Ergodic BSDEs with Lévy noise and time dependence

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22 January, 2015

Research supported by the Oxford–Man Institute

Classical Ergodic control

- The goal of Ergodic optimal control is to understand payoffs that value the future as much as the present.
- In other words, taking a separable metric space of controls $\mathcal{U}$, the value functional takes the form

$$J(x,u) = \limsup_{T\to\infty} T^{-1}\mathbb{E}^u\left[\int_0^T L(X_t^x, u_t)dt\right]$$

  where X represents the forward dynamics, and u is an $\mathcal{F}_t$-predictable $\mathcal{U}$-valued process, and L is a bounded measurable cost function.
- Given the well-established connection between classical stochastic optimal control and BSDEs, as well as between BSDEs and the theory of nonlinear expectations, it is reasonable to expect that there exists a BSDE–based framework that would prove natural for understanding ergodic optimisation in nonlinear settings.

Definition of EBSDE

- One such framework is based on Ergodic BSDEs, an extension of BSDEs introduced by Fuhrman et. al. (in our case the filtration is generated by Wiener process together with an independent Poisson random measure) taking the form

$$
\begin{aligned}
Y_t = Y_T + \int_t^T [f(X_u, Z_u, U_u) - \lambda] du - \int_t^T Z_u dW_u \\
- \int_t^T \int_B U_s(x) \tilde{N}(ds, dx)
\end{aligned}
$$

  for *all* $0 \leq t \leq T < \infty$.
- A solution is a quadruple (Y, Z, U, λ), where $\lambda \in \mathbb{R}$, Y is adapted and Z, U are predictable.
- The constant λ is the ergodic value.

Ergodic control and EBSDE

A classical connection between stochastic control and BSDEs is well developed.

Take

$$f(x, y, z, U) = \inf_u \{L(x, u) + zR(u)\}$$

where $R(u)$ represents the control of the drift. Then we can solve a BSDE with f as a driver and we obtain both optimal value and control. The same is true for the ergodic case.

Risk-averse optimal control 1/2

- One natural way to apply their theory would be to look at the case where the expectation E_x^U in the EBSDE is replaced by a dynamically consistent nonlinear expectation.
- For example we can look at a variant of risk averse ergodic control under uncertainty, where the control u does not yield a single measure $\mathbb{Q}^u$, but a family of probability measures $\{\mathbb{Q}^{u,w}\}_{w\in\mathcal{W}}$ equivalent to some reference measure $\mathbb{P}$, where $\mathcal{W}$ is an index set.

Risk-averse optimal control 2/2

In this setting we could consider the problem of minimising the supremum over a suitable class of measures, corresponding to the value functional

$$J(x,u) = \limsup_{T\to\infty} T^{-1} \sup_{w\in\mathcal{W}} \mathbb{E}_x^{u,w}\Big[\int_0^T L(X_t,u_t)dt\Big].$$

It can then be verified that $J(x,u)$ is the λ-component of the solution to the EBSDE with $f(\cdot,\cdot,\cdot)$ replaced by a suitable control-dependent driver $f(u,\cdot,\cdot,\cdot)$.

Main goals of present work

- Add jumps to the diffusion setting of Hu et. al. From the standpoint of finance it allows us to factor shocks into the model.
- Incorporate time-dependence. This will allow us to consider dynamics with seasonal components, such as business cycles.
- Show how the techniques we develop can be applied to various problems of pricing under uncertainty on the example of power plant evaluation.

Plan of the talk

1. Motivation
   - Ergodic Control
   - First look at EBSDEs
   - Main goals of present work
2. Main result
   - Forward SDE
   - Ergodic BSDEs
   - Alternative representation for λ
3. Applications
   - Classical Ergodic Control
   - Power plant evaluation

Forward SDE

Setup

The forward dynamics are given by an H-valued process X, given by the following non-autonomous mild Itô SDE:

$$X(t,\tau,x) = U(t,\tau)x + \int_\tau^t U(t,s)F_s(X(s,\tau,x))ds + \int_\tau^t U(t,s)G(s)dL(s)$$

which is a mild version of the following Cauchy problem:

$$dX_t = A(t)X_t dt + F_t(X_t)dt + G(t)dL_t, \quad X_\tau = x.$$

Assumptions

Assumption

(i) *The family* A_t *generates exponentially bounded evolution family. Their adjoints* $A^*(t)$ *also have a common domain dense in* H*.*

