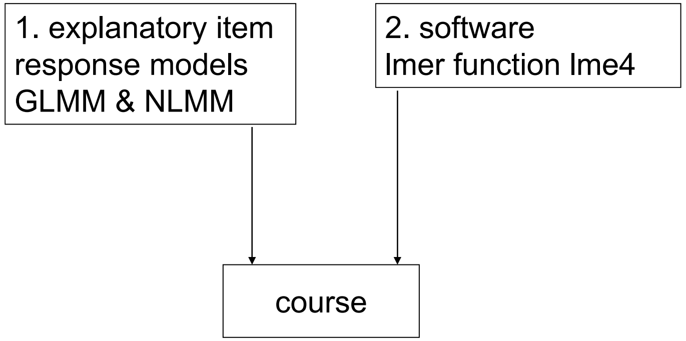
IRT models and mixed models: Theory and lmer practice

Paul De Boeck
U. Amsterdam
& K.U.Leuven

Sun-Joo Cho
Peabody College
Vanderbilt U.

NCME, April 8 2011, New Orleans

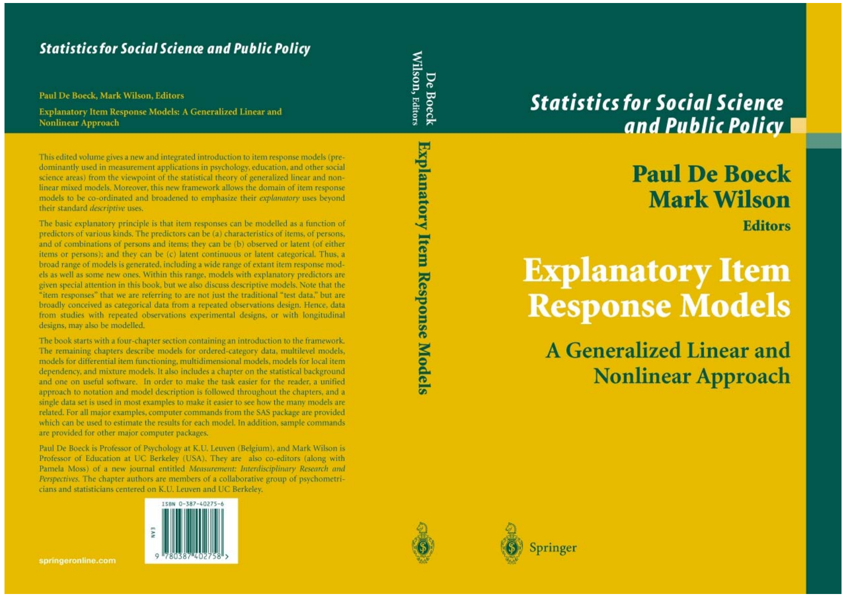


1a. Rijmen, F., Tuerlinckx, F., De Boeck, P., & Kuppens, P. (2003). A nonlinear mixed model framework for item response theory. *Psychological Methods*, *8*, 185-205.

1b. De Boeck, P., & Wilson, M. (Eds.) (2004). *Explanatory item response models: A generalized linear and nonlinear approach.* New York: Springer.

2. De Boeck, P. et al. (2011). The estimation of item response models with the lmer function from the lme4 package in R. *Journal of Statistical Software.*

Website : http://bearcenter.berkeley.edu/EIRM/



- In 1 and 2 mainly SAS NLMIXED
- In 3 lmer function from lme4

- Data
- GLMM
- Lmer function

1. Data

- setwd(“ ”)
- library(lme4)

?VerbAgg
head(VerbAgg)

24 items with a 2 x 2 x 3 design
- situ: other vs self
  two frustrating situations where an*other* person is to be blamed
  two frustrating situations where one is *self* to be blamed
- mode: want vs do
  wanting to be verbally agressive vs doing
- btype: cursing, scolding, shouting
  three kinds of being verbally agressive

e.g., “A bus fails to stop. I would want to curse” yes perhaps no

316 respondents
- Gender: F (men) vs M (women)
- Anger: the subject's Trait Anger score as measured on the State-Trait Anger Expression Inventory (STAXI)

str(VerbAgg)

Let us do the Rasch model

1. Generalized Linear Mixed Models
“no 2PL”, no 3PL
“no ordered-category data”
but many other models instead

Modeling data

- A basic principle
  Data are seen as resulting from a true part and an error part.

  binary data
  $Y_{pi} = 0,1$

  V_{pi} is continuous and not observed
  V_{pi} is a real defined on the interval $-\infty$ to $+\infty$
  $V_{pi} = \eta_{pi} + \varepsilon_{pi}$ $\varepsilon_{pi} \sim N(0,1)$ probit, normal-ogive
  $\varepsilon_{pi} \sim \text{logistic}(0,3.29)$ logit, logistic

  $Y_{pi}=1$ if $V_{pi} \geq 0$, $Y_{pi}=0$ if $V_{pi}<0$

Logistic models

- Standard logistic instead of standard normal
  Logistic model – logit model
  vs
  Normal-ogive model – probit model

  density general logistic distribution:
  $f(x)=k\ \exp(-kx)/(1+\exp(-kx))^2$

  $var = \pi^2/3k^2$

  standard logistic: k=1,
  $\sigma = \pi/\sqrt{3} = 1.814$
  setting $\sigma=1$, implies that k=1.814

  best approximation from standard normal: k=1.7
  this is the famous D=1.7 in "early" IRT formulas

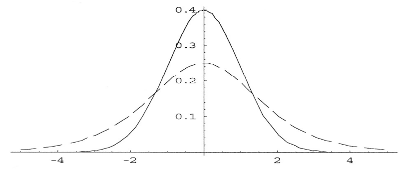


FIGURE 1.
The logistic distribution with $k = 1$ and the standard normal (solid line).

standard (k=1) logistic
vs
standard normal

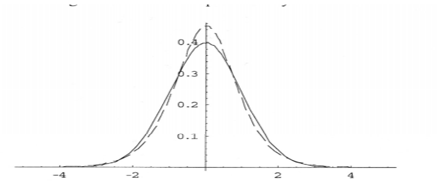
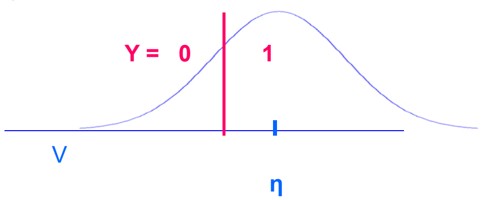


FIGURE 2.
The logistic distribution with $k = 1.8$ and the standard normal (solid line).

