

# Applied Econometrics with

Extension 1

## Financial Econometrics

Financial Econometrics

# Overview

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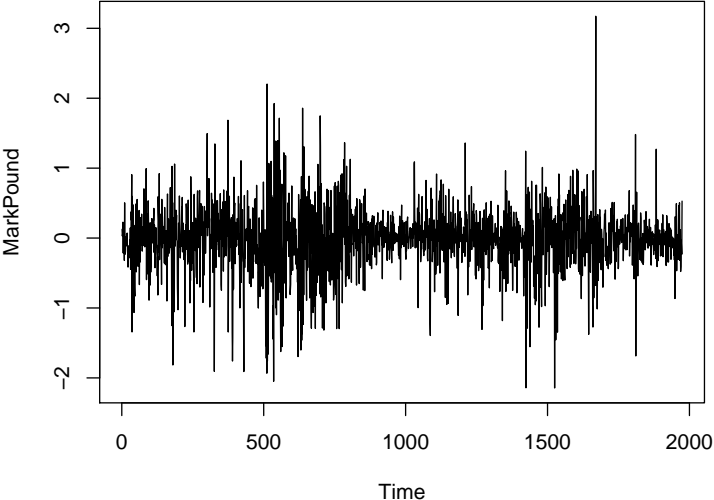
## Further packages for time series analysis

- **dse** – Multivariate time series modeling with state-space and vector ARMA (VARMA) models.
- **FinTS** – R companion to Tsay (2005).
- **forecast** – Univariate time series forecasting, including exponential smoothing, state space, and ARIMA models.
- **fracdiff** – ML estimation of ARFIMA models and semiparametric estimation of the fractional differencing parameter.
- **longmemo** – Convenience functions for long-memory models.
- **mFilter** – Time series filters, including Baxter-King, Butterworth, and Hodrick-Prescott.
- **Rmetrics** – Some 20 packages for financial engineering and computational finance, including GARCH modeling in **fGarch**.
- **tsDyn** – Nonlinear time series models: STAR, ESTAR, LSTAR.
- **vars** – (Structural) vector autoregressive (VAR) models

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# **GARCH Modelling via tseries**

# GARCH models



# GARCH models

**tseries** function `garch()` fits GARCH( $p, q$ ) with Gaussian innovations.  
Default is GARCH(1, 1):

$$y_t = \sigma_t \nu_t, \quad \nu_t \sim \mathcal{N}(0, 1) \text{ i.i.d.},$$
$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \quad \omega > 0, \alpha > 0, \beta \geq 0.$$

**Example:** DEM/GBP FX returns for 1984-01-03 through 1991-12-31

```
R> library("tseries")
R> mp <- garch(MarkPound, grad = "numerical", trace = FALSE)
R> summary(mp)
```

Call:

```
garch(x = MarkPound, grad = "numerical", trace = FALSE)
```

Model:

```
GARCH(1,1)
```

Residuals:

Min	1Q	Median	3Q	Max
-6.79739	-0.53703	-0.00264	0.55233	5.24867

# GARCH models

Coefficient(s):

	Estimate	Std. Error	t value	Pr(> t )
a0	0.0109	0.0013	8.38	<2e-16
a1	0.1546	0.0139	11.14	<2e-16
b1	0.8044	0.0160	50.13	<2e-16

Diagnostic Tests:

Jarque Bera Test

data: Residuals

X-squared = 1100, df = 2, p-value <2e-16

Box-Ljung test

data: Squared.Residuals

X-squared = 2.5, df = 1, p-value = 0.1

## Remarks:

- *Warning:* OPG standard errors assuming Gaussian innovations.
- More flexible GARCH modeling via `garchFit()` in **fGarch**.

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# **GARCH Modelling via Rmetrics**



# Rmetrics

## Rmetrics

- Initiated and mainly developed by D. Würtz (ETH, Dept. of Theoretical Physics).
- Environment for financial engineering and computational finance.
- Currently comprises some 20 packages:  
**fArma, fAsianOptions, fAssets, fBasics, fBonds, fCalendar, fCopulae, fEcofin, fExoticOptions, fExtremes, fGarch, flmport, fMultivar, fNonlinear, fOptions, fPortfolio, fRegression, fSeries, fTrading, fUnitRoots, fUtilities.**
- Unified framework, initially designed for teaching purposes.
- Unified naming conventions via standardized wrappers.  
For example, `arma()` from **stats** appears as `armaFit()`.
- We consider GARCH modelling via `garchFit()` from **fGarch**.

