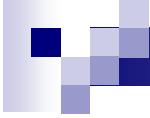


Paired-Comparison: Pattern Model

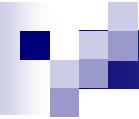
Eigenschaften von Finanzmodellen und deren
Bedeutung für den Kunden

Martin Hölbl
Birgit Miksch
David Seres



Überblick

- Abhängigkeitsannahme
- Modell
- Umfrage
 - Ausgangslage
 - Datenstruktur
- Anwendung



Abhängigkeitsannahme

- Viele Bradley-Terry Modelle nehmen (mehr oder weniger) explizit an, dass die Entscheidungen einer Versuchsperson voneinander unabhängig sind.
- Bei der wiederholten Wahl einer Versuchsperson zwischen Objekten sind Abhängigkeiten jedoch wahrscheinlich
→ Log-linear Variante des Bradley-Terry Modells das Abhängigkeiten zwischen den Beurteilungen berücksichtigt

Ausgangsmodell

- Determining the scale values of a set of J objects on a preference continuum
- O_1, O_2, \dots, O_J
- Not directly observable
- Basic BT model:

$$P\{Y_{ij} = 1 \mid \pi_i, \pi_j\} = \frac{\pi_i}{\pi_i + \pi_j}$$

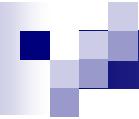
Ausgangsmodell

- Basis: Simple log-linear model for a corresponding contingency table
- Prefering obj. i over obj. j:

$$P\{Y_{ij} = 1 \mid \pi_i, \pi_j\} = \frac{\pi_i}{\pi_i + \pi_j} = \frac{\sqrt{\pi_i/\pi_j}}{\sqrt{\pi_i/\pi_j} + \sqrt{\pi_j/\pi_i}}$$

- Where π_i is defined as:

$$\pi_i = \frac{\exp\{2\lambda_i\}}{\sum_j \exp\{2\lambda_j\}}, i = 1, 2, \dots, J$$



Das Modell

- A log-linear model for
a $\binom{O}{2} \times 2$ complete 2-dimensional
(object pairs x response category)
contingency table under product
multinomial sampling

Das Modell

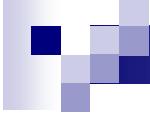
- Design matrix X :

$$X = \left(I_{\binom{J}{2}} \otimes 1_2, B \otimes h \right)$$

- $1_2' = (1, 1)$
- $h' = (1, -1)$
- $I :=$ identity matrix of order $\binom{J}{2}$
- $B :=$ paired comparison design matrix

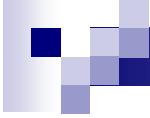
Notation

- Comparison of J objects
 $(1,2), (1,3), \dots, (1,J); (2,3), (2,4), \dots, (2,J); \dots; (J-1,J)$
 - Results can be represented by random var
- $$y_{ij} = \begin{cases} 1 & \text{if } O_i \text{ is preferred over } O_j \\ -1 & \text{if } O_j \text{ is preferred over } O_i \end{cases}$$
- $2^{\binom{J}{2}} = \ell$ response patterns (s_i); a vector of $\binom{J}{2}$ elements consisting of $\{1, -1\}$
 - $s = (y_{12}, y_{13}, \dots, y_{1J}, y_{23}, y_{24}, \dots, y_{2J}, \dots, y_{J-1,J})$



Dependent decisions

- Repeated paired comparison by the same judge → possible dependencies among the pairwise judgements
- Analysis of multiple binomial response
 $(Y_{12}, Y_{13}, \dots, Y_{1J}, Y_{23}, Y_{24}, \dots, Y_{2J}, \dots, Y_{J-1,J})$
- For identical standards in the comparison of objects i and j and objects i and k the assessment of object i is likely to be similar in both comparisons



Dependent decisions

- Similarity in evaluation introduces dependencies between the observed responses
- Paired comparison with object pairs (O_i, O_j) and $(O_i, O_k) \rightarrow$ dependency is introduced by the same object O_i involved in both pairs which is characterized by a further parameter $\theta_{ij,ik} := \theta_{ijk}$

