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Generalized M-Fluctuation Tests for Parameter Instability

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Contents

Model frame

We assume n independent observations

$$Y_i \quad \sim \quad F(\theta_i) \qquad (i = 1, \ldots, n).$$

from distribution F with k-dimensional parameter θ_i.

Observations are ordered with respect to “time” and can be vector-valued.

Extension to regression situation and dependent data: later.

Model frame

Null hypothesis:

$$H_0 : \quad \theta_i = \theta_0 \qquad (i = 1, \ldots, n).$$

Alternative:

H_1: θ_i varies over "time" i.

Philosophy

The generalized fluctuation test framework ...

"... includes formal significance tests but its philosophy is basically that of data analysis as expounded by Tukey. Essentially, the techniques are designed to bring out departures from constancy in a graphic way instead of parametrizing particular types of departure in advance and then developing formal significance tests intended to have high power against these particular alternatives." (Brown, Durbin, Evans, 1975)

Generalized fluctuation tests

- empirical fluctuation processes reflect fluctuation in
  - residuals
  - coefficient estimates
  - *M-scores* (including OLS or ML scores etc.)
- theoretical limiting process is known
- choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability α.
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.

Generalized M-fluc. processes

Consider a smooth k-dimensional score function $\psi(\cdot)$ with:

$$\mathsf{E}\{\psi(Y_i, \theta_i)\} \quad = \quad 0$$

and define the covariance matrix

$$B(\theta) \quad = \quad \mathsf{COV}_{\theta_0}\{\psi(Y, \theta)\}.$$

Generalized M-fluc. tests

A common choice for ψ is the partial derivative of some objective function Ψ

$$\psi(y, \theta) \quad = \quad \frac{\partial \Psi(y, \theta)}{\partial \theta}.$$

which includes OLS and ML.

In a misspecification context: Quasi-ML, robust M-estimation.

$$\psi(y, \theta) \quad = \quad \mathsf{min}(c, \mathsf{max}(y - \theta, -c)).$$

Instead of full likelihood use estimating equations, IV, GMM, GEE.

Theoretical M-fluc. processes

Consider the cumulative score process given by

$$W_n(t, \theta) \quad = \quad n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \psi(Y_i, \theta).$$

Theoretical M-fluc. processes

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$$W_n(t, \theta) \quad = \quad n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \psi(Y_i, \theta).$$

Under H_0 the following functional central limit theorem (FCLT) holds

$$B(\theta_0)^{-1/2} W_n(\cdot, \theta_0) \quad \xrightarrow{\text{d}} \quad W(\cdot).$$

Empirical M-fluc. processes

A suitable estimate $\widehat{\theta}_n$ of θ_0 is defined by

$$\sum_{i=1}^{n} \psi(Y_i, \widehat{\theta}_n) \quad = \quad 0.$$

Empirical M-fluc. processes

A suitable estimate $\widehat{\theta}_n$ of θ_0 is defined by

$$\sum_{i=1}^{n} \psi(Y_i, \widehat{\theta}_n) \quad = \quad 0.$$

Under H_0 the following FCLT holds

$$\widehat{B}_n^{-1/2} W_n(\cdot, \widehat{\theta}_n) \quad \xrightarrow{\mathrm{d}} \quad W^0(\cdot),$$

for some consistent covariance matrix estimate $\widehat{B}_n$.

Generalized M-fluc. tests

In an empirical sample the empirical M-fluctuation process

$$efp(t) \quad = \quad \widehat{B}_n^{-1/2} W_n(t, \widehat{\theta}_n).$$

is a $k \times n$ array. Can be aggregated to a scalar test statistic by a functional $\lambda(\cdot)$

$$\lambda \left(efp_j \left(\frac{i}{n} \right) \right),$$

where $j = 1, \ldots, k$ and $i = 1, \ldots n$.

Generalized M-fluc. tests

λ can usually be split into two components: λ_{time} and λ_{comp}.

Typical choices for λ_{time}: L_∞ (absolute maximum), mean, range.

Typical choice for λ_{comp}: L_∞, L_2.

$\Rightarrow$ can identify component and/or timing of shift. Requires different visualization techniques.