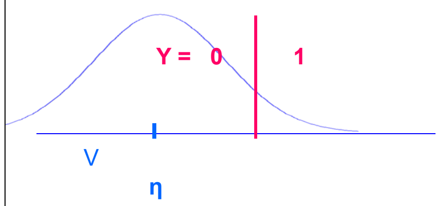
logistic k=1.8
vs
standard normal

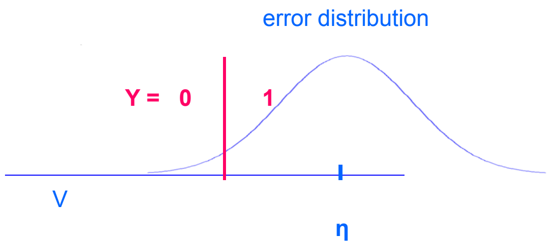
copied from Savalei, *Psychometrika* 2006

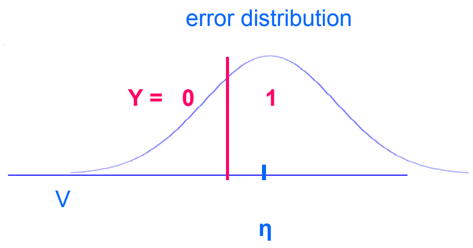
binary data



moving hat model

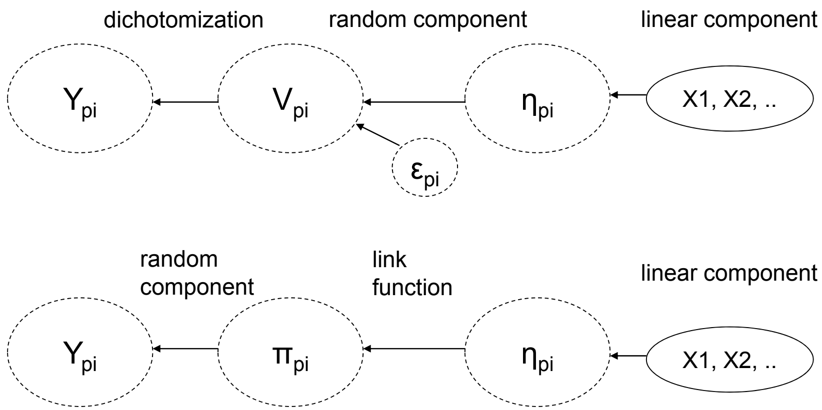




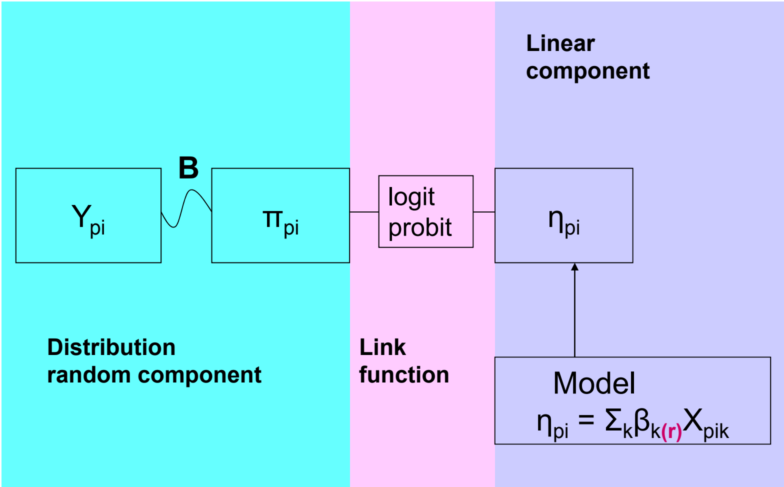


$$\eta_{pi} = \Sigma_k \beta_{k(r)} X_{pik}$$

$$V_{pi} = \Sigma_k \beta_{k(r)} X_{pik} + \varepsilon_{pi}$$



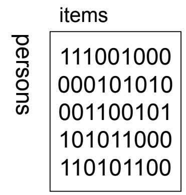
Logit and probit models



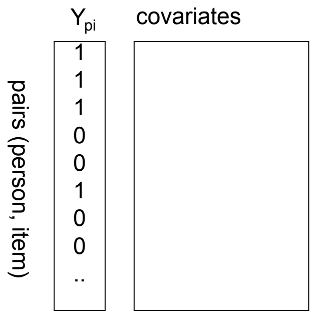
2. lmer function
   from lme4 package (Douglas Bates)
   for GLMM, including multilevel
   not meant for IRT

Long form

- Wide form is P x I array



- Long form is vector with length PxI



Content

1. Item covariate models
   1PL, LLTM, MIRT
2. Person covariate models
   JML, MML, latent regression, SEM, multilevel

Break from 12.20pm to 2pm

3. Person x item covariate models
   DIF, LID, dynamic models
4. Other
   random item models
   “impossible models”: models for ordered-category data, 2PL
5. Estimation and testing

1. Item covariate models

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1. Rasch model
1PL model

1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1	β_1 β_2 β_3 β_4	fixed
1 1 1 1	θ_p	random
Y		

$\eta_{pi} = \theta_p X_{i0} - \Sigma_k \beta_i X_{ik}$

$\eta_{pi} = \theta_p - \beta_i$

note that lmer does $+\beta_i$

$\pi_{pi} = \exp(\eta_{pi})/(1+\exp(\eta_{pi}))$

$\theta_p \sim N(0, \sigma^2_\theta)$

Note on 2PL: Explain that in 2PL the constant X_{i0} is replaced with discrimination parameters

lmer(r2 ~ …… , family=binomial("logit"), data=VerbAgg)
lmer(r2 ~ …… , family=binomial, VerbAgg) — logistic model

lmer(r2 ~ …… , family=binomial("probit"), data=VerbAgg) — normal-ogive
lmer(r2 ~ …… , family=binomial("probit"), VerbAgg) — probit model

