A framework for heteroskedasticity-robust specification and misspecification testing functions for linear models in R

Achim Zeileis, Wirtschaftsuniversität Wien
and
Giovanni Millo, R&D Dept., Generali S.p.A.

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Motivation

- Integrating the existing toolbox for econometric model specification (`lmtest`) with flexible zero-restriction testing functions, robust vs.:
  - heteroskedasticity
  - autocorrelation
  - (non-normality)
- providing the versions behaving best in small samples

A comprehensive approach

Specification testing:

$\mathcal{M}_m$ —restriction test→ test

H_0: Rβ=0 for H_0

Misspecification testing:

$\mathcal{M}_m$ —translate→ $\mathcal{M}_{aux}$ —restriction test→ test

H_0 Rβ=0 for H_0

Non-nested model comparison:

$\mathcal{M}_m$ —translate→ $\mathcal{M}_{encomp}$ —restriction test→ test

$\mathcal{M}_{alternative}$ Rβ=0 for H_0

3 ways of testing (zero-) restrictions

Wald test

Lagrange multipliers

Likelihood ratio

- asymptotically equivalent
- possibly conflicting in small samples
- in the linear model with spherical errors,

$LM \leq LR \leq W$

The HC issue in restriction testing

- Heteroskedasticity and autocorrelation invalidate standard restrictions (significance) testing
- Het. screening tests are known to have little power, so researchers are advised (MacKinnon and White, JoE 1985; Long and Ervin, JASA 2000) to use robust statistics in the first place in “suspect” situations
- The original White matrix (a.k.a. HC0) has poor small-sample properties: quasi-t tests based on it are grossly overrejecting (LE 2000)

Design principles

- Translating the theoretical approach to likelihood-based restriction testing (Wald-LM-LR) into software
- Reusing tools from package `sandwich` for covariance matrix computations
- Making the restriction testing functions reusable as computing tools for tests based on auxiliary models
- Preserving “freedom of tool choice” at every step of computation

Present situation

Base R provides:

- `summary()` method: partial t-tests
- `anova()` method: F-tests for nested model comparison

both based on $\text{vcov}(\varepsilon)=\sigma^2 I$ (and normality);

The `sandwich` package provides a general framework for HC and HAC estimators of vcov (βhat)

Robust restrictions testing

Plugging in estimators from `sandwich`, a Wald test robust vs. heteroskedasticity and autocorrelation of residuals can be implemented for restriction testing

Asymptotic and exact (F) versions of this are already available as `waldtest()` in package `lmtest`, as are robust versions of quasi-t tests (`coeftest()`)

The interface: model specification

Convenience options are available in the `waldtest()` interface: one may specify:

- a list of (nested) model objects or formulae or
- a model object (or formula) and
  - the index(es) of the regressor(s) to be tested for exclusion or
  - the name(s) of the same

The interface: covariance spec.

`coeftest()`, `waldtest()` (and thus all higher-level functions) take a vcov argument either as a matrix or as an estimating function, e.g. from `sandwich`, defaulting

- to the standard `vcov()`
- to HC3 (if `vcovHC()` is specified)

A new function implementing a particular estimator may easily be set up and supplied (see Zeileis, JSS 2004)

Work in progress: higher level tests

Some testing functions based on `waldtest()` and `coeftest()` are already available in `lmtest`:

- Granger causality
  - `grangertest()`
- Non-nested model comparison
  - `encomptest()`
  - `jtest()`

Work in progress: LM, LR

Lagrange Multiplier (score test) and Likelihood Ratio counterparts to the Wald test are in the last stages of development

We are currently experimenting with small-sample “OLS” versions of both and with LM-HC (Wooldridge, MIT WP 1987, also in his *Econometrics of cross-section and panel data*)

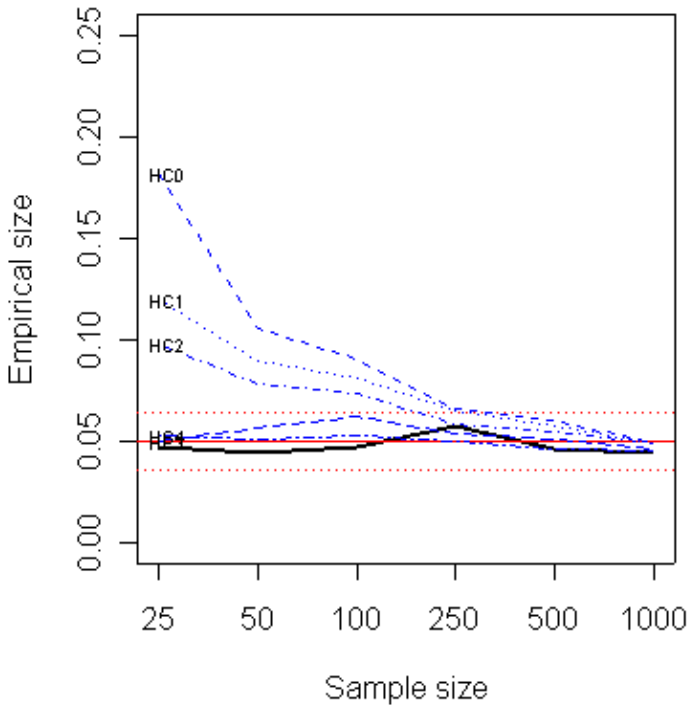
Work in progress: performance assessment