Generalized M-fluc. tests

In a regression situation:

$$\psi(Y_i, \theta_i) \quad = \quad \psi(y_i, x_i, \theta_i),$$

Assumptions:

- Zero expectation with respect to $f(y_i \,|\, x_i, \theta_i)$
- Stabilizing variances

$$\frac{1}{n} \sum_{i=1}^{n} \mathsf{COV}\{\psi(y_i, x_i, \theta_0)\} \;=\; J_n \quad \xrightarrow{\mathsf{p}} \quad J,$$

$\Rightarrow$ can be applied to (generalized) linear models.

Generalized M-fluc. tests

Special cases:

- Nyblom-Hansen test
- OLS-based CUSUM tests
- Hjort-Koning tests
- robust CUSUM test
- ...

For dependent data:

- mean function can usually be estimated consistently,
- use HAC covariance matrix estimates.

German M1 money demand

Lütkepohl, Teräsvirta, Wolters (1999) investigate the linearity and stability of German M1 money demand: stable regression relation for the time before the monetary unification on 1990-06-01 but a clear structural instability afterwards.

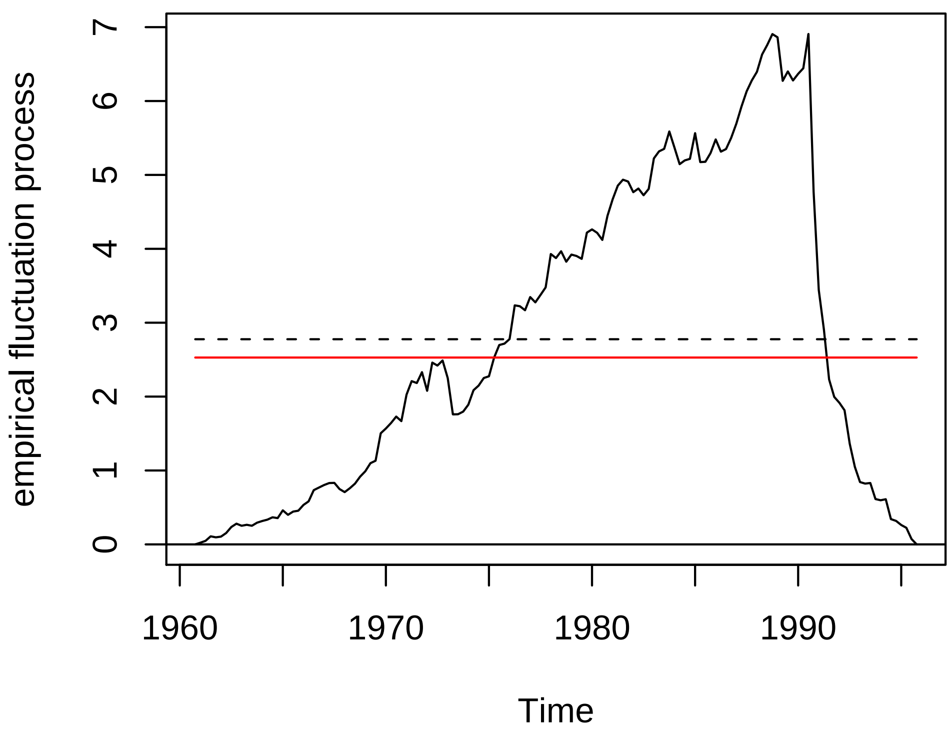
Data: seasonally unadjusted quarterly data, 1961(1) to 1995(4)

Error Correction Model (in logs) with variables:
M1 (real, per capita) m_t, price index p_t, GNP (real, per capita) y_t and long-run interest rate R_t:

$$\begin{aligned}
\Delta m_t &= -0.30\Delta y_{t-2} - 0.67\Delta R_t - 1.00\Delta R_{t-1} - 0.53\Delta p_t \\
&\quad -0.12 m_{t-1} + 0.13 y_{t-1} - 0.62 R_{t-1} \\
&\quad -0.05 - 0.13 Q1 - 0.016 Q2 - 0.11 Q3 + \hat{u}_t,
\end{aligned}$$

German M1 money demand

M-fluctuation test in linear regression estimated by OLS:



