Seminar: Rasch Models

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Part 1: Do we need the Rasch Model?

A Starter

Rasch Model

- to measure latent traits (e.g., ability or attitude)
- origin in psychology
- now applied in various disciplines:
  health, education, social sciences, economics, finance, ...

distinction: latent variables — manifest observations

latent traits are usually assessed through the responses of a sample of subjects to a set of items (questions)

- responses are scored 0/1 (dichotomous response format)
- or 0/1/2/... (polytomous response format)

Example

GLAZER STRESS CONTROL LIFESTYLE QUESTIONAIRE (extract)

I can be characterised by:

1. Never late for appointments
2. Must get things finished once started
3. Try to do more than one thing at a time. What's next?
4. Anticipate others in conversations, interrupt finishing their sentences
5. Uneasy when waiting
6. Like to meet friends at night

Answer with (maybe) YES / (maybe) NO.

Aim: obtain a measurement for Stress

usual procedure:

- sum up number of Yes-answers
- obtain scores for persons (high value – highly stressed)
- do some statistics (e.g., means for males/females)

Is this justifiable?

Questions

- **How about the sum scores?**
  - same stress for persons with same score?
  - YES to items 1 and 2 the same as YES to items 3 and 4?
  - do all items measure the same latent trait? (unidimensional)
  - do we mix up several latent traits?
- **What about the scale properties?**
  - is the sumscore interval scaled? (sum of binary variables)
  - the central limit theorem justifiable? (the i.i.d. assumption)
- **How about linearity?**
  - consider training: progress $1 \rightarrow 3$ the same as $2 \rightarrow 4$?
- **Distinction manifest observations from latent trait?**
  - manifest sum score is equated with location on latent trait

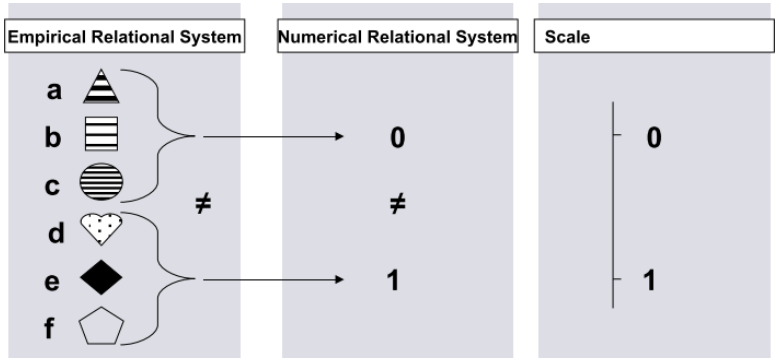
need for some thoughts on measurement!

Measurement

> Measurement is the mapping from a set of predefined objects and their empirically observable relations onto a set of numbers and their relations

- empirical relational system: $\mathcal{E} = \{A, r_1, \ldots, r_m\}$
  - A is a (finite or infinite) set of objects (e.g., subjects)
  - r_j is a relation between them concerning a certain attribute j (e.g., gender)
- numerical relational system: $\mathcal{N} = \{\mathbb{R}, r'_1, \ldots, r'_m\}$
  - $r_j \Leftrightarrow r'_j$, for all defined j

Example: measurement of gender
in $\mathcal{E}$: r_j is relation between male and female: can be $=$ or $\neq$
for $\mathcal{N}$: we can choose any two numbers, e.g., 0 and 1



Example: attribute j is fill pattern, relation r_j is equivalence again constitutes a nominal scale

but: assigning numbers to objects according to certain rules is not enough

Specific Objectivity (Rasch, 1960)

General Scientific Principle:

- scientific statements refer to comparisons between objects
- should be generalisable ('objective')

generalisable means beyond particular (experimental) situations or measurement instruments on which comparisons are based

example from classical mechanics – Newton's 2nd axiom:

force = mass × acceleration or $A_{vi} = M_v^{-1} F_i$

objects O_v, masses M_v, forces F_i, and acceleration A_{vi}

comparison of objects O_v and O_w regarding acceleration:

$$\frac{A_{vi}}{A_{wi}} = \frac{M_v^{-1} F_i}{M_w^{-1} F_i} = \frac{M_w}{M_v}$$

independent of F_i, generalizable across experimental situations

Specific Objectivity (cont'd)

in Psychology:
compare ability of subjects S_v and S_w based on test item I_i

response variable R_{vi}: generated from pair (S_v, I_i)
observation of R_{vi} may be direct (deterministic) or indirect via registration of the realisation of R_{vi} (probabilistic)

subject S_v is fully characterised (concerning R_{vi}) by latent trait parameter ξ_v, item I_i by λ_i, and R_{vi} by a reaction parameter π_{vi}

Rasch demanded: comparison of S_v and S_w with respect to ξ_v and ξ_w must be always possible once π_{vi} and π_w are known, and must be unique

a function $\pi_{vi} = f(\xi_v, \lambda_i)$ is required

Specific Objectivity (cont'd)

after some tedious mathematics requiring assumptions such as monotonicity, continuity, etc. (Fischer, 1995) we obtain

$$\pi_{vi} = \frac{exp(h(\xi_v) - \lambda_i)}{1 + \exp(h(\xi_v) - \lambda_i)}$$

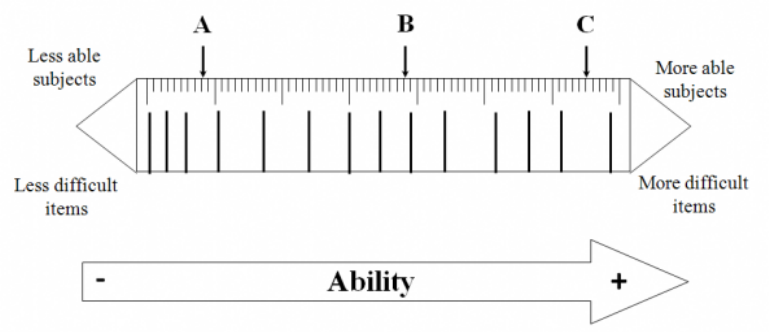
where $h(\xi_v) = \theta_v$, unique up to linear transformations $a\theta + b$, $a > 0$

using the sufficieny principle and conditional ML estimation:
the parameters ξ_v and λ_i are separable

- this allows for specific objective comparisons of subjects S_v and S_w via ξ_v and ξ_w only
the comparisons are independent of the items represented by λ_i

this is the Rasch (measurement) Model (RM)

What we would like to have



if the Rasch model holds we can compare subjects independently of the measurement instrument