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# Exchange Rate Regime Classification with Structural Change Methods

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# Overview

- Exchange rate regimes
- What is the new Chinese exchange rate regime?
- Frankel-Wei regression for de facto exchange rate regime classification
- Regime stability
  - Testing
  - Monitoring
  - Dating
- Applications: Indian exchange rate regimes
- Software

# Exchange rate regimes

The foreign exchange (FX) rate regime of a country determines how it manages its currency wrt foreign currencies. It can be

- *floating:* currency is allowed to fluctuate according to the foreign exchange market,
- *pegged:* currency is fluctuating only in a certain band, pegged to (basket of) other currencies,
- *fixed:* direct convertibility to another currency.

**Problem:** The *de facto* and *de jure* FX regime in operation in a country often differ.

$\Rightarrow$ Interest in methods for data-driven classification of FX regimes (see e.g., Reinhart and Rogoff, 2004).

# Chinese exchange rate regime

China gave up on a fixed exchange rate to the US dollar (USD) on 2005-07-21.

The People's Bank of China announced that the Chinese yuan (CNY) would no longer be pegged to the USD but to a basket of currencies with greater flexibility.

This generated a lot of interest, both in the media and the scientific literature. However, little support could be found for the announcements of the People's Bank of China.

Shah, Zeileis, Patnaik (2005) investigate the Chinese *de facto* FX regime based on the so-called *Frankel-Wei regression model* using structural change methods.