(ii) $F : \mathbb{R}^+ \times H \to H$ *is a uniformly bounded family of measurable maps with common domain* $D(F)$ *which is dense in* H*.*

(iii) $\{G_t\}_{t \geq 0}$ *is a uniformly bounded family of linear operators in* $L(H, H)$ *with common domain* $D(G)$ *dense in* H *and with bounded inverses.*

(iv) *The linear operator* $U(t, \cdot)G(\cdot)$ *is uniformly bounded in the Hilbert–Schmidt norm.*

(v) $(\Omega, \mathcal{F}, \mathbb{P})$ *is a complete probability space, and the pair* $(W, \tilde{N})$ *that comes from the Itô–Lévy decomposition of* L *has a predictable representation property in the filtration* $\{\mathcal{F}_t\}_{t \geq 0}$*.*

(vi) *Coefficients* $A(t)$*,* $F(t, \cdot)$ *and* $G(t)$ *are* T^*- *periodic for some* $T^* \geq 0$*, that is, for example,* $A(t + T^*) = A(t)$*.*

Evolution family

Definition

An exponential bounded evolution family on H is a two-parameter family $\{U(t,s)\}_{t\geq s}$ of bounded linear operators on H such that we have

- $U(s,s) = I$ and $U(t,s)U(s,r) = U(t,r)$ for $r \leq s \leq t$
- for each $x \in H$ $(t,s) \to U(t,s)x$ is continuous on $s \leq t$
- there exists $M > 0$ and $\mu > 0$ such that $||U(t,s)||_{op} \leq Me^{-\mu(t-s)}$ for $s \leq t$

Remark

By 'generates' we mean that for $0 \leq s \leq t$ *we have*

$$\frac{d}{dt}U(t,s)x = A(t)U(t,s)x.$$

Coupling estimate

Theorem

Let the family $F : \mathbb{R}^+ \times H \to H$ *satisfy the assumptions above. In particular,* $\{F_t\}_{t \geq 0}$ *is uniformly bounded. Then there exist constants* $C > 0$ *and* $\rho > 0$ *such that*

$$|P(\tau, t)[\psi](x) - P(\tau, t)[\psi](y)| \leq C(1 + ||x||^2 + ||y||^2)e^{-\rho(t-\tau)}||\psi||_0$$

where

$$P(s, t)[f](x) := \mathbb{E}\big[f(X(t, s, x))\big],$$

is a two-parameter semigroup associated with the forward SDE, and constants C *and* ρ *depend on* $\sup_{t \geq 0, u \in H} F_t(u)$*, but are independent of* τ*.*

Idea of proof 1/2

We focus on the case $\tau = 0$. We consider the coupled process

$$Y_t = \begin{cases} X_t^y, & t < T, \\ X_t^x, & t \geq T. \end{cases}$$

where $T := \inf\{s : X_s^x = X_s^y\}$ is the first meeting time of X^x and X^y. We notice that Y and X^y have the same law. Now, for any bounded $\phi : H \to \mathbb{R}$,

$$\begin{aligned} |\mathcal{P}_t[\phi](x) - \mathcal{P}_t[\phi](y)| &= |\mathbb{E}\phi(X_t^x) - \mathbb{E}\phi(Y_t)| \\ &= \left|\mathbb{E}\left([\phi(X_t^x) - \phi(Y_t)]\mathbf{1}_{\{T>t\}}\right)\right| \\ &\leq 2 \sup_{x \in H} |\phi(x)| \mathbb{P}(T > t) \end{aligned}$$

and by Chernoff's inequality, for any $\rho > 0$

$$\mathbb{P}(T > t) \leq \mathbb{E}\left[e^{\rho T}\right] e^{-\rho t}$$

Idea of proof 2/2

- Show that there exists an exponential bound on the waiting time for any given solution to enter a "small set", in our case a fixed ball around the origin.
- Show that we can construct a successful coupling of two solutions with different initial conditions (within the same fixed ball) on a given time interval, in the sense that we can produce two processes that have corresponding laws and meet with probability that is bounded below.
- Use these arguments over and over again to prove that the laws of two solutions with different initial conditions get exponentially close with time.

Recurrence

We prove the recurrence result for processes satisfying the forward SDE. We use this result later to prove uniqueness of the Markovian solution.

