On the Estimation of Credit Exposures Using Regression-Based Monte-Carlo Simulation

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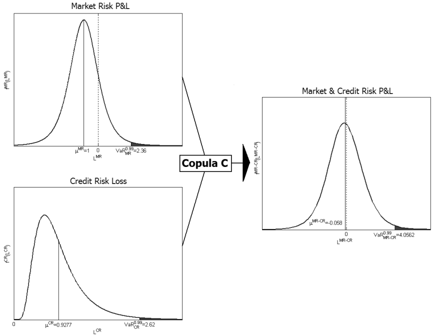
Broader Background: Market & Credit Risk Aggregation

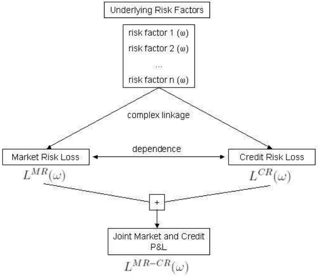
- General motivation: firmwide risk aggregation of market and credit risks - Top-Down vs. Bottom-Up Approach
- Top-Down approach is based on the concept of Copulas and specifies dependence at risk type level



Bottom-Up Approach

- Bottom-up methodology captures dependence at an elementary risk driver level
- Several steps that need to be considered:
  - (a) identification of risk factors;
  - (b) use of historical data to determine the relationships between risk drivers;
  - (c) simulation of the most appropriate model for the risk factors.
- Appropriate risk drivers
  - for market risk: equity prices, interest rates, swap rates, bond prices, FX rates, commodity prices, credit spreads,...
  - for credit risk: default trends and rates, expected default frequencies (EDFs),credit spreads, recovery rates, macroeconomic variables,...

Bottom-Up Approach



- Implications: consistent valuation for all type of products accounting for dependencies among risk drivers is needed; as well as a proper definition of a joint bottom-up loss variable for market and credit risk.

Introduction

- Risk management function has advanced significantly in recent years
- Three credit risk components: default indicator, exposure at default, loss given default
- Problem: to quantify credit exposure for complex products with no analytical (closed-form) solution in a scenario conistent way
- Scarce literature: conditional valuation approach by Lomibao and Zhu (2005), Phykhtin and Zhu (2006,2007) using the Brownian brigde technique
- Our approach: modified version of least-squares Monte-Carlo technique by Longstaff and Schwartz (2001)

Notation

- Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ be a filtered probability space with underlying stochastic process $(X_t)_{t\geq 0}$, $X_t \in \mathbb{R}^d$, $\mathcal{F}_t = \sigma(X_s,\ 0 \leq s \leq t)$, $\mathbb{P}$ is physical prob. measure, assume there exists EMM $\mathbb{Q} \sim \mathbb{P}$
- $V_t,\ t \geq 0$: value of a financial product over time
- $NPV_t = \mathbb{E}^{\mathbb{Q}}[C_{ashflows}(t, T)|\mathcal{F}_t]$: net present value of outstanding cash flows to some counterparty, $C_{ashflows}(t, T)$, $0 \leq t \leq T$: a claim's discounted net cash flow between time t and T.
- Distinction between market and credit risk exposure

Market Exposure

Definition (Market Exposure)

We define market exposure ME_t at time t as the future market value of the product, i.e.

$$ME_t = V_t, \quad t \geq 0.$$

Moreover, the time-t maximum likely market exposure (MLME) at some confidence level α is given by

$$MLME_t^{\alpha} = \inf\{x : \mathbb{P}[ME_t > x] \leq 1 - \alpha\}. \tag{1}$$

Then the expected market exposure (EME) at time t as seen from time zero is given by

$$EME_t = \mathbb{E}^{\mathbb{P}}[ME_t] \quad (= \mathbb{E}^{\mathbb{P}}[ME_t \,|\mathcal{F}_0]). \tag{2}$$

Credit Exposure

Definition (Credit Exposure)

We define the credit exposure CE_t at time t to a counterparty as its zero-floored expected discounted outstanding cash flows, i.e.

$$CE_t = \max(NPV_t, 0), \quad t \geq 0. \tag{3}$$

Analogously to (1) and (2), we define

$$MLCE_t^{\alpha} = \inf\{x : \mathbb{P}[CE_t > x] \leq 1 - \alpha\} \quad t \geq 0,$$

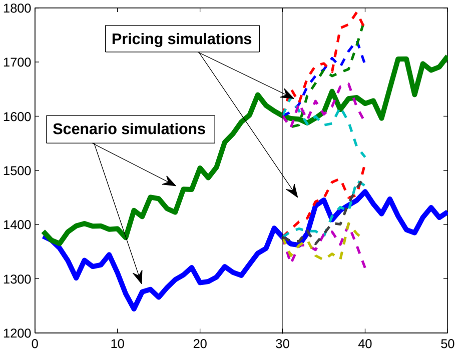
and

$$ECE_t = \mathbb{E}^{\mathbb{P}}[CE_t] \quad t \geq 0.$$

Provided the counterparty defaults at time t, i.e. on $\{\tau = t\}$, corresponds to the so-called exposure at default (EAD).

