C.s for all J firms.

Since the integral required to find expected share must be computed via a simulation approximation, there is no closed form formula for the Nash Equilibrium price.
Two approaches:

1. Simultaneously minimize all FOCs
2. Iterate profit maximization

We use both.

## The Outside Option - Is it necessary?

Can you consistently estimate the logit preference parameters w/o the no-choice or "outside" option (i.e. with "forced" choice data)?

Yes, this is possible. The critical question is whether you can interpret the price coefficient as the marginal utility of income.

In theory, at least, when a respondent makes price trade-offs against features in forced choice, the respondent does consider the value of the money saved by lower price profiles.

## The Outside Option - Is it necessary?

## For computing equilibrium prices?

Here we need to allow for the explicit possibility that addition/deletion of the product feature will increase/decrease total demand.

We conclude outside option is necessary.
Price coefficient can be estimated with forced-choice data but the price elasticities are biased.

However, the devil is in the details.

## Price and Price Sensitivity

Pseudo WTP, real WTP, and economic value analyses are all critically dependent on the price coefficient.

This made all that more difficult with models of heterogeneous respondents and by the improper use of MCMC methods, e.g. computing respondentlevel "estimates."

Without constraints, a large fraction (upwards of 25 per cent) of price coefficient draws can be positive. Gets even worse with dummy variable coding of price levels.

## Problems for Equilibrium Pricing

Computation of equilibrium prices imposes even more strenuous demands on the conjoint analysis.
We form market level demand by integrating over (summing over) all draws of the logit model parameters.
Certainly, we cannot have positive price coefficient draws. In this case, the "optimal" pricing policy will be to "fire" all of customers except for the ones with positive price coefficients and then jack up price to whatever limits are used in the optimization search.
There are also comparable problems with small negative price coefficients.

## Conjoint Design for Equilibrium Computations

The requirement that conjoint data can be used to compute sensible equilibrium prices imposes a particular set of requirements:

1. The price coefficient must be estimated precisely.
2. The competitive set must be realistic.
3. Cost data may be required.
4. The outside option must be considered and implemented carefully.
As people start to compute pseudo-WTP and move toward equilibrium prices, the importance of high quality data will become ever more apparent.

## Computing Posterior Predictive Distributions

The hierarchical model postulates a distribution of logit parameters over the population. For example,

$$
\beta_{i} \sim N\left(\mu, V_{\beta}\right)
$$

Market Share is given by

$$
\operatorname{MktShr}\left(j \mid \mu, V_{\beta}\right)=\int \operatorname{Pr}(j \mid \beta) \phi\left(\beta \mid \mu, V_{\beta}\right) d \beta
$$

## Computing Posterior Predictive Distributions

Market Share determines firm profits and, therefore, equilibrium prices are a function of normal distribution parameters.

$$
p^{*}=f\left(\mu, V_{\beta}\right)
$$

Our analysis of the conjoint data provides the posterior distribution, $\mu, V_{\beta} \mid$ data, $h$.
Thus, we are able to simulate the posterior predictive distribution of optimal prices.

## An Illustration

We have advocated using equilibrium pricing methods to establish an economic value for a product feature.

We illustrate our method using a conjoint study of the Point and Shoot Digital Camera market.

Critical components of design are:

1. Outside Good via dual response
2. Set of major brands

## Digital Camera Conjoint Design

Attributes:

1. Brand: Canon, Sony, Nikon, Panasonic
2. Pixels: $10,16 \mathrm{mp}$
3. Zoom: 4x, 10x optical
4. Video: HD (720p), Full HD (1080p) \& mike
5. Swivel Screen: No, Yes
6. WiFi: No, Yes
7. Price: \$79-\$279


## Digital Camera Conjoint Analysis

Dual Response:
a. which of 4 profiles do you prefer?
b. would you buy at the stated price?

16 choice sets each with 5 (including outside) alternatives.

Screened to those who own a point and shoot digital camera. Should we screen to those in the market for same?

501 completed surveys. 468 after removal of "straight-line" (always pick same option, or always high price).

## HB-CBC computations

Price enters linearly with single coefficient.
Constrained to be $<0$ via re-parameterization.
Bayesm HB routine, rhierMnlMixture.
All coefficient draws are made from the predictive distribution given model hyper-parameters.

$$
\begin{aligned}
& \beta^{\star} \sim N\left(\bar{\beta}, V_{\beta}\right) \\
& \beta=\left[\begin{array}{c}
\beta_{- \text {price }}^{\star} \\
-\exp \left(\beta_{\text {price }}^{\star}\right)
\end{array}\right]
\end{aligned}
$$

## Priors Can Matter!

$$
\beta_{p}=-\exp \left(\beta_{p}^{*}\right)
$$

Beta_p_star
Beta_p
"Standard" diffuse prior settings are not reasonable. Implies a high prior
probability of low price sensitivity


## Priors Can Matter! <br> $$
\beta_{p}=-\exp \left(\beta_{p}^{*}\right)
$$

A tighter but more
reasonable prior.

Beta_p_star


Beta_p


## Part-Worths

Lot's of price insensitive people!
Should we censor sample so remove those who don't appear to be paying attention?

Industry standard practice.