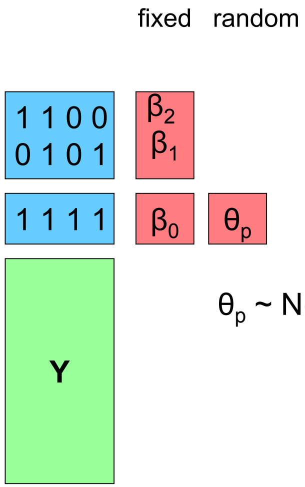
……
item + (1 |id), — first item is intercept, other item parameters are differences with first $\beta_0 = \beta_1, \beta_2 - \beta_1, \beta_3 - \beta_1$, ..
or
-1 + item + (1 |id) no intercept, only the common item parameters
item + (1 |id)

item is item factor
id is person factor
1 is 1-covariate
(a|b) effect of a is random across levels of b

- to avoid correlated error output:

  print(modelname, cor=F)

2. LLTM model



$\eta_{pi} = \theta_p X_{i0} - \Sigma_k \beta_k X_{ik}$

$\eta_{pi} = \theta_p - \Sigma_k \beta_k X_{ik}$

$\theta_p \sim N(0, \sigma^2_\theta)$

-1+mode+situ+btype+(1|id), family=binomial, VerbAgg

contrasts

treatment dummy	sum effect	helmert	poly
00	1 0	-1 -1	linear
10	0 1	1 -1	quadratic
01	-1-1	0 2	

without intercept always

100

010

001

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- <u>lmer treatment coding with intercept</u>

want	other	curse	0 0 0 0
want	other	scold	0 0 1 0
want	other	shout	0 0 0 1
want	self	curse	0 1 0 0
want	self	scold	0 1 1 0
want	self	shout	0 1 0 1
do	other	curse	1 0 0 0
do	other	scold	1 0 1 0
do	other	shout	1 0 0 1
do	self	curse	1 1 0 0
do	self	scold	1 1 1 0
do	self	shout	1 1 0 1

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- <u>lmer treatment coding without intercept</u>

want	other	curse	1 0 0 0 0
want	other	scold	1 0 0 1 0
want	other	shout	1 0 0 0 1
want	self	curse	1 0 1 0 0
want	self	scold	1 0 1 1 0
want	self	shout	1 0 1 0 1
do	other	curse	0 1 0 0 0
do	other	scold	0 1 0 1 0
do	other	shout	0 1 0 0 1
do	self	curse	0 1 1 0 0
do	self	scold	0 1 1 1 0
do	self	shout	0 1 1 0 1

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btype	treatment	sum	helmert	mode	treatment	sum	helmert
curse	0 0	1 0	-1-1	want	0	1	-1
scold	1 0	0 1	1-1	do	1	-1	1
shout	0 1	-1-1	0 2				

main effects and interactions

✓ mode:btype is for cell means independent of coding

✓ dummy coding
main effects: mode+btype or
C(mode,treatment) + C(btype,treatment)
main effects & interaction: mode*btype or
C(mode,treatment) *C(btype,treatment)
✓ effect coding
main effects: 1+C(mode,sum)+ C(btype,sum)
main effects & interaction: C(mode,sum)*C(btype,sum)

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3. LLTM + error model

remember there are two items per cell

$\eta_{pi} = \theta_p - \Sigma_k \beta_k X_{ik} + \varepsilon_i$

fixed random

1 1 0 0
0 0 1 1
0 1 0 1

β_1
β_2
β_3

1 1 1 1

θ_p ε_i

Y

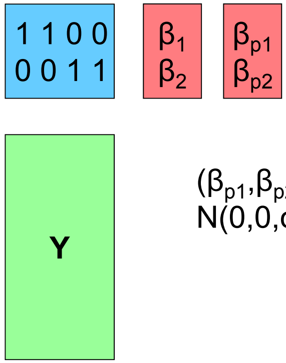
$\theta_p \sim N(0, \sigma^2_\theta)$
$\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$

lmer(r2 ~ mode + situ + btype + (1 |id) + (1|item),
or
lmer(r2 ~ - 1 + mode + situ + btype + (1 |id) + (1|item),
family=binomial, VerbAgg)

- two types of multidimensional models
  - random-weight LLTM
  - multidimensional 1PL

4. Random-weight LLTM

fixed random

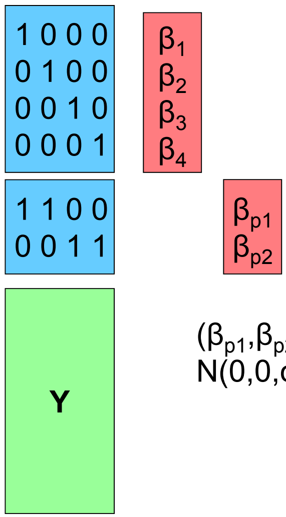


$\eta_{pi} = \Sigma_k \beta_{pk} X_{ik} - \Sigma_k \beta_k X_{ik}$

$(\beta_{p1}, \beta_{p2}) \sim N(0, 0, \sigma^2_{\theta 1}, \sigma^2_{\theta 2}, \sigma_{\theta 1 \theta 2})$

lmer(r2 ~ mode + situ + btype + (-1 + mode|id), family=binomial, VerbAgg)