Simulations within Simulations

- MC-simulation has become the favorite tool for pricing complex financial instruments
- Calculation of future exposures: rather easy for products with closed-form solution;
  simulate underlying risk drivers X_t under $\mathbb{P}$ (scenarios) and insert them into the corresponding formulas for V_t or NPV_t (pricing).
- If there is no closed-form solution $\rightarrow$ computational infeasible, due to additional simulations (under $\mathbb{Q}$) for pricing.
- Solution: approximate conditional expectations for V_t or NPV_t

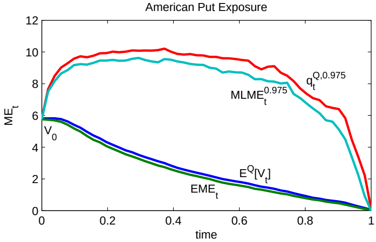

Pricing simulations
Scenario simulations
1800
1700
1600
1500
1400
1300
1200
0
10
20
30
40
50

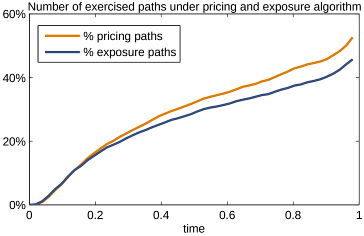
American Put

- The American put option price V_t at time t is calculated according to

$$V_t = \sup_{\tau \geq t} \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_t^{\tau} r_s ds} (K - S_\tau) | \mathcal{F}_t \right], \tag{4}$$

  with optimal stopping times $\tau \in \mathcal{T}$
- LSMC: Estimate optimal stopping time τ^* by backward induction comparing at each point in time the intrinsic value I_t (exercise value) and the extrinsic value F_t (continuation value),
- For the purpose of modeling exposures, we also need to account for the change of measure $\frac{d\mathbb{Q}}{d\mathbb{P}}$


American Put Exposure
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
time
ME_t
$q_t^{Q,0.975}$
$MLME_t^{0.975}$
V_0
$E^Q[V_t]$
EME_t


Number of exercised paths under pricing and exposure algorithm
60%
40%
20%
0%
0
0.2
0.4
0.6
0.8
1
time
% pricing paths
% exposure paths

Exposure Algorithm

- Additional Notation:
  $C_{ashflows}(t, T) = D(t, T)h(X_T)$ single cash-flow with
  discount factor: $D(t, T) = e^{-\int_t^T r_s ds}$
  payoff function: $h(X_t)$
  zero-coupon bond prices: $B(t, T) = \mathbb{E}^{\mathbb{Q}}[D(t, T)|\mathcal{F}_t]$
  change of measure density: $M_t := \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t}$

Algorithm (Exposure algorithm)

(1) *Simulate L independent paths* $X_{t_k}^{(l)}$, $k = 0, \ldots, K$, $l = 1, \ldots, L$, *of the underlying process under the* $\mathbb{P}$ *probability.*

Algorithm (Exposure Algorithm cont.)

(2) *At terminal date* $t_K = T$, *set* $V_T^{(l)} = h(X_T^{(l)})$ *for* $l = 1, \ldots, L$, *and define the stopping time* $\tau_K := T$.

(3) *Apply backward induction, i.e.* $k+1 \rightarrow k$ *for* $k = K-1, \ldots, 1$

(a) *estimate extrinsic or continuation values* $F_{t_k}^{(l)} = B^{(l)}(t_k, \tau_{k+1})\tilde{F}_{t_k}^{(l)}$ *for* $l = 1, \ldots, L$, *where* $\tilde{F}_{t_k}^{(l)}$ *is estimated by regressing discounted measure-rebased values*

$$\frac{D^{(l)}(t_k, \tau_{k+1})}{B^{(l)}(t_k, \tau_{k+1})} \frac{M_{\tau_{k+1}}^{(l)}}{M_{t_k}^{(l)}} V_{\tau_{k+1}}^{(l)}$$

on appropriate basis functions;

Algorithm (Exposure Algorithm cont.)

(3) (b) *define a new stopping time* $\tau_k^{(l)}$ *according to the stopping rule; e.g. for the American option*
$\tau_k^{(l)} = \min\{m \in \{k, k+1, \dots, K\} | I_{t_m}^{(l)} \geq F_{t_m}^{(l)},\ I_{t_m}^{(l)} > 0\}$;

(c) *for each path* $l = 1, \dots, L$ *set* $NPV_{t_k}^{(l)} = \max\{I_{t_k}^{(l)}, F_{t_k}^{(l)}\}$ *and* $NPV_{t_m}^{(l)} = 0$ *for* $t_m > \tau_k^{(l)}$.