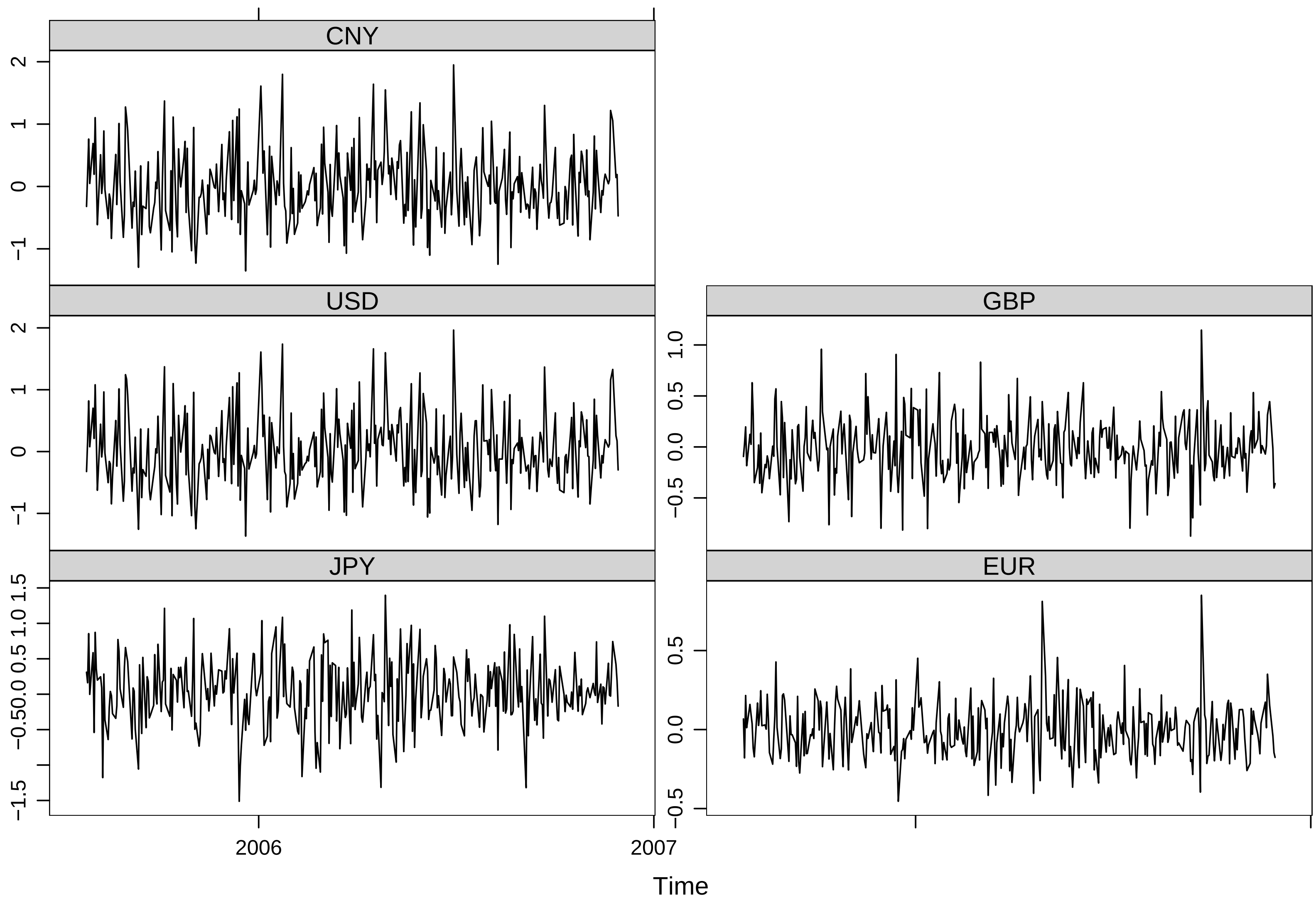
# Frankel-Wei regression

The Frankel-Wei model (Haldane and Hall 1991, Frankel and Wei 1994) is the popular workhorse for de facto FX regime classification. It is a linear regression based on log-returns of cross-currency exchange rates (with respect to some floating reference currency).

Fitting the model for CNY with regressors USD, JPY, EUR and GBP (all wrt CHF) based on data up to 2005-10-31 ($n = 68$) shows that a plain USD peg is still in operation:

$$\begin{aligned} \mathsf{CNY}_i &= -0.005 + 0.9997\,\mathsf{USD}_i + 0.005\,\mathsf{JPY}_i \\ &\quad -0.014\,\mathsf{EUR}_i - 0.008\,\mathsf{GBP}_i + \hat{u}_i. \end{aligned}$$

# Frankel-Wei regression



# Frankel-Wei regression

```
Call:
fxlm(formula = CNY ~ USD + JPY + EUR + GBP, data = window(cny,
    end = as.Date("2005-10-31")))

Residuals:
      Min        1Q    Median        3Q       Max
-0.065697 -0.021036  0.001147  0.021440  0.069985

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.004782   0.003688  -1.297    0.199
USD          0.999653   0.008779 113.868   <2e-16 ***
JPY          0.004668   0.010669   0.437    0.663
EUR         -0.014238   0.026516  -0.537    0.593
GBP         -0.007744   0.014568  -0.532    0.597
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02953 on 63 degrees of freedom
Multiple R-Squared: 0.9979,        Adjusted R-squared: 0.9978
F-statistic:  7577 on 4 and 63 DF,  p-value: < 2.2e-16
```

# Regime stability

**Questions:**

1. Is this model for the period 2005-07-26 to 2005-10-31 stable or is there evidence that China kept changing its FX regime after 2005-07-26? (*testing*)
2. Depending on the answer to the first question:
   - Does the CNY stay pegged to the USD in the future (starting form November 2005? (*monitoring*)
   - When and how did the Chinese FX regime change? (*dating*)

# Regime stability

**In practice:** Rolling regressions are often used to answer these questions by tracking the evolution of the FX regime in operation.

**More formally:** Structural change techniques can be adapted to the Frankel-Wei regression to estimate and test the stability of FX regimes.

**Problem:** Unlike many other linear regression models, the stability of the error variance (fluctuation band) is of interest as well.

**Solution:** Employ an (approximately) normal regression estimated by ML where the variance is a full model parameter.

# Regime stability

The Frankel-Wei regression is essentially a standard linear regression model

$$y_i \quad = \quad x_i^\top \beta \; + \; u_i$$

with coefficients $\beta$ and error variance $\sigma^2$.

The corresponding estimating functions for the parameters are

$$\begin{aligned} \psi_\beta(y, x, \beta) &= (y - x^\top \beta)\, x, \\ \psi_{\sigma^2}(y, x, \beta, \sigma^2) &= (y - x^\top \beta)^2 - \sigma^2. \end{aligned}$$

To test the stability of the parameters $\beta$ and $\sigma^2$, it can be assessed whether the empirical estimating functions differ systematically from their zero mean.