# GARCH modeling via `garchFit()`

**Example:** DEM/GBP FX returns for 1984-01-03 through 1991-12-31

```
R> library("fGarch")  
R> mp_gf <- garchFit(~garch(1,1), data = MarkPound, trace = FALSE)  
R> summary(mp_gf)
```

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~garch(1, 1), data = MarkPound,  
trace = FALSE)
```

Mean and Variance Equation:

```
data ~ garch(1, 1)
```

```
<environment: 0x5613f8a63330>
```

```
[data = MarkPound]
```

Conditional Distribution:

```
norm
```

Coefficient(s):

mu

omega

alpha1

beta1

# GARCH modeling via garchFit()

-0.0061903 0.0107614 0.1531341 0.8059737

Std. Errors:  
based on Hessian

Error Analysis:

	Estimate	Std. Error	t value	Pr(> t )
mu	-0.006190	0.008462	-0.732	0.464447
omega	0.010761	0.002838	3.793	0.000149
alpha1	0.153134	0.026422	5.796	6.8e-09
beta1	0.805974	0.033381	24.144	< 2e-16

Log Likelihood:  
-1107 normalized: -0.5606

Description:  
Thu Oct 12 13:41:42 2017 by user: zeileis

Standardised Residuals Tests:

	Statistic	p-Value
Jarque-Bera Test	R	Chi <sup>2</sup> 1060
		0

# GARCH modeling via `garchFit()`

Shapiro-Wilk Test	R	W	0.9623	0
Ljung-Box Test	R	Q(10)	10.12	0.4299
Ljung-Box Test	R	Q(15)	17.04	0.3163
Ljung-Box Test	R	Q(20)	19.3	0.5026
Ljung-Box Test	R <sup>2</sup>	Q(10)	9.063	0.5262
Ljung-Box Test	R <sup>2</sup>	Q(15)	16.08	0.3769
Ljung-Box Test	R <sup>2</sup>	Q(20)	17.51	0.6198
LM Arch Test	R	TR <sup>2</sup>	9.771	0.636

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
1.125	1.137	1.125	1.129

## Remarks:

- Benchmark data set for GARCH(1, 1), see McCullough and Renfro (*J. Economic and Social Measurement* 1998).  
`garchFit()` hits the benchmark.
- Note that constant included by default (not possible with **tseries**).
- Standard errors are from the Hessian.

## More on `garchFit()`

`garchFit()` **provides**

- ARMA models with GARCH-type innovations
- Various innovation distributions: Gaussian,  $t$ , GED, including skewed generalizations.
- Several algorithms for maximizing log-likelihood, default is `nlminb`.
- Two methods for initializing recursions.

# ARMA models with GARCH components

Mean equation is ARMA

$$y_t = \mu + \sum_{t-i}^m \phi_i y_{t-i} + \sum_{t-j}^n \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Variance equation for GARCH( $p, q$ ) is

$$\begin{aligned}\varepsilon_t &= \sigma_t \nu_t, \\ \nu_t &\sim \mathcal{D}_\vartheta(0, 1) \text{ i.i.d.}, \\ \sigma_t^2 &= \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{t-j}^q \beta_j \sigma_{t-j}^2.\end{aligned}$$

# ARMA models with APARCH components

Mean equation is ARMA

$$y_t = \mu + \sum_{t-i}^m \phi_i y_{t-i} + \sum_{t-j}^n \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Variance equation for APARCH( $p, q$ ) is

$$\begin{aligned}\varepsilon_t &= \sigma_t \nu_t, \\ \nu_t &\sim \mathcal{D}_\vartheta(0, 1) \text{ i.i.d.}, \\ \sigma_t^\delta &= \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{t-j}^q \beta_j \sigma_{t-j}^\delta.\end{aligned}$$

where  $\delta > 0$  and the leverage parameters  $-1 < \gamma_i < 1$ .

APARCH comprises various GARCH-type models, including ARCH, GARCH, Taylor/Schwert-GARCH, GJR-GARCH, TARCH, NARCH, log-ARCH, ...

# ARMA models with APARCH components

**More complex example:** Ding, Granger, Engle (*J. Emp. Fin.* 1993)  
MA(1)-APARCH(1,1) model for S&P 500 returns (17055 observations)