German M1 money demand

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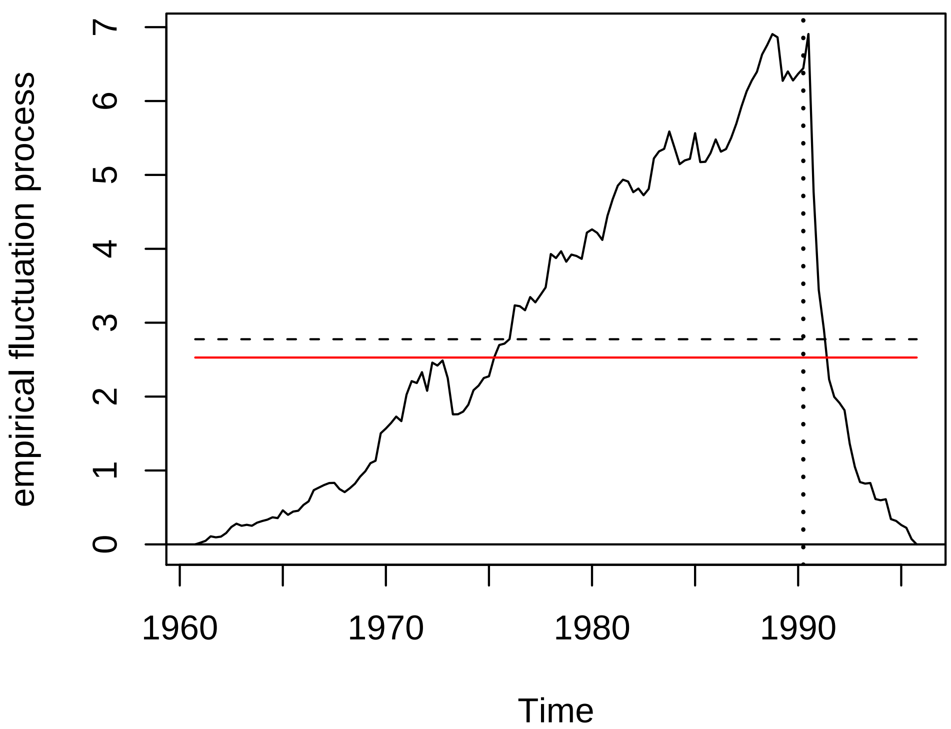