# Testing

**More formally:** Based on the empirical estimating functions

$$\hat{\psi}_i \quad = \quad (\psi_\beta(y_i, x_i, \hat{\beta}), \psi_{\sigma^2}(y_i, x_i, \hat{\beta}, \hat{\sigma}^2))^\top$$

capture deviations in the scaled cumulative sum process

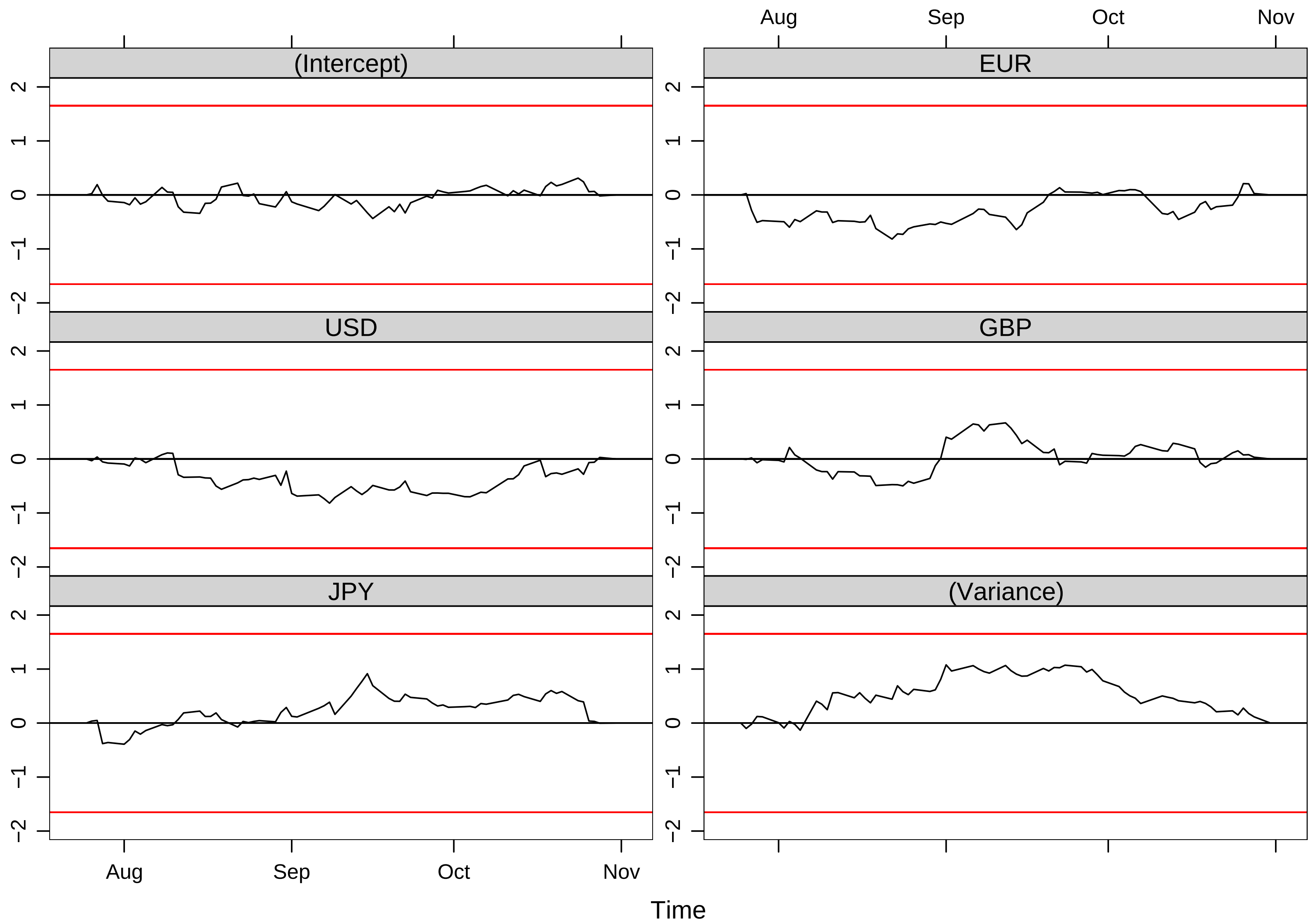
$$W_n(t) \quad = \quad \hat{B}^{-1/2}\, n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \hat{\psi}_i \qquad (0 \le t \le 1).$$

For this process a functional central limit theorem (FCLT) holds:

$$W_n(\cdot) \quad \xrightarrow{\mathrm{d}} \quad W^0(\cdot),$$

where $W^0(t)$ ($t \in [0, 1]$) is a standard Brownian bridge. The process $W_n(\cdot)$ is the basis for inference: various types of statistics can be computed that capture systematic deviations from zero.

# Testing



(Intercept)
EUR
USD
GBP
JPY
(Variance)
2
1
0
−1
−2
Aug
Sep
Oct
Nov
Time

# Testing

This corresponds to using a double maximum statistic

$$\max_{j=1,\dots,k} \max_{i=1,\dots,n} |W_n(i/n)_j|$$

which is here $1.078$ ($p = 0.73$). Alternatively, use:

- Nyblom-Hansen test using a Cramér-von Mises functional

$$n^{-1} \sum_{i=1}^{n} ||W_n(i/n)||_2^2 \,,$$

- Andrews' sup*LM* test

$$\sup_{t \in \Pi} \frac{||W_n(t)||_2^2}{t(1-t)}.$$

See Zeileis (2005) for a unified approach.