Theorem

For any $x_0, x \in H$, $s \geq 0$ *and for any fixed* $\epsilon > 0$, *we define*

$$\tau := \inf\{t \geq s : X(t,s,x) \in B_\epsilon(x_0)\}.$$

Then the process $X(\cdot, s, x)$ *satisfies*

$$\mathbb{P}(\tau > T) \to 0 \quad as \quad T \to \infty.$$

Idea of proof

Assumption

For notational simplicity we assume that we start at time $\tau = 0$. Then the process $Z_A(t)$ defined by

$$Z_A(t) = \int_0^t U_s G(s) dL_s.$$

spans the entire space H, that is, $\mathbb{P}(Z_A(t) \in B) > 0$ for all $t > 0$ and any open $B \in H$.

Lemma

If the process $Z_A(t)$ satisfies Assumption above for all $t > 0$, then the forward process X_t^x is irreducible. In other words

$$\mathbb{P}(X_t^x \in B_\epsilon(z)) > 0$$

for any $t > 0$, $z \in H$, $\epsilon > 0$.

Summary, forward SDE

- Under certain assumptions there exists a unique solution to the forward SDE.
- The laws of two solutions with different starting points converge exponentially in time uniformly over the family of bounded nonlinearities.
- Under additional assumption the solution is irreducible and recurrent.

Backward SDE

The EBSDE

We recall that we are interested in the equation

$$Y_t = Y_T + \int_t^T [f(X_u, Z_u, U_u) - \lambda] du - \int_t^T Z_u dW_u \\ - \int_t^T \int_B U_s(x) \tilde{N}(ds, dx)$$

for *all* $0 \leq t \leq T < \infty$. We would like to know whether a solution exists and what are its properties.

Definition

Henceforth we assume that the driver of a BSDE with jumps is a function $f : \Omega \times \mathbb{R}^+ \times \mathbb{R} \times H \times \mathcal{L}^2(B, \mathcal{B}, \nu) \to \mathbb{R}$.

Assumption

- *f is "monotonic" w.r.t y: $\exists \alpha \in \mathbb{R}$ such that* $\forall t \geq 0, \forall y, y' \in \mathbb{R}, \forall z \in H, \forall u \in \mathcal{L}^2(B, \mathcal{B}, \nu)$

$$(y - y')(f(\omega, t, y, z, u) - f(\omega, t, y', z, u)) \leq \alpha |y - y'|^2 \quad \mathbb{P} - a.s.$$

- *f is Lipschitz w.r.t. z and u. That is* $\exists K \geq 0 : \forall t \in [0, T], \forall y \in \mathbb{R}, \forall z, z' \in H^*, \forall u, u' \in \mathcal{L}^2(B, \mathcal{B}, \nu)$

$$|f(\omega, t, y, z, u) - f(\omega, t, y, z', u')| \leq K ||z - z'|| + K \left(\int_B |u(v) - u'(v)|^2 \nu(dv) \right)^{1/2}$$

Assumption

- *f is continuous w.r.t y and the exists an* $\mathbb{R}^+$*-valued process* $(\phi_t)_{0\leq t\leq T}$ *such that* $\mathbb{E}\left(\int_0^T \phi_s^2 ds\right) < \infty$ *and*

$$|f(\omega,t,y,z,u)| \leq \phi_t + K\left(|y| + ||z|| + \int_B |u(v)|^2 \nu(dv)\right)^{1/2}$$

- *There exists* $-1 < C_1 \leq 0$ *and* $C_2 \geq 0$ *such that* $\forall x \in H$, $\forall z \in H$, $\forall u, u' \in \mathcal{L}^2(B, \mathcal{B}, \nu, \mathbb{R})$ *we have*

$$f(\omega,t,z,u) - f(\omega,t,z,u') \leq \int_B (u(v) - u'(v))\gamma^{\omega,t,z,u,u'}(v)\nu(dv),$$

where $\gamma^{\omega,t,z,u,u'} : \Omega \times B \to \mathbb{R}$ *is measurable and satisfies*

$$C_1(1 \wedge ||x||) \leq \gamma^{\omega,t,z,u,u'}(x) \leq C_2(1 \wedge ||x||).$$

Existence

The following existence theorem for finite horizon BSDEs with jumps is due to Royer (2006, see also Barles et al '94, Situ '97). In that paper the case of finite-dimensional Brownian motion is considered. The extension to the case where W is a Q-Wiener processes is immediate.