German M1 money demand

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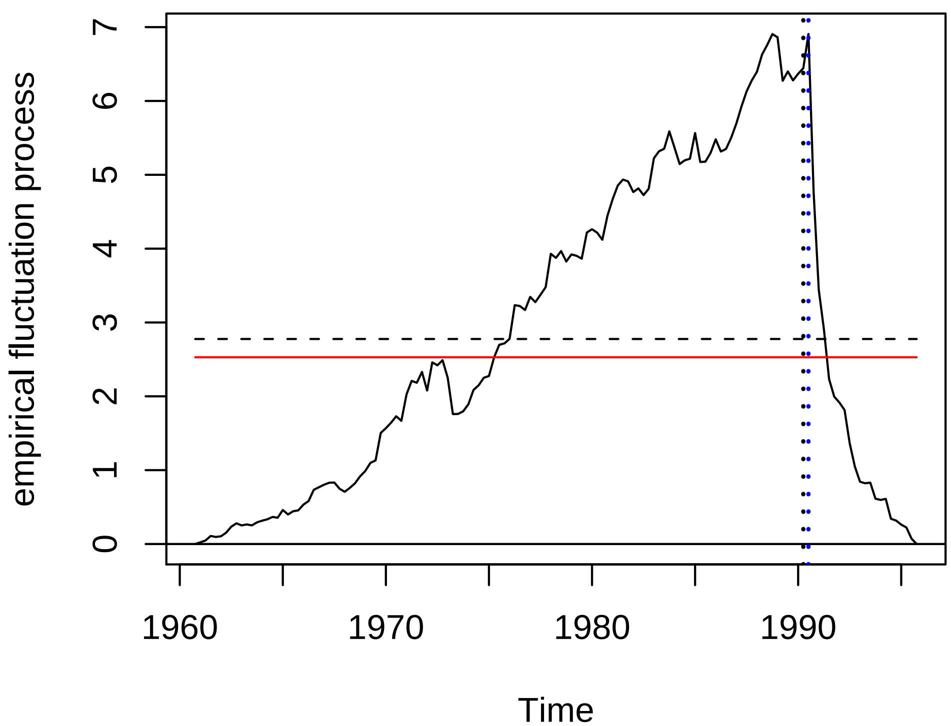
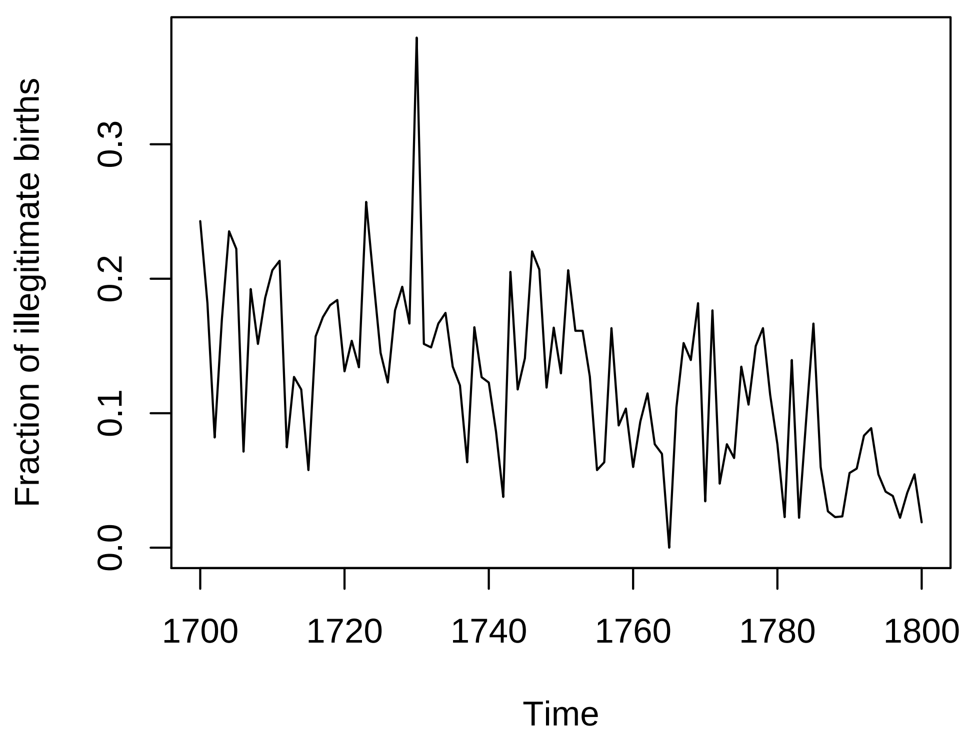