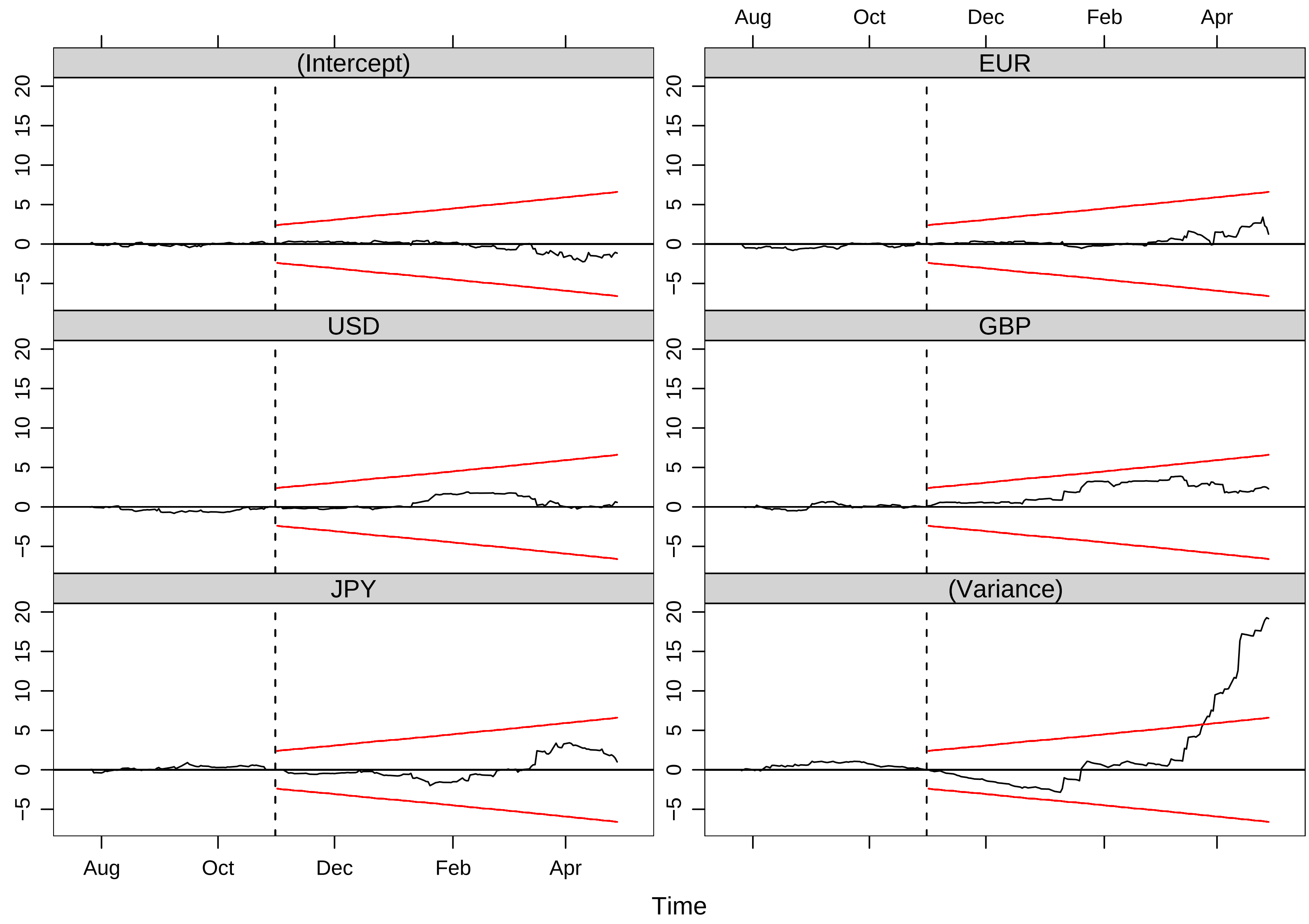
# Monitoring

The same ideas can be used to test whether incoming observations $i > n$ conform with an established model—this is referred to as monitoring of linear regression models (Chu *et al.*, 1996, Horváth *et al.*, 2004).