Theorem

Under the above assumptions there exists a unique solution $(Y, Z, U) \in (\mathcal{S}^2 \times \mathcal{L}^2(W) \times \mathcal{L}^2(\nu))$ to the equation

$$Y_t = \eta + \int_t^T f(\omega, u, Y_u, Z_u, U_u)du - \int_t^T Z_u dW_u - \int_t^T \int_B U_s(x)\tilde{N}(ds, dx),$$

where $\eta \in \mathcal{L}^2(\mathcal{F}_T)$ is the terminal condition.

Infinite horizon discounted BSDEs 1/2

Theorem

Let $\alpha > 0$ *and* $f : \Omega \times \mathbb{R}^+ \times H \times \mathbb{R} \to \mathbb{R}$ *be such that*

- f *satisfies the assumptions above*
- $|f(\omega, t, 0, 0)|$ *is uniformly bounded by* $C \in \mathbb{R}$

Then there exists an adapted solution (Y, Z, U)*, with* Y *càdlàg and* $Z \in \mathcal{L}^2(W)$*,* $U \in \mathcal{L}^2(\tilde{N})$ *to the infinite horizon equation*

$$
\begin{aligned}
Y_T = Y_t &- \int_t^T (-\alpha Y_u + f(\omega, u, Z_u, U_u))du + \int_t^T Z_u^* dW_u \\
&+ \int_t^T \int_B U_s(x)\tilde{N}(ds, dx),
\end{aligned}
$$

for all $0 \le t \le T < \infty$*, satisfying* $|Y_t| \le C/\alpha$*, and this solution is unique among bounded adapted solutions.*

Infinite horizon discounted BSDEs 2/2

Theorem

Furthermore, if (Y^T, Z^T, U^T) denotes the (unique) adapted solution to

$$Y_t^T = \int_t^T (-\alpha Y_u^T - f(\omega, u, Z_u^T, U_u^T))du - \int_t^T (Z_u^T)dW_u - \int_t^T \int_B U_s(x)\tilde{N}(ds, dx)$$

then $\lim_{T\to\infty} Y_t^T = Y_t$ a.s., uniformly on compact sets in t.

The proof is carried out by standard techniques: change of measure + Tanaka's formula for general semimartingales (Jacod) many times.

Markovian solution

Assumption

(Markovian structure) *In the sequel we will assume that the driver f is Markovian, that is*

$$f(\omega, t, Z_t, U_t) = \bar{f}(X_t(\omega), Z_t, U_t)$$

for some $\bar{f}$. For convenience we simply write f for $\bar{f}$.

Let $(Y^{\alpha,x,s}, Z^{\alpha,x,s}, U^{\alpha,x,s})$ be the unique bounded solution to the discounted BSDE in infinite horizon

$$Y_T^{\alpha,x,s} = Y_t^{\alpha,x,s} - \int_t^T (-\alpha Y_u^{\alpha,x,s} + f(X(t,s,x), Z_u^{\alpha,x,s}, U_u^{\alpha,x,s}))du + \int_t^T (Z_u^{\alpha,x,s})dW_u + \int_t^T \int_B U_u^{\alpha,x,s}(x)\tilde{N}(du,dx),$$

for $0 \leq s \leq t \leq T < \infty$. We denote $v^\alpha(s,x) = Y_s^{\alpha,x,s}$.

Convergence

With the notation established above, and using the result on convergence of laws, we can prove the following theorem:

Theorem

For an arbitrary $x_0 \in H$, there exist bounds C' and C such that

$$|v^\alpha(s,x) - v^\alpha(s,x_0)| < C'(1 + \|x\|^2 + \|x_0\|^2), \qquad \alpha|v^\alpha(s,x)| < C$$

holds uniformly in x, s and α. Hence there exists a sequence $\alpha_n \to 0$ such that

$$(v^{\alpha_n}(s,x) - v^{\alpha_n}(s,x_0)) \to v(s,x) \quad \text{and} \quad \alpha_n v^{\alpha_n}(s,x) \to \lambda$$

for all $s \geq 0$, $x \in H$ and for some bounded function $v : \mathbb{R}^+ \times H \to \mathbb{R}$ and some $\lambda \in \mathbb{R}$.