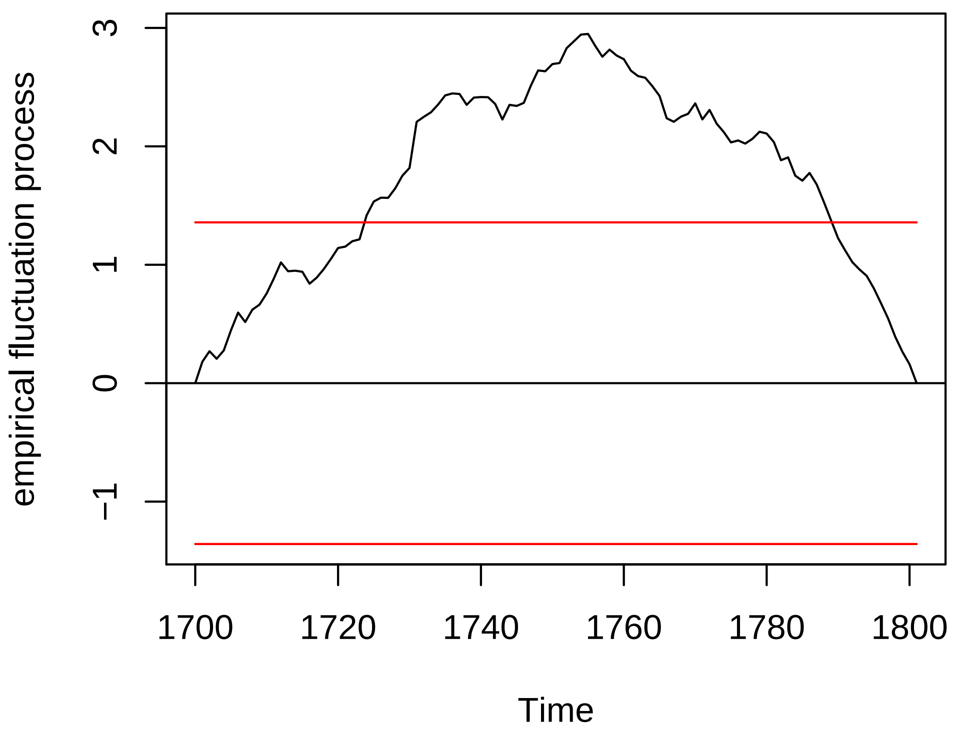
Illegitimate births

Fraction of illegitimate births in Großarl, Austria (1700–1800).



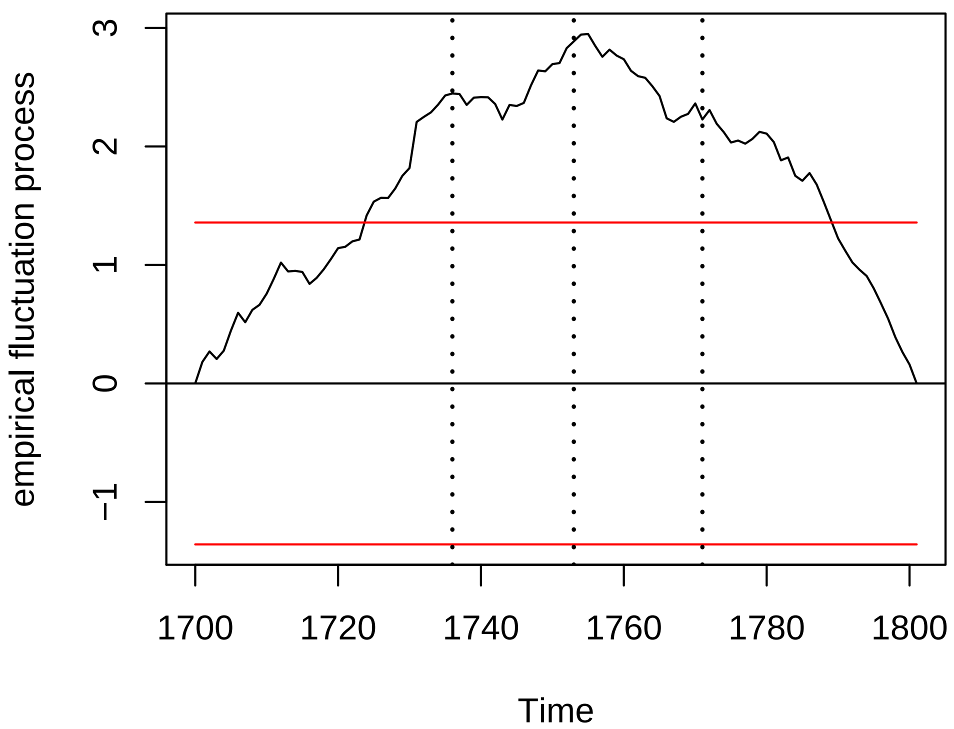
Illegitimate births

M-fluctuation test in binomial GLM estimated by ML:



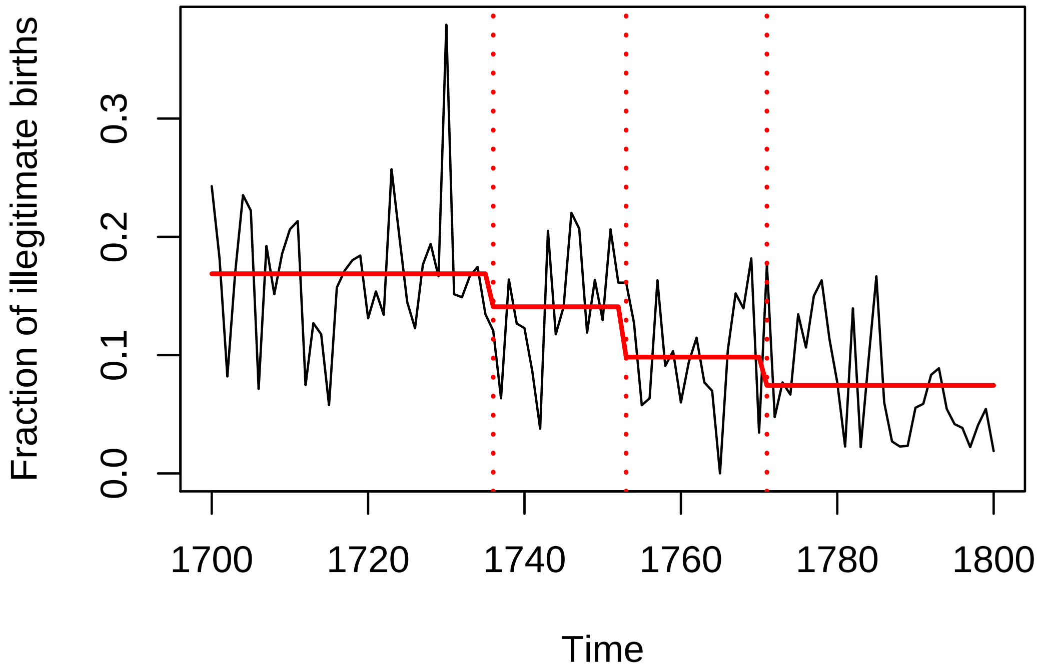
Illegitimate births

M-fluctuation test in binomial GLM estimated by ML:



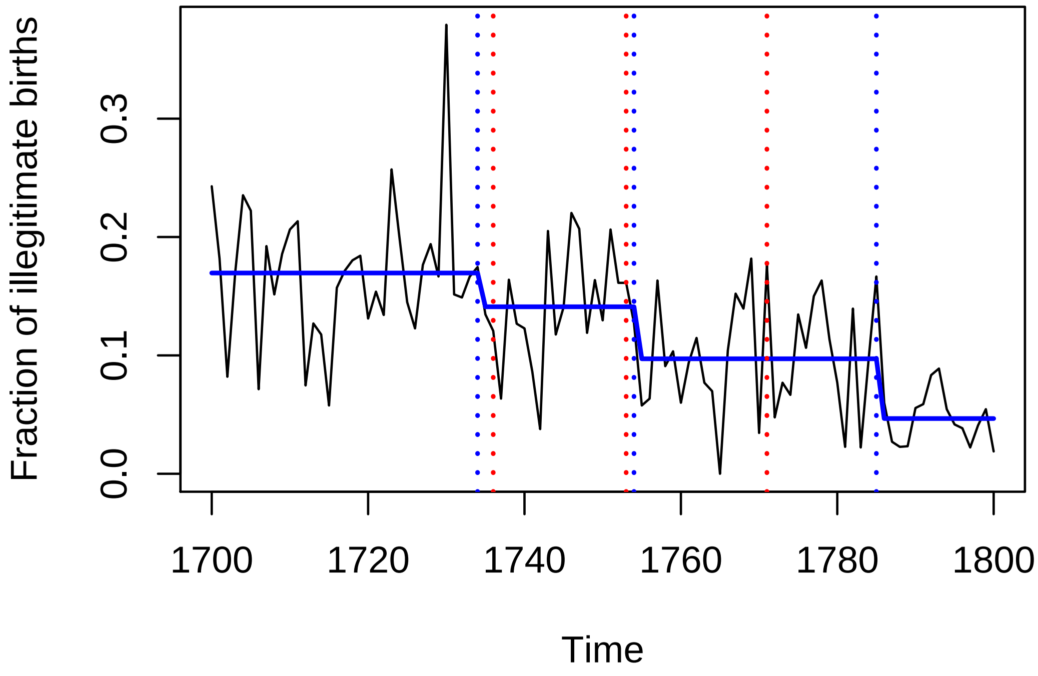
Illegitimate births

Fitted model with exogenous breakpoints (1736, 1753, 1771).



