prefmod: modelling preferences using a paired comparison approach

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Response-format	Modell		Designmatrix	Modell-schätzung	Notes
echte Paar-vergleiche	LLBT	Daten	`llbt.design()`	`glm(),gnm()`	1,2,(3),4, (5)
		Daten		`llbt.fit()`	1,3,5
	Pattern	Daten	`patt.design()`	`glm(),gnm()`	2,4,(5),6
		Daten	⟶	`pattPC.fit()`	1,3,(5),6
Rankings	Pattern	Daten	`patt.design()`	`glm(), gnm()`	2,4,(5)
		Daten	⟶	`pattR.fit()`	1,3,5
Ratings (Likert)	Pattern	Daten	`patt.design()`	`glm(),gnm()`	2,4,(5)
		Daten	⟶	`pattL.fit()`	1,3,5,6

(1) NAs
(2) R standard Output
(3) größere Anzahl Vergleiche (Objekte)
(4) Objektvariablen
(5) metrische Personenvariablen
(6) Dependencies

LLBT – Design Matrix

```
> data(cemspc)
> head(cemspc)
  V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11 V12 V13 V14 V15 ENG SEX
1  0  0 NA  2  2  2  0  0  1   0   0   0   1   0   1   1   2
2  0  0 NA  0  2  2  0  2  2   2   0   2   2   0   2   1   1
3  1  0 NA  0  0  2  0  0  1   0   0   0   1   0   1   1   2
4  0  0 NA  0  2  0  0  0  0   0   0   0   0   0   2   1   1
5  0  0 NA  2  2  2  2  2  2   0   0   0   0   0   2   2   2
6  2  2 NA  0  0  0  2  2  2   2   0   0   0   0   2   1   2

> des <- llbt.design(cemspc, nitems = 6, cov.sel = "ENG")
> head(des)
    y mu g0 g1 g2 o1 o2 o3 o4 o5 o6 ENG
1 129  1  1  0  0  1 -1  0  0  0  0   1
2  20  1  0  1  0  0  0  0  0  0  0   1
3  73  1  0  0  1 -1  1  0  0  0  0   1
4 167  2  1  0  0  1  0 -1  0  0  0   1
5  16  2  0  1  0  0  0  0  0  0  0   1
6  39  2  0  0  1 -1  0  1  0  0  0   1
```


LLBT – Design Matrix

Usage:

```
llbt.design(obj, nitems = NULL, objnames = "",
    blnCasewise = FALSE, cov.sel = "",
    blnGLIMcmds = FALSE, glimCmdFile = "", outFile = "")
```

LLBT – mit glm()

```
> eng <- factor(des$ENG)
> res <- glm(y ~ o1 + o2 + o3 + o4 + o5 + o6 + eng:(o1 + o2 +
+     o3 + o4 + o5 + o6) + mu * eng, family = poisson, data = des)
> res$coefficients
(Intercept)          o1          o2          o3          o4          o5
     4.2248      1.0696      0.5771      0.1371      0.1762      0.1339
         o6         mu2         mu3         mu4         mu5         mu6
         NA     -0.1919     -0.3055     -0.1709      0.0264      0.0788
        mu7         mu8         mu9        mu10        mu11        mu12
    -0.1936      0.0148      0.0793      0.0787     -0.2708     -0.0288
       mu13        mu14        mu15        eng2     o1:eng2     o2:eng2
     0.0730      0.0689      0.0733     -1.0528     -0.0521     -0.1786
    o3:eng2     o4:eng2     o5:eng2     o6:eng2    eng2:mu2    eng2:mu3
     0.0098      0.2452     -0.1001          NA      0.0775     -0.0542
   eng2:mu4    eng2:mu5    eng2:mu6    eng2:mu7    eng2:mu8    eng2:mu9
     0.1797      0.0973      0.0202      0.0178      0.0652      0.0403
  eng2:mu10   eng2:mu11   eng2:mu12   eng2:mu13   eng2:mu14   eng2:mu15
    -0.0043      0.0755      0.1005      0.0437     -0.0034      0.0502
```


LLBT – using gnm()

```
> eng <- factor(des$ENG)
> res2 <- gnm(formula = y ~ o1 + o2 + o3 + o4 + o5 + o6 + eng:(o1 +
+     o2 + o3 + o4 + o5 + o6), eliminate = eng:mu, family = poisson,
+     data = des)

> res2
Call:
gnm(formula = y ~ o1 + o2 + o3 + o4 + o5 + o6 + eng:(o1 + o2 +
    o3 + o4 + o5 + o6), eliminate = eng:mu, family = poisson,
    data = des)

Coefficients of interest:
      o1        o2        o3        o4        o5        o6   eng2:o1
 1.06960   0.57713   0.13713   0.17625   0.13390        NA  -0.05207
 eng2:o2   eng2:o3   eng2:o4   eng2:o5   eng2:o6
-0.17865   0.00984   0.24522  -0.10011        NA

Deviance:            1175
Pearson chi-squared: 1030
Residual df:         50
```


LLBT – using llbt.fit()