**Basic assumption:** The model parameters are stable in the history period $i = 1, \dots, n$.

The same empirical fluctuation process $W_n(t)$ is updated in the monitoring period ($t > 1$ corresponding to $i > n$) using suitable boundaries (Zeileis, 2005).

# Monitoring



(Intercept)
EUR
USD
GBP
JPY
(Variance)
20
15
10
5
0
−5
Aug
Oct
Dec
Feb
Apr
Time

# Monitoring

This signals a clear increase in the error variance which is picked up by the monitoring procedure on 2006-03-27.

However, all other regression coefficients did not change significantly, signalling that a USD peg is still in operation.

Using data from the extended period up to 2006-12-01, we fit a segmented model to determine where and how the model parameters changed.

# Dating

Bai and Perron (2003) describe a strategy for estimating breakpoints in linear regressions based on the residual sum of squares:

$$\Psi_{\mathsf{RSS}}(\beta) \quad = \quad \sum_{i=1}^{n} (y_i - x_i^\top \beta)$$

For this additive objective function, a dynamic programming algorithm that evaluates all potential $m$-partitions (i.e., with $m$ breakpoints) is available. It is an application of Bellman's principle of optimality.

**Problem:** Dating based on $\Psi_{\mathsf{RSS}}(\cdot)$ does not exploit changes in the error variance (only regression coefficients).