5. multidimensional 1PL model

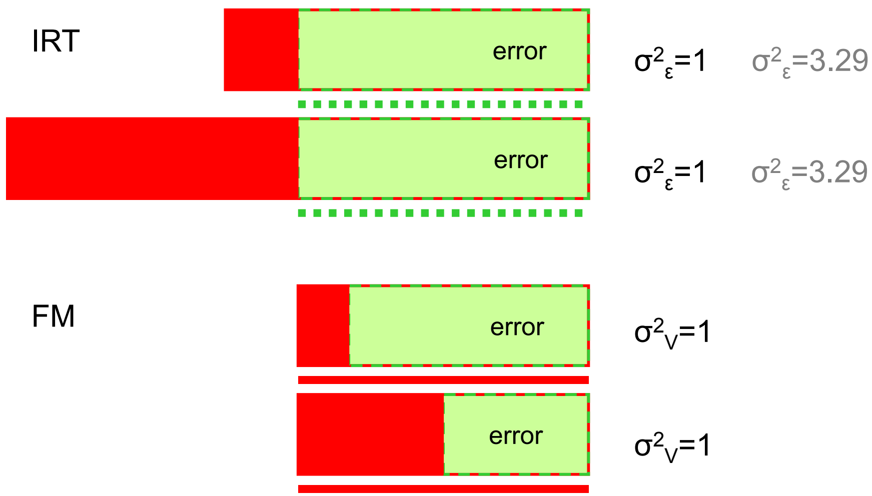


$\eta_{pi} = \Sigma_k \beta_{pk} X_{ik} - \beta_i$

$(\beta_{p1}, \beta_{p2}) \sim N(0, 0, \sigma^2_{\theta 1}, \sigma^2_{\theta 2}, \sigma_{\theta 1 \theta 2})$

Note on factor models, how they differ from IRT models
Note on rotational positions

variance partitioning



- item covariate based multidimensional models
  a non-identified model
  and four possible identified models

	1	2	3		4
~~1 0 1 0~~	1 0 0	1 0 0	1 0	1 0	1 0 0 0
~~1 0 1 0~~	1 0 0	1 0 0	1 0	1 0	1 0 0 0
~~1 0 0 1~~	1 0 1	1 0 1	1 0	0 1	0 1 0 0
~~1 0 0 1~~	1 0 1	1 0 1	1 0	0 1	0 1 0 0
~~0 1 1 0~~	1 1 0	0 1 0	0 1	1 0	0 0 1 0
~~0 1 1 0~~	1 1 0	0 1 0	0 1	1 0	0 0 1 0
~~0 1 0 1~~	1 1 1	0 1 1	0 1	0 1	0 0 0 1
~~0 1 0 1~~	1 1 1	0 1 1	0 1	0 1	0 0 0 1

1 -1 + item + (mode + situ|id)
2 -1 + item + (-1 + mode + situ|id)
3 -1 + item + (-1 + mode |id) + (-1 + situ |id)
4 -1 + item + (mode:situ|id)

Illustration of non-identified model

```
VerbAgg$do=(VerbAgg$mode==“do”)+0
VerbAgg$want=(VerbAgg$mode==“want”)+0
VerbAgg$self=(VerbAgg$mode==“self”)+0
VerbAgg$other=(VerbAgg$mode==“other”)+0
mMIR1=lmer(r2~-1+item+
    (-1+do+want+self+other|id),family=binomial,VerbAgg)
mMIR2=lmer(r2~-1+item+
    (-1+want+do+self+other|id),family=binomial,VerbAgg)
```

compare with identified model

```
mMIR3=lmer(r2~-1+item+(-1+mode+situ|id), family=binomial, VerbAgg)
```

-1 + item + (mode + situ + btype |id)
-1 + item + (-1 + mode + situ + btype |id)
-1 + item + (-1 + mode |id) + (-1 + situ |id) + (-1 + btype |id)
-1 + item + (mode:situ:btype |id)

how many dimensions?

rotations

```
VerbAgg$do=(VerbAgg$mode==“do”)+0.
VerbAgg$want=(VerbAgg$want==“want”)+0.
VerbAgg$dowant=(VerbAgg$mode==“do”)-1/2.
```

1. simple structure orthogonal
(-1+do|id)+(-1+want|id)
2. simple structure correlated
(-1+mode|id)
3. general plus bipolar
(dowant|id)
4. general plus bipolar uncorrelated
(1|id)+(-1+dowant|id)

2 and 3 are equivalent
1 and 4 are constrained solutions
all four are confirmatory

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estimation of person parameters and random effects in general

three methods
- ML maximum likelihood – flat prior
- MAP maximum a posteriori – normal prior, mode of posterior
- EAP expected a posteriori – normal prior, mean of posterior, and is therefore a prediction

irtoys does all three
lmer does MAP
ranef(model)
se.ranef(model) for standard errors

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2. Person covariate models

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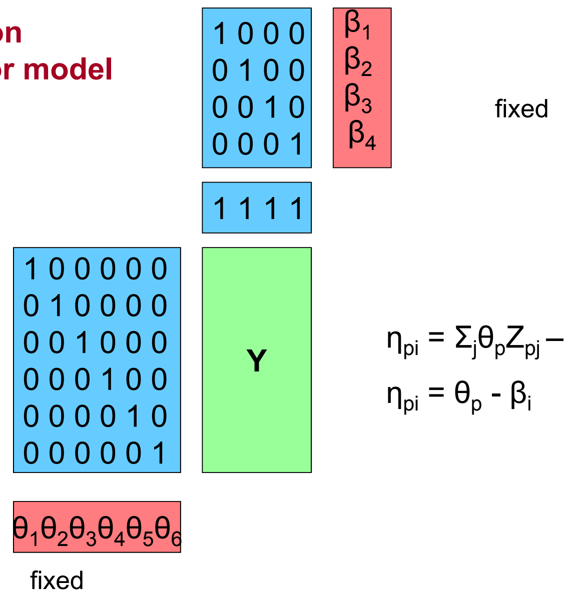
1. Person indicator model JML



$\eta_{pi} = \Sigma_j \theta_p Z_{pj} - \Sigma_k \beta_i X_{ik}$

$\eta_{pi} = \theta_p - \beta_i$

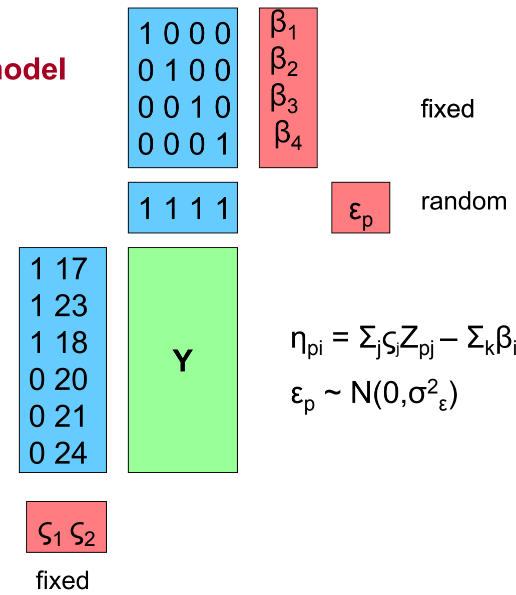
-1 + item + id + (1 |item)

four models

- fixed persons & fixed items JML
- random persons & fixed items MML
- fixed persons & random items
- random persons & random items
  fixed-effect fallacy in experimental psychology
  treating stimuli as fixed