Swivel Screen Part-Worth


## Elasticity Structure

| Price $\backslash$ Mkt Share | $\mathrm{MS}_{\text {Sony }}$ | $\mathrm{MS}_{\text {Canon }}$ | $\mathrm{MS}_{\text {Nikon }}$ | $\mathrm{MS}_{\text {Pansonic }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $P_{\text {Sony }}$ | -1.69 | .47 | .36 | .34 |
| $P_{\text {Canon }}$ | .49 | -1.79 | .40 | .45 |
| $P_{\text {Nikon }}$ | .39 | .41 | -1.62 | .31 |
| $P_{\text {Panasonic }}$ | .44 | .55 | .37 | -1.75 |

Price sensitive (those are very high price elasticities for a feature conjoint) but brand loyal.

$$
\frac{\partial M k t S h r_{j}}{\partial \ln p_{i}}=\frac{\partial}{\partial \ln p_{i}} \int \operatorname{Pr}(j \mid \beta) p\left(\beta \mid \mu_{\beta^{\prime}} V_{\beta}\right) d \beta
$$

## Equilibrium Prices

Start with products representing four major brands (CanonlSonylNikonIPanasonic). Set all attributes at lowest levels and turn "Swivel Screen" off.
Compute equilibrium in this market.
Add Swivel Screen to the Sony product profile and keep others off. Compute equilibrium prices and shares.

Difference in prices is a measure of market value for the feature. Note: this will depend on a cost assumption (here set at MC of $\$ 75$ and the assumption that the marginal cost of SS is $\$ 5$ ) as well as the set of alternative competing products.

## Change in Eq Prices



## Change in Equilibrium Profits (\% Change)

About a 35 per cent increase in Sony Profits.

Would be over 80 percent without price response!


## Pseudo-WTP

We compute posterior distribution of the E[p-WTP].
Mean is $\$ 63.21$.
Posterior dispersion is huge.
Sample size of around 500 is not informative enough!


## WTB or Change in Shares

Mean is about 5.5 share point increase, holding prices constant.

Change in
Shares allowing for price adjustment is about 1 per cent.

## A "Lost Profits" Damages Analysis

In patent litigation, there is a need to compute the profits lost to an alleged infringer.

Assume Nikon owns patent for the Swivel Screen technology and Sony infringes.

We compute two equilibria:

1. Nikon alone offers the SS
2. Both Sony and Nikon offer SS.

## A "Lost Profits" Damages Analysis



## First-Stage Prior Sensitivity

First stage of the prior is what non-Bayesian call the random coefficient distribution.

Even with diffuse hyper-parameter settings, the assumption of a normal first stage is strong. Induces a type of shrinkage due to the thin tails of the normal prior.

This is critical for equilibrium calculations as we use the normal distribution to represent the distribution of preferences.

## Mixtures of Normals

Mixtures of Normals can approximate any distribution whose density at the boundary of support is zero.

In particular, we might expect:

1. There is a set of respondents who don't pay attention and answer "at random."
2. The distribution of brand coefficients could be ver non-normal (some love the brand, most are indifferent).
3. Price coefficient distribution may have a price insensitive "component"

## Mixtures of Normals

Price Part-Worth


Swivel Screen Part-Worth


One Component

Price Part-Worth


Swivel Screen Part-Worth


Three Component

## Mixtures of Normals

Does it make a difference to equilibrium calculations?

Yes, these are sensitive to assumption of normality.
Mixture of normals fits to the entire sample yields a small mass of respondents with very low price sensitivity and gives rise to very high equilibrium prices.

We remove all respondents with a very low marginal likelihood (equivalent to guessing) and compute elasticity structure and equilibrium prices.

## Mixtures of Normals - Elasticity Structure

| Price $\backslash$ Mkt Share | $\mathrm{MS}_{\text {Sony }}$ | $\mathrm{MS}_{\text {Canon }}$ | $\mathrm{MS}_{\text {Nikon }}$ | $\mathrm{MS}_{\text {Pansonic }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $P_{\text {Sony }}$ | -2.58 | .77 | .05 | 1.07 |
| $P_{\text {Canon }}$ | .91 | -2.70 | .09 | .72 |
| $P_{\text {Nikon }}$ | .15 | .20 | -1.47 | .11 |
| $P_{\text {Panasonic }}$ | 1.33 | .75 | .05 | -2.75 |

Higher own elasticities
Nikon stands out with lower own elasticity and small cross elasticities. Due to small mass of respondents who have both large Nikon partworths and low price sensitivity

## Mixtures of Normals - Changes in Eq Prices

|  | Sony | Canon | Nikon | Panasonic |
| :---: | :---: | :---: | :---: | :---: |
| W/O SS | $\$ 117.66$ | $\$ 136.88$ | $\$ 247.25$ | $\$ 114.30$ |
| W SS | $\$ 140.94$ | $\$ 135.31$ | $\$ 249.15$ | $\$ 114.26$ |
| $\Delta$ | $\$ 23.27$ | $-\$ 1.58$ | $-\$ 1.90$ | $-\$ 0.04$ |

Very similar results to one-component normal model but lower in magnitude due to higher elasticities.

## Conclusions

We advocate the use of equilibrium calculations as a way of valuing product features.
p-WTP overstates price premium by about 100 per cent.

WTB overstates change in market share by about 550 per cent.
The use of conjoint data as the basis for equilibrium pricing calculations establishes a high bar for data quality, information content, and survey design.

## Further Reading

"Economic Valuation of Product Features," SSRN working paper.
"Valuation of Patented Product Features," forthcoming, Journal of Law and Economics.