Existence

Theorem

Let v and λ be constructed as above. We also set $x_0 = 0$ for the sake of simplicity. Then if we define $Y_t^x = v(t, X_t^x)$, there exist processes Z^x and U^x such that the quadruple (Y^x, Z^x, U^x, λ) solves the EBSDE

$$Y_t^x = Y_T^x + \int_t^T [f(X_u^x, Z_u^x, U_u^x) - \lambda] du - \int_t^T (Z_u^x)^* dW_u - \int_t^T \int_B U_s^x(x) \tilde{N}(ds, dx).$$

Moreover, if there exists any other solution (Y', Z', U', λ') that satisfies

$$|Y_t'| < c_x(1 + \|X_t^x\|^2)$$

for some constant c that may depend on x, then $\lambda = \lambda'$.

Uniqueness

In order to establish uniqueness we look at the class of Markovian solutions $Y_t = v(t, X_t)$ for which

$$v(t,x) = v(t + T^*, x) \quad \forall t > 0, x \in H$$

Theorem

Let (Y, Z, U, λ) and (Y', Z', U', λ') be two Markovian solutions to the EBSDE, in the sense that $Y_t = v(t, X_t^x)$, $Y'_t = v'(t, X_t^x)$ for some continuous functions $v, v' : \mathbb{R}^+ \times H \to \mathbb{R}$. If v, v' satisfy polynomial growth in the second argument, and $v'(0,0) = v(0,0)$, then $v = v'$ $\mathbb{P} - a.s.$

Invariant measure

We define the one parameter semigroup associated with the forward equation by

$$P_s u(t,x) := \mathbb{E} u(t+s, X(t+s,t,x)).$$

Then, using results established by Knäble we obtain the following:

Lemma

There exists a unique invariant measure for the semigroup P. In other words for every bounded measurable function u such that $u(t+T^*,x) = u(t,x)$ for each $t>0$ and $x \in H$ we have:

$$\int_{[0,T^*]\times H} P_s u(t,x)\mu(dt,dx) = \int_{[0,T^*]\times H} u(t,x)\mu(dt,dx).$$

Representation

We recall that the Markovian solution to the EBSDE (10) is T^* - periodic, that is the quadruple (Y, Z, U, λ) has a representation (v, ξ, ψ, λ), where v, ξ and ψ are T^*-periodic in time.

Theorem

The value λ in the EBSDE solution (v, ξ, ψ, λ) satisfies

$$\lambda = \int_{[0,T^*]\times H} f(x, \xi(t,x), \psi(t,x))\mu(dt, dx),$$

where μ is the unique invariant measure.

Summary, EBSDEs

- There exists a unique bounded solution to the infinite time horizon discounted BSDE.
- There exists a solution to the Markovian EBSDE.
- In a certain class this solution is unique.
- λ is the long term average.

Applications

Classical control 1/3

Denoting $L : H \times \mathcal{U} \to \mathbb{R}$ a bounded measurable cost function such that

$$|L(x,u) - L(x',u)| \leq C\|x - x'\|,$$

for some $C > 0$. We consider the problem of minimising

$$J(x_0,u) = \limsup_{T\to\infty} T^{-1}\mathbb{E}^{u,T}\Big[\int_0^T L(X_t,u_t)dt\Big],$$

over the space $\mathcal{U}$ of controls, which is a separable metric space and $u_t(\omega)$ takes values in $\mathcal{U}$. We further assume that under $\mathbb{P}^{u,T} \sim \mathbb{P}$ the dynamics of the controlled process X on $[0,T]$ are given by

$$dX_t = (A(t)X_t + F_t(X_t))dt + R(u_t)dt + \Big(\int_B \gamma(u(t),y)\nu(dy)\Big)dt + G(t)dL_t.$$

We further assume that $\|R(u)\| \leq C'$ and γ satisfies all the assumptions.

- We define the Hamiltonian

$$f(x,z,r) = \inf_{u\in\mathcal{U}} \Big\{ L(x,u) + zR(u) + \int_B \gamma(u,\xi) r(\xi) \nu(d\xi) \Big\},$$

where $x \in H$, $z \in H^*$ and $r : B \to \mathbb{R}$.