(4) *Calculate the estimated exposure at time* $t = 0$ *by*

$$NPV_0 = \frac{\sum_{1 \leq l \leq L} D^{(l)}(0, \tau_1) M_{\tau_1}^{(l)} V_{\tau_1}^{(l)}}{L}.$$

(5) *Set* $CE_t^{(l)} = \max(NPV_t^{(l)}, 0)$ *for the estimated credit exposures.*

Decomposition and Truncation

- Let $D_1, \ldots, D_d$ be a partition of the state-space of X_τ
- Assuming interest rates to be zero, the continuation value at time t is given by $F_t = \mathbb{E}^{\mathbb{P}}[\frac{M_\tau}{M_t} h(X_\tau) | \mathcal{F}_t]$.
- Applying Bayes theorem we can decompose the continuation value:

$$F_t = \sum_{i=1}^{d} \underbrace{\mathbb{E}^{\mathbb{P}}[\frac{M_\tau}{M_t} h(X_\tau) | X_\tau \in D_i, \mathcal{F}_t]}_{=:I} \cdot \underbrace{\mathbb{P}[X_\tau \in D_i | \mathcal{F}_t]}_{=:II}.$$

- Truncation of I by setting minimum and maximum values for this partition
- Decomposition of II can be achieved by multi-nomial regression techniques

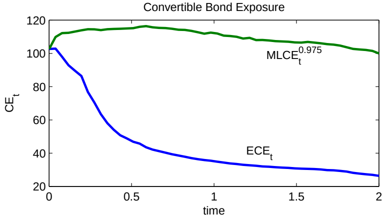
Convertible Bond

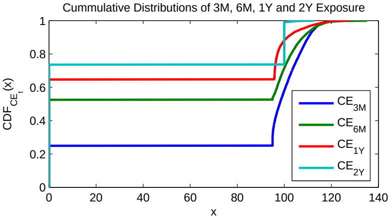
- Setup: Convertible bond with multiple exercise decisions (voluntary and forced conversion, put and call options)
- Underlying stochastic driver: dividend paying stock with dynamics

$$dS_t = (\mu - \delta)S_t dt + \sigma S_t dW_t^{\mathbb{P}} \quad S_0 = 100$$

- Exercise can take place at each point in time; maturity: 2 years.

2Y Convertible Bond Exposures

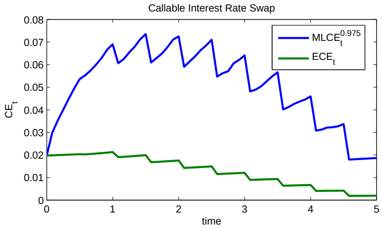


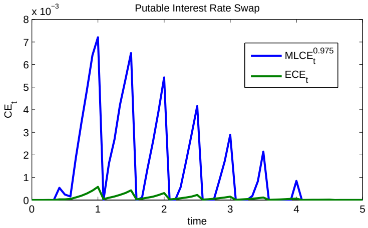


Cancelable Interest Rate Swaps

- A callable (putable) swap is an interest rate swap (IRS), where the fixed rate payer (receiver) has the right, but not the obligation to terminate the contract at pre-determined dates during the swaps lifetime
- Specification: short-rate r_t is governed by a two-factor Vasicek model
- 5-year cancelable IRS contract, which can be exercised each half a year;

5Y Cancelable Interest Rate Swaps





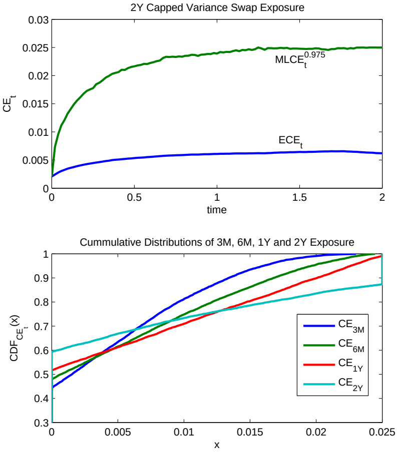
Variance Swap

- A standard variance swap pays off the difference between the annualized realized variance σ_R^2 and a pre-specified strike K
- Additional feature: capped by $\sigma_{Cap}^2 = 0.075$; i.e. the payoff becomes

$$\min\left(\underbrace{\frac{1}{T}\int_0^T v_s ds}_{\sigma_R^2}, \sigma_{Cap}^2\right) - K,$$

  where v_t denotes the instantaneous variance rate, $K = 0.05$.
- v_t is modelled by Heston model with stochastic central tendency

2Y Capped Variance Swap



Conclusion

- Presented approach allows to estimate credit exposures without closed-form formulas by backward induction
- The algorithm incorporates change of measure technique, and partitions the state space of the payoff function
- Practical feature: simple and easy to implement
- However, comparison of performance with other techniques such as non-parametric approaches using Malliavin calculus are desirable.

References

- F.E. Longstaff and E.S. Schwartz, *Valuing American Options by Simulation: A Simple Least-Squares Approach*, Review of Financial Studies, Vol. 14, No. 1, pp. 113-147, 2001.
- D. Lomibao and S. Zhu, *A Conditional Valuation Approach for Path-Dependent Instruments*, Counterparty Credit Risk Modeling, Risk Books, London, 2005.
- M. Pykhtin and S. Zhu, *Measuring Counterparty Credit Risk for Trading Products under Basel II*, in The Basel II Handbook (2nd edition), edited by M. K. Ong, Risk Books, London, 2006.
- M. Pykhtin and S. Zhu, *A Guide to Modelling Counterparty Credit Risk*, GARP Risk Review, July/August, 2007.