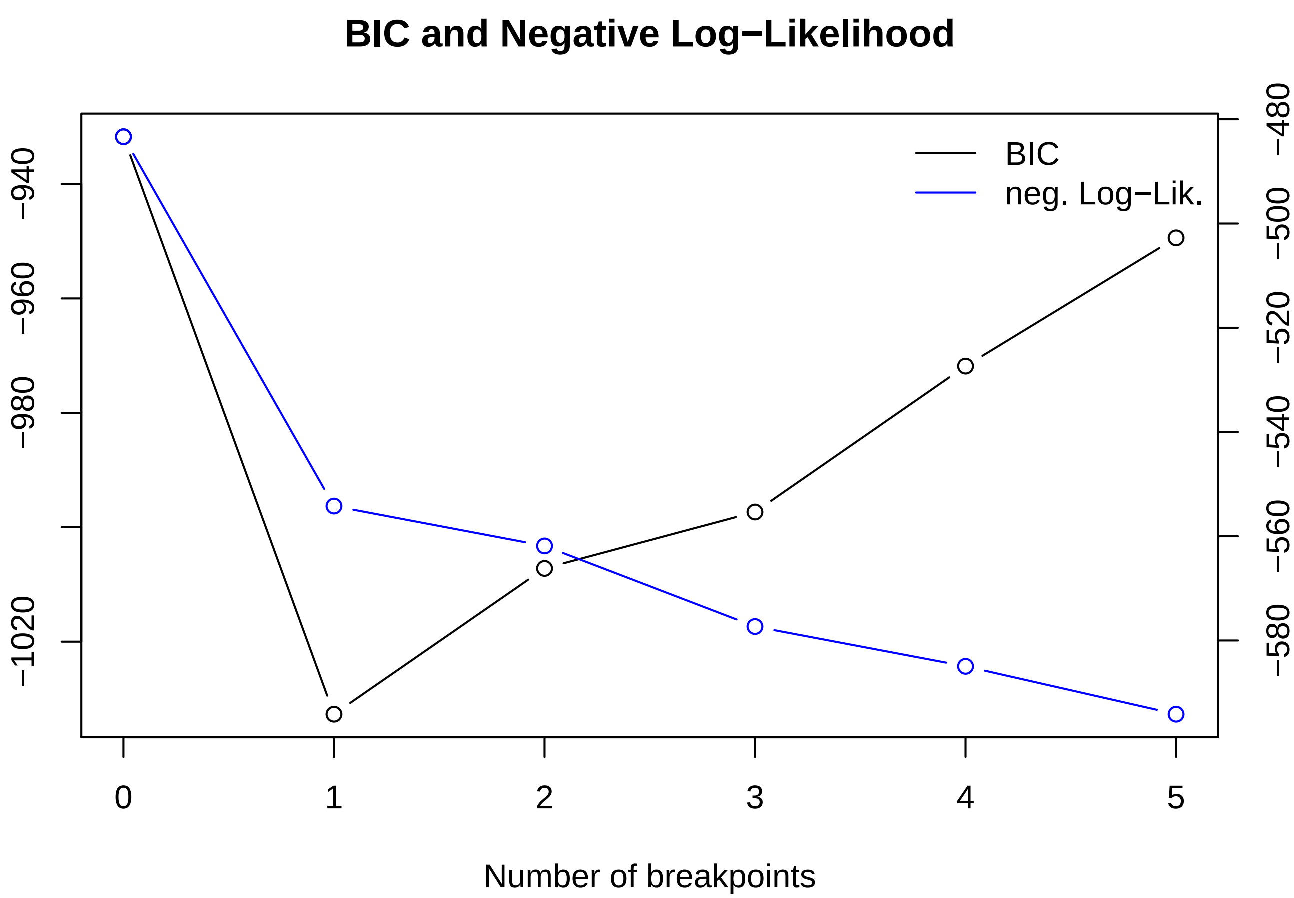
# Dating

For the Frankel-Wei regression, we employ the same dynamic programming algorithm based on a different additive objective function: the (negative) log-likelihood from a normal model so that changes in the variance are also captured:

$$\Psi_{\mathsf{loglik}}(\beta, \sigma) \quad = \quad - \sum_{i=1}^{n} \log \left( \sigma^{-1} \phi \left( \frac{y_i - x_i^\top \beta}{\sigma} \right) \right).$$

For a fixed given number of breaks $m$, the optimal breaks (wrt $\Psi_{\mathsf{loglik}}(\cdot, \cdot)$) can be found. To determine the number of breaks, standard techniques for model selection can be applied here, e.g., information criteria or sequential tests etc.

# Dating



BIC and Negative Log−Likelihood
BIC
neg. Log−Lik.
−940
−960
−980
−1020
−480
−500
−520
−540
−560
−580
0
1
2
3
4
5
Number of breakpoints

# Dating

The estimated breakpoint (maximizing the segmented likelihood) is 2006-03-14.

The corresponding parameter estimates are

| | (Intercept) | USD | JPY | EUR | GBP | (Std. Error) |
|---|---|---|---|---|---|---|
| 2005-07-26--2006-03-14 | -0.005 | 0.999 | 0.005 | -0.015 | 0.007 | 0.028 |
| 2006-03-15--2006-11-29 | -0.016 | 0.993 | 0.009 | -0.009 | -0.025 | 0.074 |

and correspond to a

- very tight USD peg,
- slightly relaxed USD peg.

# Indian FX regimes

To show how this methodology can be employed in practice, the evolution of the Indian FX regime starting from 1993-04-01 is analyzed.

All functionality is available within the R system for statistical computing using the **strucchange** package and a set of convenience interfaces in the package **fxregime**.

```
R> library("fxregime")
R> data("FXRatesCHF", package = "fxregime")
R> inr <- fxreturns("INR", frequency = "weekly",
+     data = window(FXRatesCHF,
+         start = as.Date("1993-04-01")))
```

# Indian FX regimes

A simple convenience interface to `lm()` is used for fitting the regression for the full sample period (1993-04-01 to 2006-12-01):