-1 + item + id + (1|item)
-1 + item + (1|id)
-1 + id + (1|item)
(1|id) + (1|item)

2. Latent regression model



$\eta_{pi} = \Sigma_j \varsigma_j Z_{pj} - \Sigma_k \beta_i X_{ik} + \varepsilon_p$

$\varepsilon_p \sim N(0,\sigma^2_{\varepsilon})$

F = man
M = woman

-1 + item + Anger + Gender + (1|id)
-1 + item + Anger:Gender +(1|id)
-1 + item + Anger*Gender+(1|id)

heteroscedasticity

VerbAgg$M=(VerbAgg$Gender==“M”)+0.
VerbAgg$F=(VerbAgg$Gender==“F”)+0.

Heteroscedastic 1
(-1+Gender|id)
parameters is not correct

Heteroscedastic 2
(-1+M|id)+(-1+F|id)
parameters is correct

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differential effects

effect of Gender differs depending on the dimension

-1+item+Gender:mode+(-1+mode|id)
-1+item+Gender*mode+(-1+mode|id)
do not work

-1+Gender:mode+(1|item)+(-1+mode|id)
Gender*mode+(1|item)+(-1+mode|id)
C(Gender,sum)*C(mode,sum)+(1|item)+(-1+mode|id)

do work

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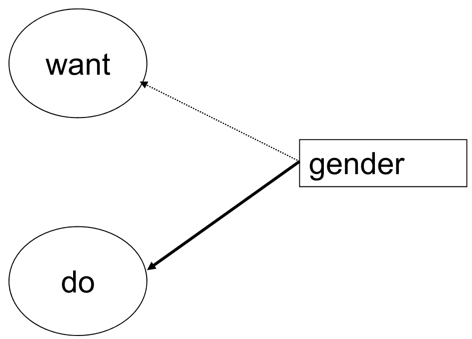
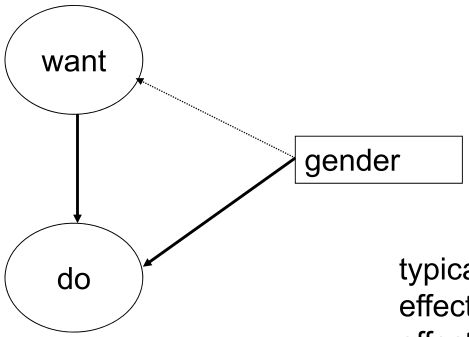
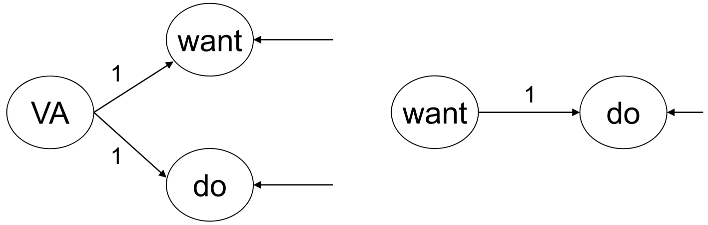




typical of SEM are effects of one random effect on another

SEM with lmer




```
VerbAgg$do=(VerbAgg$mode==“do”)+0.
VerbAgg$want=(VerbAgg$mode==“want”)+0.
```

```
-1+item+(1|id)+(-1+want|id)+(-1+do|id)
-1+item+(1|id)+(-1+do|id)
```

3. Multilevel models

typical of multilevel models is that effects are random across nested levels

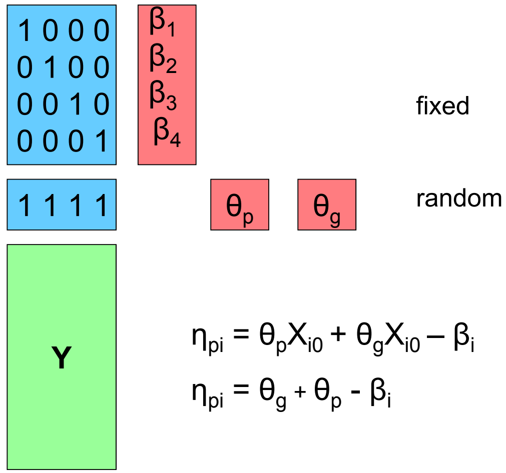
(nested) person partitions
- educational measurement: classes – schools
- cross-cultural psychology: countries
- health: neighborhoods, cities, regions

nested item partitions

crossed person partitions
- crossed between-subject factors

crossed item partitions
- crossed within-subject factors

Multilevel model



$\eta_{pi} = \theta_p X_{i0} + \theta_g X_{i0} – \beta_i$

$\eta_{pi} = \theta_g + \theta_p - \beta_i$

-1 + item + (1|id) + (1|group)

use Gender as group
in order to illustrate

heteroscedastic model

-1 + item + (-1+group |id) + (1|group)

try with Gender for group

multilevel factor model

The dimensionality and covariance structure can differ depending on the level

use Gender as group
in order ro illustrate

-1 + item + (1|id) + (1|group)

-1 + item + (-1+mode|id) + (-1+mode|group)

try with Gender for group

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3. Person-by-item covariate models

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- covariates of person-item pairs