Illegitimate births

Fitted model with estimated breakpoints (1734, 1754, 1785).



Illegitimate births

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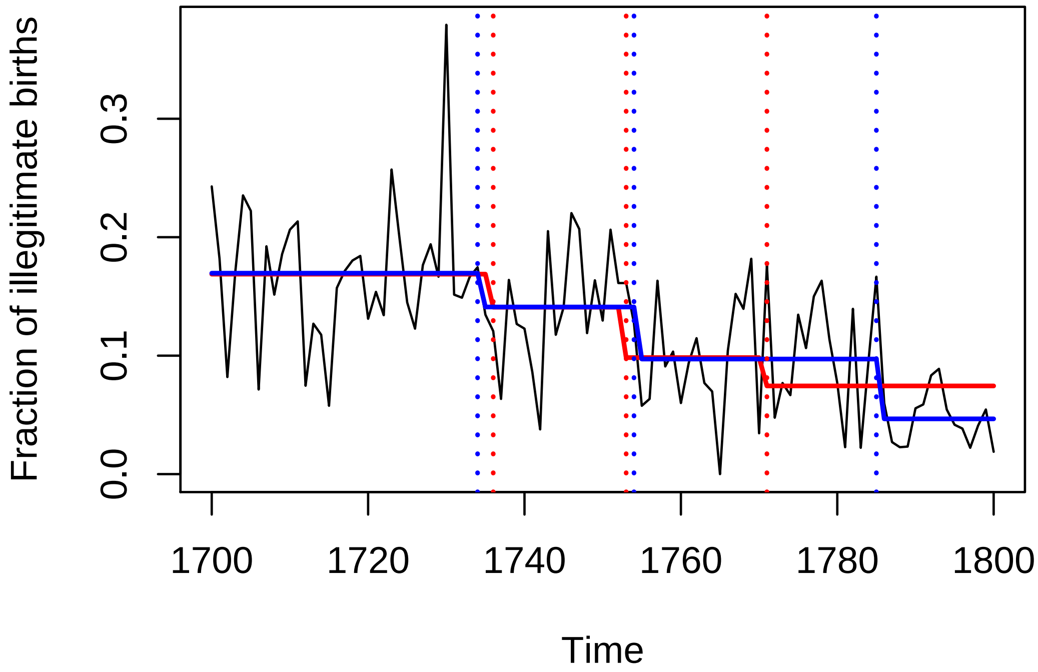
Fitted model with estimated breakpoints (1734, 1754, 1785).



Software

All methods implemented in the R system for statistical computing and graphics

`http://www.R-project.org/`

in the contributed package `strucchange`.

Both are available under the GPL (General Public Licence) from the Comprehensive R Archive Network (CRAN):

`http://CRAN.R-project.org/`

Software

```
R> M1.model <- dm ~ dy2 + dR + dR1 + dp + ecm.res + season
R> scus <- efp(M1.model, type = "Score-CUSUM", data = GermanM1)
R> plot(scus, functional = "meanL2")
```