Dependent decisions

- Special case $J = 3$:

$$p(y_{12}, y_{13}, y_{23}) = C^{**} \left(\frac{\sqrt{\pi_1}}{\sqrt{\pi_2}} \right)^{y_{12}} \left(\frac{\sqrt{\pi_1}}{\sqrt{\pi_3}} \right)^{y_{13}} \left(\frac{\sqrt{\pi_2}}{\sqrt{\pi_3}} \right)^{y_{23}} \times \\ \exp \{ \theta_{1,23} y_{12} y_{13} + \theta_{2,13} y_{12} y_{23} + \theta_{3,12} y_{13} y_{23} \}$$

Dependent decisions

■ Using matrix notation:

all response pattern = $(1, X, W)\beta$

$X = YB$

$\beta' = (\gamma, \lambda_1, \lambda_2, \lambda_3, \theta_{1,23}, \theta_{2,13}, \theta_{3,12})$

W = interaction effects

$1 = (2^{\frac{J}{2}} \times 1)$

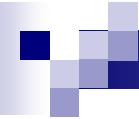
$Y = (2^{\frac{J}{2}} \times \binom{J}{2})$ design matrix

$B = (\binom{J}{2} \times J)$ paired comparison design matrix.

Subject –specific covariate

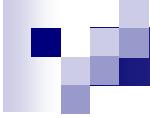
- Replacing λ and θ by a linear predictor
- Linear predictor: $\lambda_j \rightarrow \lambda_j + \lambda_{j\ell}$

$$\begin{aligned}\ln m(y_{12}, y_{13}, y_{23} | \ell) = & \gamma_\ell + (y_{12\ell} + y_{13\ell})(\lambda_1 + \lambda_{1\ell}) \\ & + (y_{23\ell} - y_{12\ell})(\lambda_2 + \lambda_{2\ell}) + (-y_{13\ell} - y_{23\ell})(\lambda_3 + \lambda_{3\ell}) \\ & + \theta_{1,23} y_{12\ell} y_{13\ell} + \theta_{2,13} y_{12\ell} y_{23\ell} + \theta_{3,12} y_{13\ell} y_{23\ell}.\end{aligned}$$



Ausgangslage Behavioral Finance

- Anleger und Finanzdienstleister nehmen Anlageentscheidungen unterschiedlich wahr
 - ⇒ Ineffizienzen
- „homo oeconomicus“
 - Unrealistische Annahme
- ⇒ Einbezug von verhaltenswissenschaftlichen Aspekten
 - ⇒ Situative Faktoren



Datensatz (I)

- Angaben zum Umfeld der Anlageentscheidung
 - Wirtschaftliche Entwicklung der nächsten 2 Jahre
 - Zukunftsaussichten des Kapitalmarktes
 - Anteil des Investierten Betrages am Gesamtvermögen
 - Eingeschätzte Höhe des angelegten Betrages
- Persönliche Angaben
 - Alter
 - Familienstand

Datensatz (II) - Paarvergleich

■ Original Datensatz (8 Objekte)

Ausprägungen Eigenschaften	1	2	3
Risiko	gering	hoch	
Ertrag	gering	hoch	
Laufzeit	kurz	lang	
Anzulegender Betrag	gering	mittel	hoch

■ Verwendete Daten (6 Objekte)

Ausprägungen Eigenschaften	1	2	3
Risiko	gering	hoch	
Anzulegender Betrag	gering	mittel	hoch



Anwendung

→ R

Datensatz (III)

- Wirtschaft
 - 0 positiv
 - 1 stabil
 - 2 negativ
- Kapitalmarkt
 - 0 positiv
 - 1 negativ
- Betrag/Vermögen
 - 0: < 10%
 - 1: 11 – 25%
 - 2: 26 – 40%
 - 3: > 40%
- Betragshöhe
 - 0: gering
 - 1: durchschnittlich
 - 2: hoch
- Alter
 - 0: < 25
 - 1: 26 - 35
 - 2: 36 - 55
 - 3: 56 - 65
 - 4: > 65
- Familienstand
 - 0: alleine
 - 1: Partnerschaft/Ehe (keine Kinder)
 - 2: Partnerschaft/Ehe (Kinder < 18)
 - 3: Partnerschaft/Ehe (Kinder > 18)