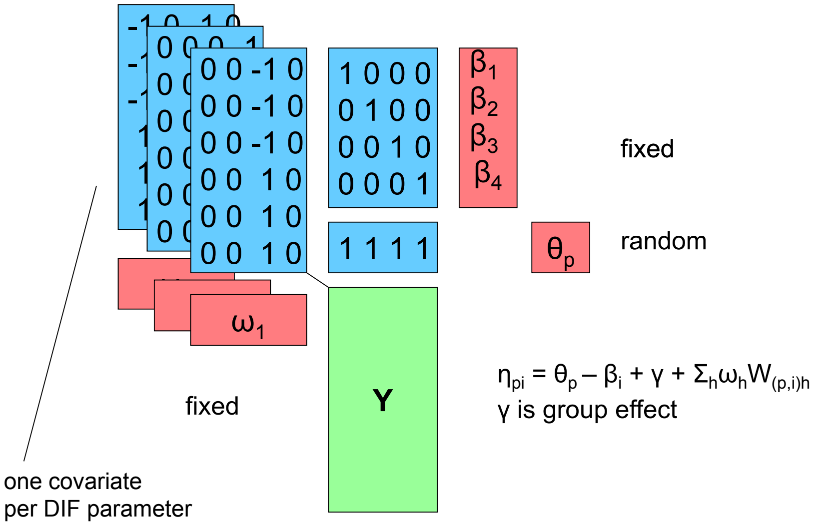
  external covariates

  e.g., differential item functioning
  an item functioning differently depending on the group
  person group x item
  e.g., strategy information per pair person-item

  internal covariates

  responses being depending on other responses

  e.g., do responses depending on want responses
  local item dependence – LID;
  e.g., learning during the test, during the experiment
  dynamic Rasch model

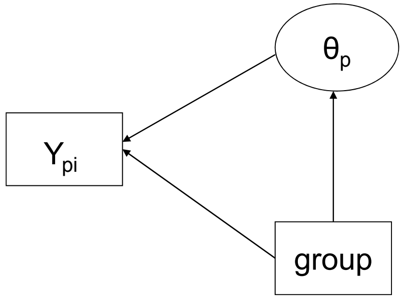


$\eta_{pi} = \theta_p - \beta_i + \gamma + \Sigma_h \omega_h W_{(p,i)h}$

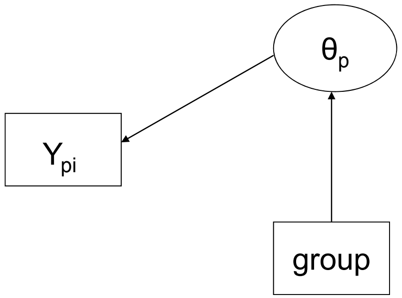
γ is group effect

1. DIF model Differential item functioning

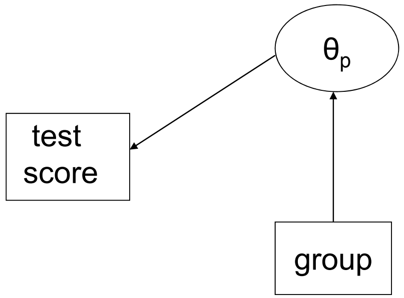
unfair (because of DIF)



fair (no DIF)



fair (lack of differential test funtioning)



gender DIF for all do items of the curse and scold type

```
dif=with(VerbAgg, factor( 0 + ( Gender=="F" & mode=="do" & btype!="shout") ) )
```

```
-1 +item + Gender + dif + (1|id)
```

random across persons

```
-1 +item + Gender + dif + (1 + dif|id)
```

F = man
M = woman

dummy coding vs contrast coding
(treatment vs sum or helmert) makes
a difference for the item parameter estimates

DIF approaches

difficulties in the two groups – *equal mean abilities*

```
VerbAgg$M=(VerbAgg$Gender=="M")+0.
VerbAgg$F=(VerbAgg$Gender=="F")+0.
-1+Gender:item+(-1+M|id)+(-1+F|id)
```

simultaneous test of all items – *equal mean difficulties*

```
-1+C(Gender,sum)*C(item,sum)+(-1+M|id)+(-1+F|id)
```

-- *difference with reference group*

```
-1+Gender*item+(-1+M|id)+(-1+F|id)
```

itemwise test

```
VerbAgg$i1=(VerbAgg$item=="S1wantcurse")+0.
VerbAgg$2=(VerbAgg$item=="S1WantScold")+0.
...
```

(pay attention to item labels)

e.g., item 3

```
-1+Gender+i1+i2+i4+i5…+i24+Gender*i3+(-1+M|id)+(-1+F|id)
```

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result depends on equating
therefore a LR test is recommended

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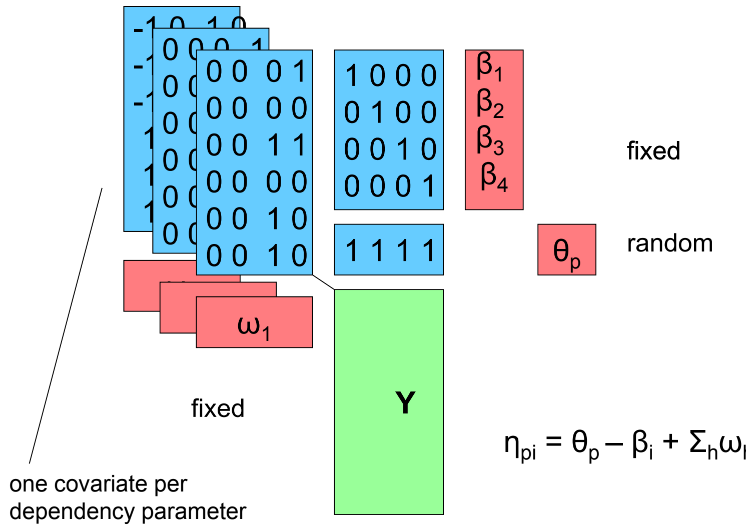
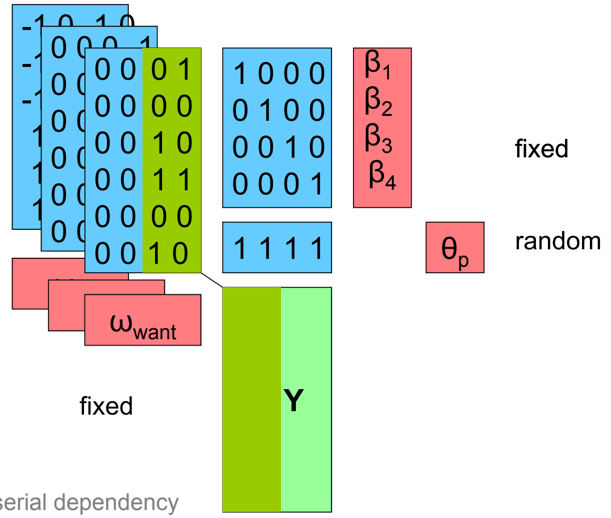