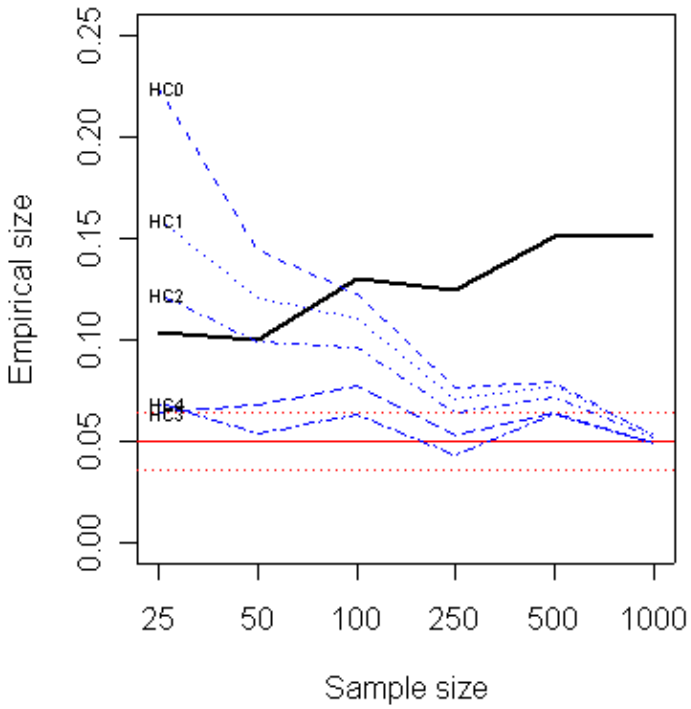
Assessment of small-sample behaviour and HC-robustness in terms of:

- empirical vs. nominal test size
- empirical power

through montecarlo simulations based on Long and Ervin's design (cit., 2000)

A first glance at the results: Wald F-test on hcn4 vs. hc123c





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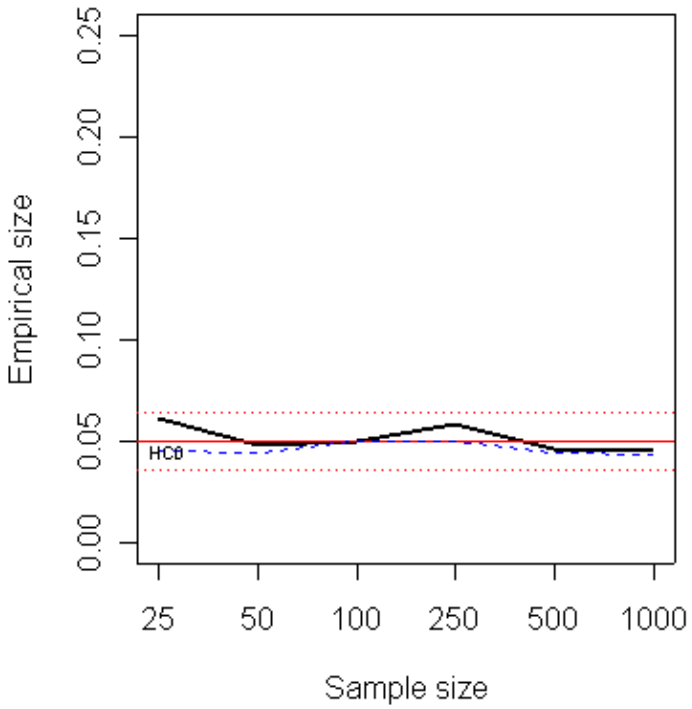

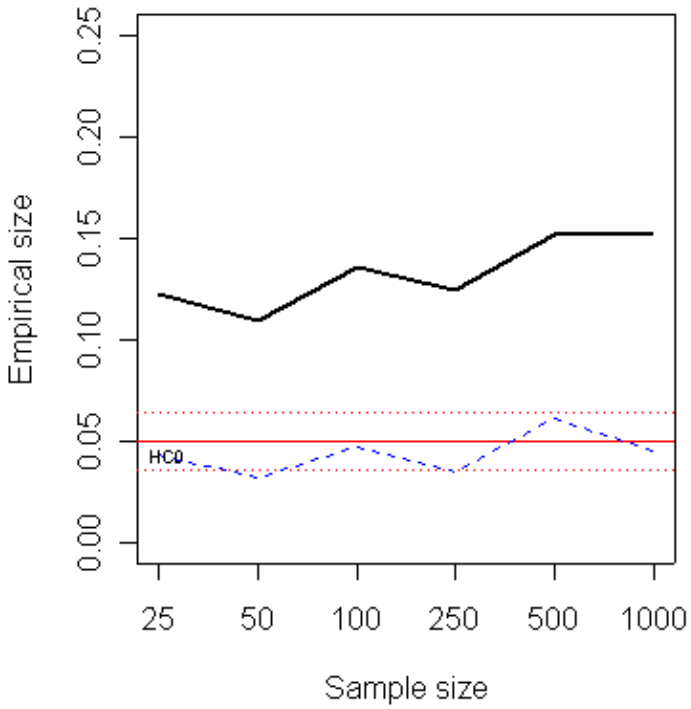

Summing up

- HC and HAC zero-restriction testing functions for linear regression models are available on CRAN (package `lmtest`) in the Wald test implementation
- These may be reused in higher-level spec. tests
- Through HC3 and HC4 corrections, performance looks satisfactory even in small samples
- LM and LR alternatives, though less straightforward in design, are under development
- We hope we made it easier to follow MacKinnon and White's advice (in 1985, cit.) to employ HC tests whenever heteroskedasticity is suspected