Usage

```
llbt.fit(y, Xmodel, q, ncat, maxiter = 100)
```


LLBT – using llbt.fit()

```
> mfr <- llbt.design(cemspc, nitems = 6, objnames = c("lo",
+     "pa", "mi", "sg", "ba", "st"), blnCasewise = TRUE)
> mm <- model.matrix(~lo + pa + mi + sg + ba, data = mfr)
> X <- mm[, -1]
> p <- ncol(X)
> ncat <- 3
> q <- length(levels(mfr$mu)) * length(levels(mfr$CASE))
> llbt.fit(mfr$y, X, q, ncat)
Results of llbt.fit:

Deviance: 9010
Residual df = 9085
Number of iterations: 22

   Estimate Std. Error    z   P(z)
lo     1.05      0.047 22.6 0.0000
pa     0.53      0.044 12.1 0.0000
mi     0.14      0.043  3.2 0.0006
sg     0.24      0.042  5.7 0.0000
ba     0.11      0.042  2.5 0.0058
```

eliminate – treating nuisance parameters

IWLS $\quad \hat{\beta}^{(t+1)} = (\tilde{X}^T \hat{W}^{(t)} \tilde{X})^{-1} \tilde{X}^T \hat{W}^{(t)} \hat{z}^{(t)}$

partition critical part $\tilde{X} = (X|X_*)$

- X . . . covariates for model parameters
- X_* . . . covariates for nuisance parameters

$$\tilde{X}^T \hat{W}^{(t)} \tilde{X} = A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} X^T \hat{W}^{(t)} X & X^T \hat{W}^{(t)} X_* \\ X_*^T \hat{W}^{(t)} X & X_*^T \hat{W}^{(t)} X_* \end{pmatrix}$$

for A^{-1} we need inverses of
A_{22} and $(A_{11} - A_{12} A_{22}^{-1} A_{21})$

A_{22} is diagonal, even very large models can be fitted

PATTERN MODEL – design matrix

```
> rgbdat <- read.table("RGB_PC.dat", header = TRUE)
> dsgn <- patt.design(rgbdat, 3, objnames = c("R", "G", "B"))

> head(dsgn, 3)
   y R  G  B
1 10 2  0 -2
2  5 2 -2  0
3  0 0  0  0
> m1 <- glm(y ~ R + G + B, family = poisson, data = dsgn)
> m1
Call:  glm(formula = y ~ R + G + B, family = poisson, data = dsgn)

Coefficients:
(Intercept)            R            G            B
      1.196        0.510        0.288           NA

Degrees of Freedom: 7 Total (i.e. Null);  5 Residual
Null Deviance:            29
Residual Deviance: 18         AIC: 44
```


PATTERN MODEL – design matrix

Usage

```
patt.design(obj, nitems = NULL, objnames = "", resptype = "paircomp",
     blnRevert = FALSE, cov.sel = "", blnIntcovs = FALSE,
     blnGLIMcmds = FALSE, glimCmdFile = "", outFile = "", intFile = "")
```


PATTERN MODEL – fitting utility for PCs

```
> pattPC.fit(cemspc, nitems = 3)
Results of pattern model for paired comparison

Call:
pattPC.fit(obj = cemspc, nitems = 3)

Deviance:  348
log likelihood:  -790

no of iterations:  6  (Code: 1 )

   estimate    se    z p-value
o1     0.93 0.069 13.6       0
o2     0.45 0.070  6.4       0
```

PATTERN MODEL – fitting utility for PCs

```
> pattPC.fit(cemspc, nitems = 3, NItest = TRUE)
Results of pattern model for paired comparison

Call:
pattPC.fit(obj = cemspc, nitems = 3, NItest = TRUE)

Deviance:  348
log likelihood:  -790

no of iterations:  16  (Code: 1 )

       estimate    se     z p-value
o1        0.953 0.080 11.93    0.00
o2        0.457 0.074  6.15    0.00
mis.o1   -0.084 0.172 -0.49    0.63
mis.o2   -0.067 0.218 -0.31    0.76
```


PATTERN MODEL – fitting utility for PCs

```
> m1 <- pattPC.fit(cemspc, nitems = 3, formel = ~1, elim = ~SEX)
> m2 <- pattPC.fit(cemspc, nitems = 3, formel = ~SEX, elim = ~SEX)
> m2
Results of pattern model for paired comparison

Call:
pattPC.fit(obj = cemspc, nitems = 3, formel = ~SEX, elim = ~SEX)

Deviance:  372
log likelihood:  -784
eliminated term(s):  ~SEX

no of iterations:  14  (Code: 1 )

        estimate    se   z p-value
o1          0.71 0.092 7.8   0e+00
o2          0.34 0.096 3.5   4e-04
o1:SEX2     0.47 0.140 3.4   7e-04
o2:SEX2     0.24 0.141 1.7   9e-02
```


PATTERN MODEL – fitting utility for PCs

Usage

```
pattPC.fit(obj, nitems, formel = ~1, elim = ~1, resptype = "paircomp",
    obj.names = NULL, undec = FALSE, ia = FALSE, NItest = FALSE,
    NI = FALSE, MISalpha = NULL, MIScommon = FALSE, MISbeta = NULL,
    pr.it = FALSE)
```


PATTERN MODEL – fitting utility for Rankings

```
> load("conf.Rdata")
> conf[5:8, ]
  PREIS AUSSEN MARKE TECHNIK HERKUNFT INNEN SEX ALTER M.PREF
5    NA      2     4      NA        3     1   1     2      2
6    NA      2     4       3       NA     1   1     1      1
7     3      2     1      NA       NA    NA   2     2      3
8     6      2     4       5        3     1   1     1      2
> mR <- pattR.fit(conf, nitems = 6)
```

PATTERN MODEL – fitting utility for Rankings

```
> mRsex <- pattR.fit(conf, nitems = 6, formel = ~SEX, elim = ~SEX)

> worthmat <- patt.worth(mRsex)
> worthmat
Worthmatrix:

         SEX1 SEX2
PREIS    0.16 0.15
AUSSEN   0.21 0.18
MARKE    0.18 0.18
TECHNIK  0.17 0.19
HERKUNFT 0.11 0.12
INNEN    0.17 0.18
```


PATTERN MODEL – fitting utility for Rankings

```
> plotworth(worthmat)
```