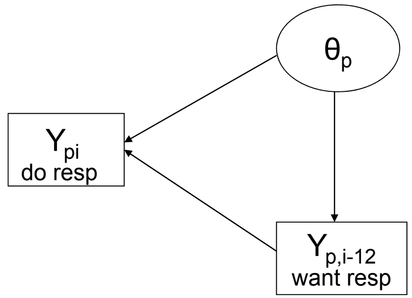
$$\eta_{pi} = \theta_p - \beta_i + \Sigma_h \omega_h W_{(p,i)h}$$

2. LID model local item dependence



Note on serial dependency and stationary vs non-stantionary models (making use of random item models)

$$\eta_{pi} = \theta_p - \beta_i + \omega_{want} X_{i,do} Y_{p,i-12}$$



dep = with(VerbAgg, factor ((mode==“do”)*(r2 [mode==“want”]==“Y”)))

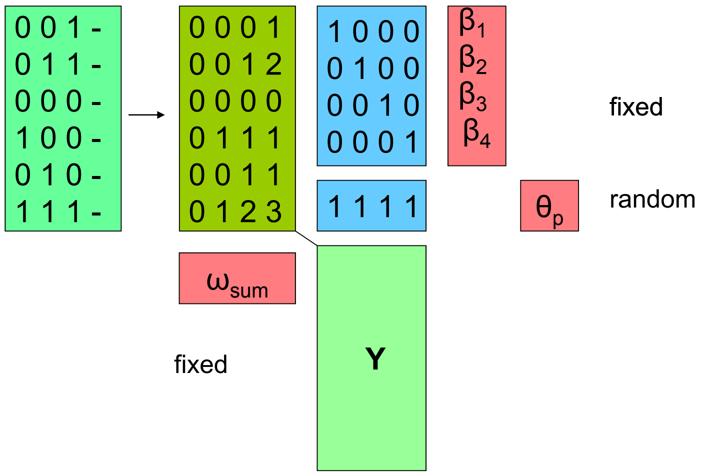
-1 + item + dep + (1|id)

random across persons
-1 + item + dep + (dep|id)

other forms of dependency

which other forms of dependency do you think are meaningful?
and how to implement them?

Remove for two examples



3. Dynamic Rasch model

$\eta_{pi} = \theta_p - \beta_i + \omega_{sum} W_{(p,i)sum}$

```
long = data.frame(id=VerbAgg$id, item=VerbAgg$item, r2=VerbAgg$r2)
wide=reshape(long, timevar=c("item"), idvar=c("id"), dir="wide")[,-1]=="Y"
prosum=as.vector(t(apply(wide,1,cumsum)))
```

-1 + item + prosum + (1|id)
random across persons
-1 + item + prosum + (1+prosum|id)

Preparing a new dataset

- Most datasets have a wide format

Dataset
1 0 0 0 0 0 a
0 1 1 0 0 0 b
0 1 0 1 0 1 c
1 1 1 1 1 0 a
1 1 0 0 1 1 b
1 1 1 0 0 0 c
0 1 1 1 0 0 a
1 0 0 0 1 1 b
Type these data into a file “datawide.txt”

S-J

From wide to long

```
widedat=read.table(file="datawide.txt")
widedat$id=paste("id", 1:8, sep="")
or
widedat$id=paste("id",1:nrow(widedat),sep="")
library(reshape)
long=melt(widedat, id=7:8)
names(long)=c("con","id","item","resp")
```

S-J

Change type

from factor to numeric

```
long$connum=as.numeric(factor(long[,1]))
```

from numeric to factor

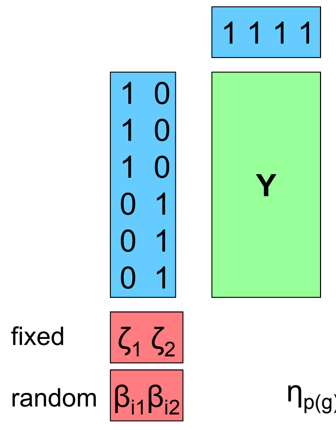
```
long$confac=factor(long[,5])
```


4a. Ordered-category data
4b. Random item models

NCME, April 8 2011, New Orleans

a. Models for random item effects

MRIP model
Multiple Random Item



$$\eta_{p(g)i} = \theta_{p(g)}X_{i0} - \Sigma_j \beta_{ij} Z_{pj} + \zeta_g$$

-1 + Gender + (-1+Gender|id) + (-1+Gender|item)

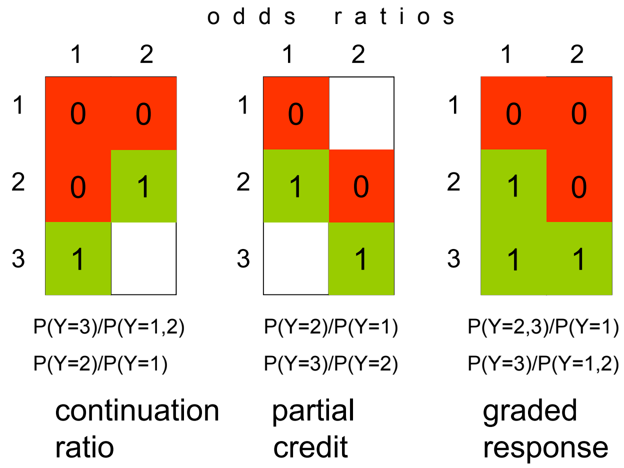
S-J

b. Ordered-category data

Models for ordered-category data

three types of odds ratios (green vs red)