- Immediately we notice that $f(x,0,0)$ is bounded. It is also easy to check that f satisfies all our assumptions.
- Therefore, the EBSDE with driver $f(x,z,r)$ admits a unique (in the class of processes with polynomial growth) Markovian solution (Y,Z,U,λ).
- If infimum in the Hamiltonian is attained, there exists (assuming the continuum hypothesis) a measurable function $\kappa : H \times H^* \times \mathcal{L}^2(B,\mathcal{B},\nu,\mathbb{R}) \to \mathcal{U}$ such that

$$f(x,z,r) = L(x,\kappa(x,z,r)) + zR(\kappa(x,z,r)) + \int_B \gamma(\kappa(x,z,r),\xi) r(\xi)\nu(d\xi).$$

In the notation established above we have the following theorem:

Theorem

Let the quadruple (Y, Z, U, λ) be the unique Markovian solution satisfying $|Y_t| \leq c(1 + \|X_t\|^2)$ for all $t \geq 0$ and some $c > 0$. Then the following hold:

(i) *For an arbitrary control $u \in \mathcal{U}$ we have $J(x_0, u) = \lambda$ if and only if*

$$f(X_t, Z_t, U_t) = L(X_t, u(t)) + Z_t R(u(t)) + \int_B \gamma(u(t), \xi) r(\xi) \nu(d\xi)$$

$d\mathbb{P} \times dt - a.e.$.

(ii) *If the infimum is attained in the Hamiltonian, then the control $\bar{u}(t) = \kappa(X_t, Z_t, U_t)$ verifies $J(x_0, \bar{u}) = \lambda$.*

Power plant evaluation

Setup

Definition

We denote by $\{E(t)\}_{t\geq 0}$ and $\{G(t)\}_{t\geq 0}$ the electricity and gas price processes respectively. We assume that a power plant allows its owner to convert gas into electricity instantaneously, generating profit if the spark spread $E(t) - cG(t) > 0$, where c is some conversion constant.

We assume that $X := E - cG$ has the following dynamics:

$$dX_t = \theta_t(\kappa_t - X_t)dt + G(t)\Big[dW_t - \int_B x\tilde{N}(dt, dx)\Big], \quad X_\tau = x,$$

where $B = \mathbb{R}\backslash\{0\}$, $\{\theta_t\}_{t\geq 0}$ is a positive process that describes the rate of mean reversion, $\{\kappa_t\}_{t\geq 0}$ is a non-negative process of the mean and $\tilde{N}$ is a compensated Poisson random measure on $\mathbb{R}_+ \times B$ with the compensator $\eta(dt, dx) = dt\nu(dx)$.

The problem

A logical way for evaluation would be to estimate the average yearly profit from the power plant, namely

$$\lambda = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\int_t^T (X_s)^+ ds,$$

where $(x)^+ := \max(x, 0)$. However, we assume that the parameters of the dynamics are not known exactly. Therefore our goal is to solve the risk averse problem by finding the worst case value of λ. In other words:

$$\lambda = \inf_{u\in\mathcal{U}} \lim_{T\to\infty} \frac{1}{T}\mathbb{E}^u\int_t^T (X_s^u)^+ ds,$$

where $\mathcal{U}$ denotes the space of admissible controls.

Controlled dynamics

- We assume that under the measure $\mathbb{P}^u \sim \mathbb{P}$ the dynamics of X are given by

$$dX_t = \theta_t(\kappa_t - X_t)dt + R(X_t, u(t))dt + \int_B \gamma(u(t), y)\nu(dy)dt + G(t)\big[dW_t - \int_B x\tilde{N}(dt, dx)\big].$$

  where we control the rate of mean reversion through R and the rate of spikes through γ.
- In order to make the model more realistic, without loss of clarity one can also consider the problem of minimising a generalised functional

$$\lambda = \inf_{u\in\mathcal{U}} \lim_{T\to\infty} \frac{1}{T}\mathbb{E}^u \int_t^T L(X_s, u(s))ds,$$

  where $L(x, u)$ incorporates a penalty corresponding to the perceived likelihood of the parameters being realised.

Connection to EBSDEs

We denote the Hamiltonian

$$f(x,z,r) = \inf_{u\in\mathcal{U}} \Big\{ L(x,u) + zR(x,u) + \int_B \gamma(u,\xi) r(\xi)\nu(d\xi) \Big\},$$

and proceed to solve the EBSDE with the driver f. This gives us the worst case scenario average yearly profit of the plant.

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