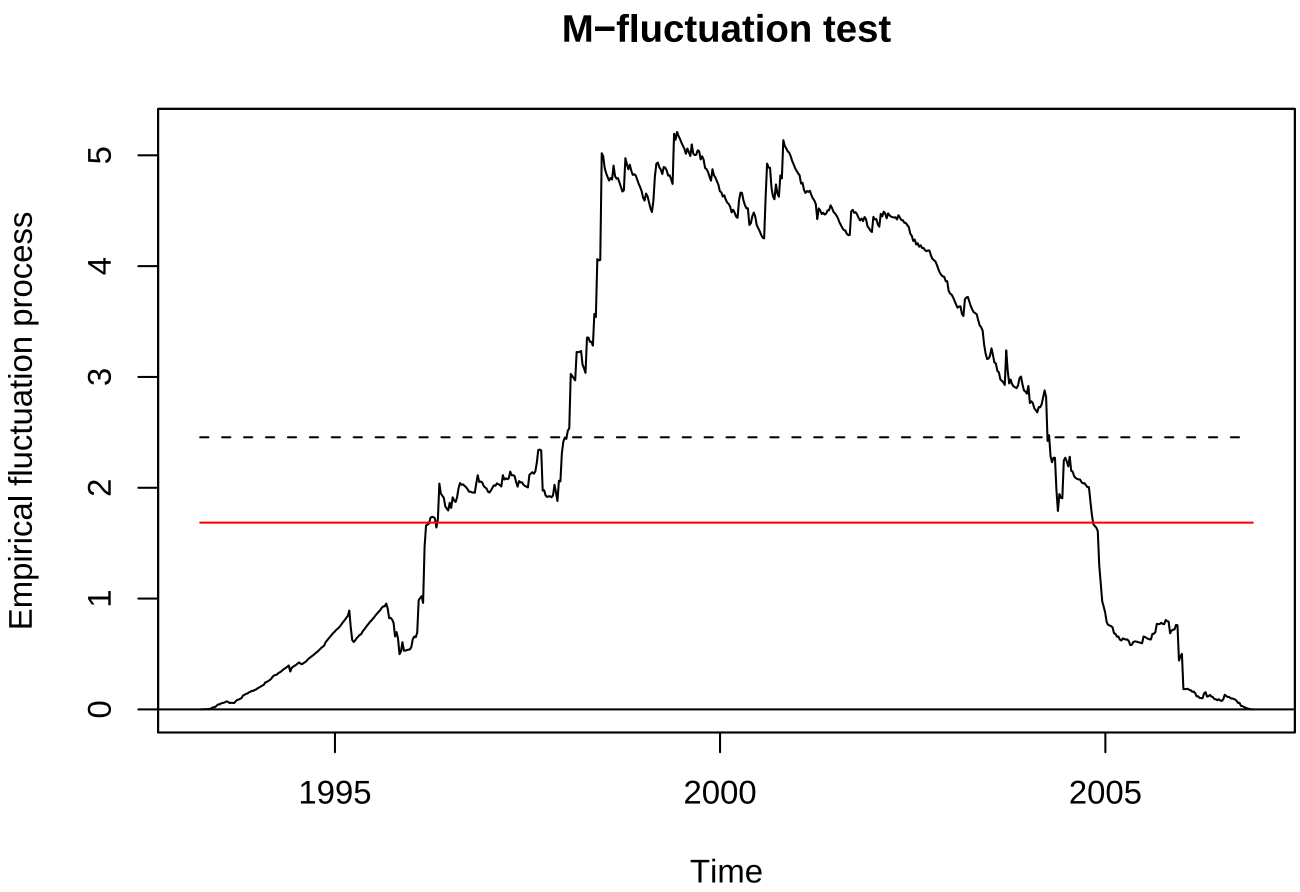
```
R> inr_lm <- fxlm(INR ~ USD + JPY + EUR + GBP,
+     data = inr)
```

which is subsequently assessed using the Nyblom-Hansen test

```
R> inr_efp <- gefp(inr_lm, fit = NULL)
R> plot(inr_efp, functional = meanL2BB)
```

leading to a test statistic of $2.456$ ($p < 0.001$).