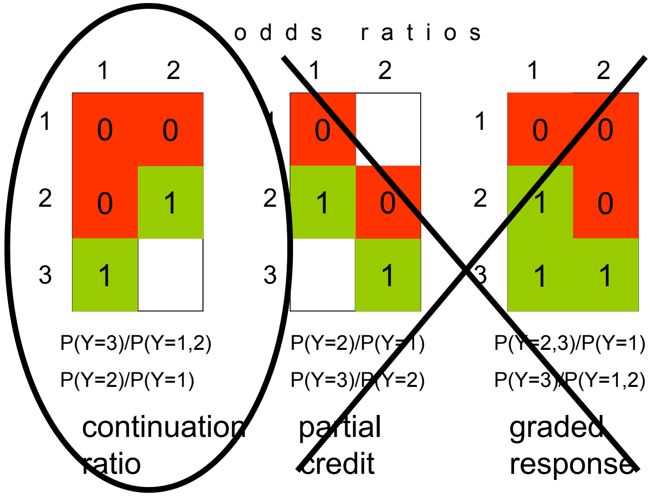
for example, three categories, two odds ratios



Models for ordered-category data

three types of odds ratios (green vs red)

for example, three categories, two odds ratios



Continuation ratio – Tutz model

P(Y=3) follows Rasch model	$P_1(\theta_1)$
P(Y=2|Y≠3) follows Rasch model and is independent of P(Y=3)	$P_2(\theta_2)$

$P(Y=3)$	$P_1(\theta_1)$
$P(Y=2)=P(Y\neq3)P(Y=2\mid Y\neq3)$	$(1-P_1(\theta_1)) \times P_2(\theta_2)$
$P(Y=1)=P(Y\neq3)P(Y\neq2\mid Y\neq3)$	$(1-P_1(\theta_1)) \times (1-P_2(\theta_2))$

Continuation ratio model is similar to discrete survival model

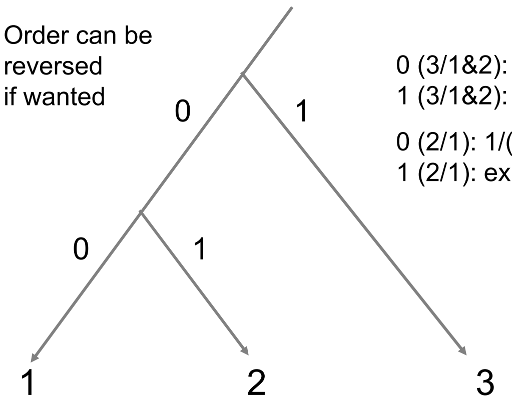
Choices are like decisive events in time

A one indicates that the event occurs, so that later observations are missing

A zero indicates that the event has not yet occured, so that later observations are possible

Tutz model choice tree

Order can be reversed if wanted



0 (3/1&2): $1/(1+\exp(\theta_{p1}-\beta_{i1}))$
1 (3/1&2): $\exp(\theta_{p1}-\beta_{i1})/(1+\exp(\theta_{p1}-\beta_{i1}))$

0 (2/1): $1/(1+\exp(\theta_{p2}-\beta_{i2}))$
1 (2/1): $\exp(\theta_{p2}-\beta_{i2})/(1+\exp(\theta_{p2}-\beta_{i2}))$

	3/1&2	2/1
1:	0	0
2:	0	1
3:	1	-

00: $1 \;/\; (1+\exp(\theta_{p1}-\beta_{i1})+\exp(\theta_{p2}-\beta_{i2})+\exp(\theta_{p1}+\theta_{p2}-\beta_{i1}-\beta_{i2}))$
01: $\exp(\theta_{p2}-\beta_{i2}) \;/\; (1+\exp(\theta_{p1}-\beta_{i1})+\exp(\theta_{p2}-\beta_{i2})+\exp(\theta_{p1}+\theta_{p2}-\beta_{i1}-\beta_{i2}))$
1- : $\exp(\theta_{p1}-\beta_{i1}) \;/\; (1 + \exp(\theta_{p1}-\beta_{i1}))$

the partial credit tree sits behind this screen extend dataset: replace each item response with two, except when missing:

1 00
2 01
3 1-

transformation can be done using `Tutzcoding` function in `R`.

VATutz=Tutzcoding(VerbAgg, “item”, “resp”)

label for

recoded responses: tutz

subitems: newitems

subitem factor: category

estimation of common model

modelTutz=lmer(tutz~-1+newitem+(1|id), family=binomial,VATutz)

more Tutz models

rating scale version
  -1+item+category+(1|id)

gender specific rating scale model
  -1+C(Gender,sum)*C(category,sum)+item+(1|id)

multidimensional: subitem specific dimensions
  -1+newitem+(-1+category|id)
  -1+item+category+(-1+category|id)   rating scale version

- much more is possible with MRIP
  one can consider each random item profile as a latent item variable (LIV)

  e.g., a double random Tutz model
  1+(-1+category|item)+(-1+category|id)

5. Estimation and testing

NCME, April 8 2011, New Orleans

Estimation

- Laplace approximation of integrand

issue: integral is not tractable
solutions
1. approximation of integrand, so that it is tractable
2. approximation of integral
   Gaussian quadrate: non-adaptive or adaptive
3. Markov chain Monte Carlo

differences
- underestimation of variances using 1
- much faster using 1
- 1 is not ML, but most recent approaches are close

- approximation of integrand:
  PQL, PQL2, Laplace6
  MLwiN: PQL2
  HLM: Laplace6
  GLIMMIX: PQL
  lmer: Laplace
  Laplace6>Laplace>PQL2>PQL
- approximation of the integral
  SAS NLMIXED, gllamm, ltm, and many other adaptive or nonadaptive
- MCMC
  WinBUGS, mlirt

Other R-programs

- **ltm** (Rizopoulos, 2006)
  1PL, 2PL, 3PL, graded response model
  included in **irtoys**
  Gaussian quadrature
- **eRm** (Mair & Hatzinger, 2007)
  Rasch, LLTM, partial credit model, rating scale model
  conditional maximum likelihood -- CML
- **mlirt** (Fox, 2007)
  2PNO binary & polytomous, multilevel

irtoys

calls among other things ltm

Illustration of ltm with irtoys

testing

problems

- strictly speaking no ML
- testing null hypothesis of zero variance
  LR Test does not apply
  - conservative test
  - mixture of $\chi^2(r)$ and $\chi^2(r+1)$ with mixing prob ½

  m0=lmer(..
  m1=lmer(..
  anova(m0,m1)

z-tests

AIC, BIC $AIC = dev + 2N_{par}$
$BIC = dev + \log(P)N_{par}$