```
R> sp_ap <- garchFit(~ arma(0,1) + aparch(1,1),  
+ data = ts(100 * sp500dge), trace = FALSE)
```

Excerpt from `summary(sp_ap)`:

Std. Errors:  
based on Hessian

Error Analysis:

	Estimate	Std. Error	t value	Pr(> t )
mu	0.020595	0.006342	3.247	0.00116
ma1	0.144709	0.008346	17.338	< 2e-16
omega	0.009991	0.001066	9.373	< 2e-16
alpha1	0.083792	0.004343	19.293	< 2e-16
gamma1	0.374182	0.028027	13.351	< 2e-16

Results broadly agree with original paper (p. 99, eq. (19)), where algorithm was BHHH. (Note: percentage returns!)



# ARMA models with APARCH components

## Further ARCH-type models:

Taylor-Schwert ARCH (compare Ding, Granger, Engle, eq. (16))

```
R> sp_tsarch <- garchFit(~ arma(0,1) + garch(1,1), delta = 1,  
+ data = ts(100 * sp500dgc), trace = FALSE)
```

Threshold ARCH (TARCH)

```
R> sp_tarch <- garchFit(~ arma(0,1) + garch(1,1), delta = 1,  
+ leverage = TRUE, data = ts(100 * sp500dgc), trace = FALSE)
```

GJR-GARCH

```
R> sp_tarch <- garchFit(~ arma(0,1) + garch(1,1), delta = 2,  
+ leverage = TRUE, data = ts(100 * sp500dgc), trace = FALSE)
```

# ARMA models with APARCH components

## Specifying innovation distributions:

`cond.dist` – specification of conditional distributions allowing for "dnorm", "dged", "dstd", "dsnrm", "dsged", "dsstd". Three of these ("dsnrm", "dsged", "dsstd") are skewed. – Thus

GARCH(1,1) with Student- $t$  (shape parameter estimated)

```
R> sp_garch_std <- garchFit(~ garch(1,1), cond.dist = "dstd",  
+ data = ts(100 * sp500dge), trace = FALSE)
```

GARCH(1,1) with Student- $t_3$  (shape parameter fixed at 3)

```
R> sp_garch_std3 <- garchFit(~ garch(1,1),  
+ cond.dist = "dstd", shape = 3, include.shape = FALSE,  
+ data = ts(100 * sp500dge), trace = FALSE)
```

GARCH(1,1) with Laplace (a GED with shape fixed at 1)

```
R> sp_garch_ged <- garchFit(~ garch(1,1),  
+ cond.dist = "dged", shape = 1, include.shape = FALSE,  
+ data = ts(100 * sp500dge), trace = FALSE)
```

# ARMA models with APARCH components

## Further remarks:

- More details regarding fitting process, defaults, etc. upon setting `trace = TRUE`
- `plot()` method offers 12 types of plots: time series, conditional std. dev., ACF of obs. and squared obs., residuals, ACF of residuals and squared residuals, etc.

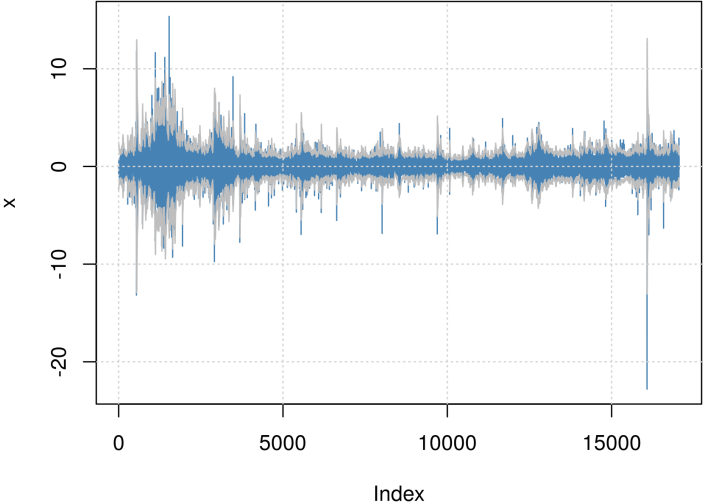
Example: (ARMA-APARCH cont'd)

Series with superimposed conditional std. dev. is

```
R> plot(sp_ap, which = 3)
```

# ARMA models with APARCH components

## Series with 2 Conditional SD Superimposed



Financial Econometrics

# Extensions

# Additional tools for financial engineering

- Portfolio management: **fPortfolio**, **portfolio** offer portfolio selection and optimization.
- Risk management:
  - Classical Value-at-Risk: **VaR**.
  - Extreme Value Theory models: **evd**, **evdbayes**, **evir**, **extRemes**, **ismec**, **POT**.
  - Multivariate modeling: **fCopulae**, **copula**, **fgac**
- High-frequency data: **realized**.

More complete overview in CRAN Task View Empirical Finance at

<http://CRAN.R-project.org/view=Finance>