# Indian FX regimes



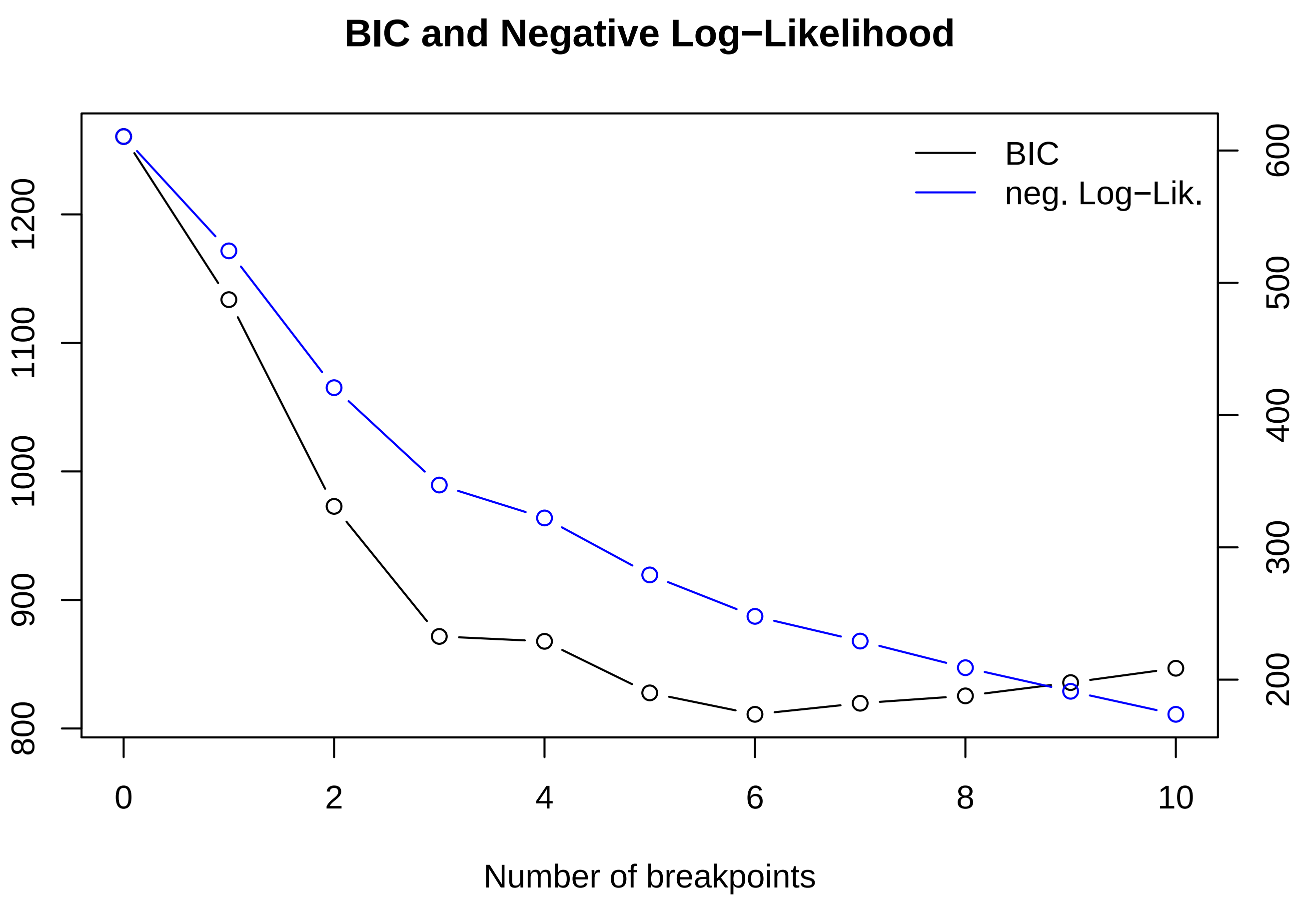
# Indian FX regimes

Given the clear evidence of structural instability of the FX regime, it should be determined what reasonable breakpoints are:

```
R> inr_reg <- fxregimes(INR ~ USD + JPY + EUR +
+     GBP, data = inr, h = 20, breaks = 10)
R> plot(inr_reg)
```

The BIC would select $m = 6$ breakpoints. However, given the kink in the BIC curve, it seems to be reasonable to inspect the $m = 3$ breakpoints model as well.

# Indian FX regimes



# Indian FX regimes

| | (Intercept) | USD | JPY | EUR | GBP | (Std. Error) |
|---|---|---|---|---|---|---|
| 1993-04-09--1995-03-03 | -0.006 | 0.972 | 0.023 | 0.011 | 0.020 | 0.157 |
| 1995-03-10--1998-08-21 | 0.161 | 0.943 | 0.067 | -0.026 | 0.042 | 0.924 |
| 1998-08-28--2004-03-19 | 0.019 | 0.993 | 0.010 | 0.098 | -0.003 | 0.275 |
| 2004-03-26--2006-12-01 | -0.020 | 0.746 | 0.240 | 0.203 | 0.087 | 0.530 |

revealing the following FX regimes:

1. tight USD peg,
2. flexible USD peg,
3. tight USD peg,
4. flexible basket peg.

The solution with $m = 6$ breakpoints is, in fact, similar. Only the second regime is partitioned into further segments.

# Indian FX regimes

| | (Intercept) | USD | JPY | EUR | GBP | (Std. Error) |
|---|---|---|---|---|---|---|
| 1993-04-09--1994-05-27 | -0.014 | 0.981 | 0.020 | 0.004 | 0.044 | 0.098 |
| 1994-06-03--1995-08-25 | 0.017 | 0.885 | 0.000 | 0.196 | 0.078 | 0.286 |
| 1995-09-01--1996-08-09 | 0.174 | 1.167 | 0.353 | -0.791 | -0.036 | 1.224 |
| 1996-08-16--1997-08-15 | 0.011 | 1.010 | -0.013 | -0.102 | 0.035 | 0.197 |
| 1997-08-22--1998-08-21 | 0.365 | 0.704 | -0.043 | 0.672 | -0.043 | 0.967 |
| 1998-08-28--2004-03-19 | 0.019 | 0.993 | 0.010 | 0.098 | -0.003 | 0.275 |
| 2004-03-26--2006-12-01 | -0.020 | 0.746 | 0.240 | 0.203 | 0.087 | 0.530 |

# Software

All methods are implemented in the R system for statistical computing and graphics

`http://www.R-project.org/`

in the contributed packages **strucchange** available from CRAN and **fxregime** which is under development at R-Forge.

`http://CRAN.R-project.org/`
`http://R-Forge.R-project.org/`

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