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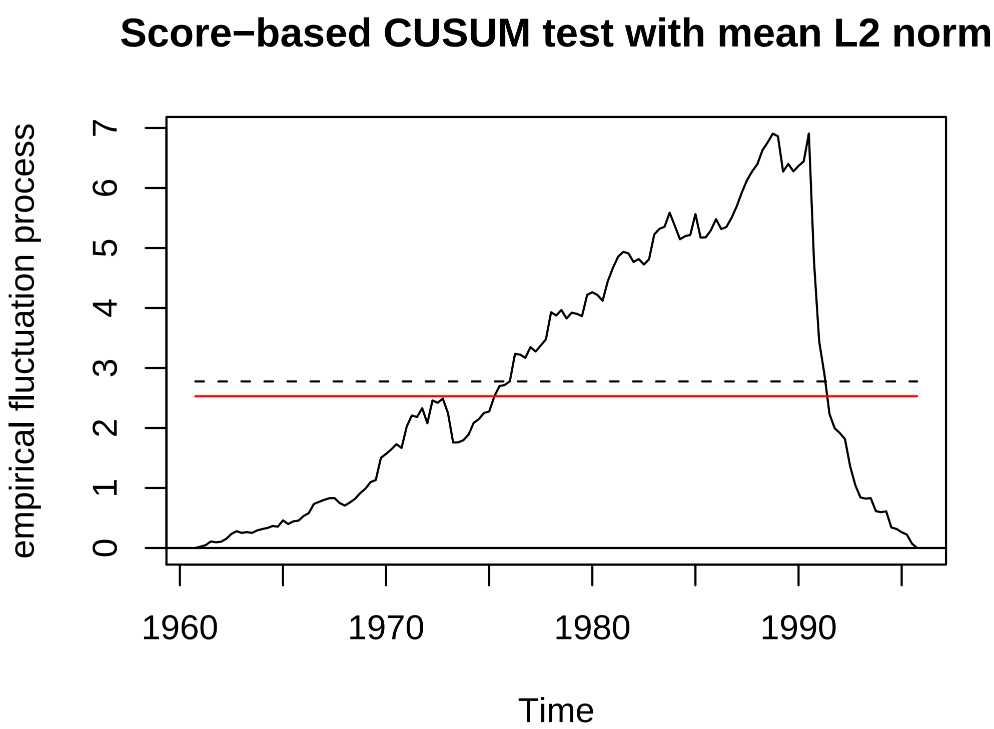
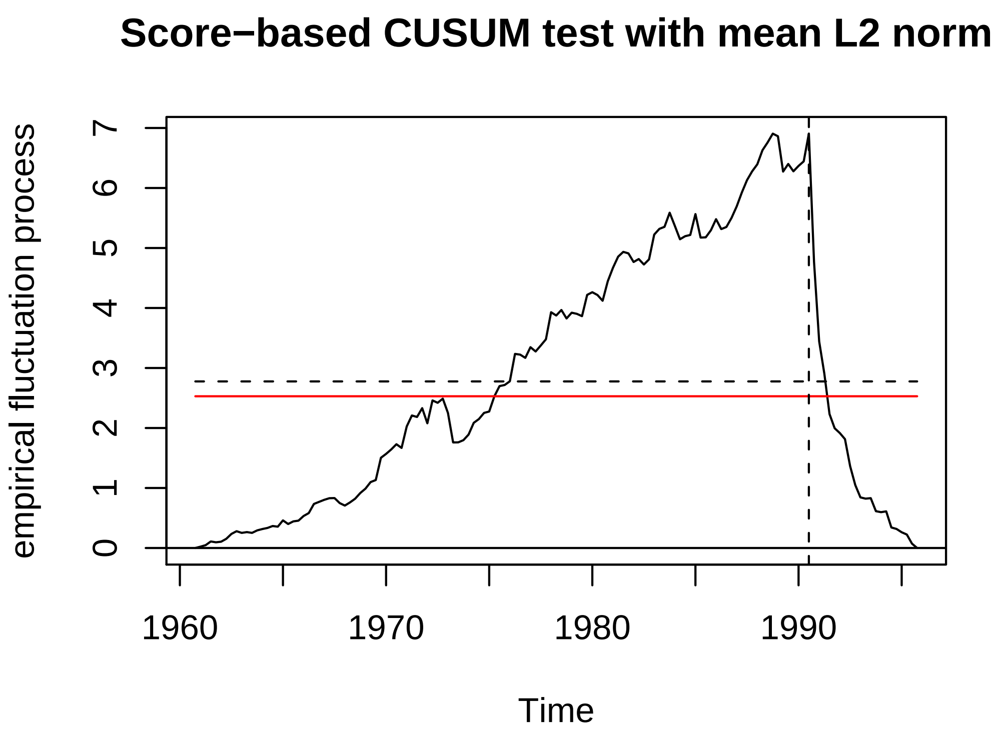


Software

```
R> bp <- breakpoints(M1.model, data = GermanM1)
R> bp.bic <- breakpoints(bp)
R> lines(bp.bic)
```



Software

```
R> sctest(scus, functional = "meanL2")

        Score-based CUSUM test with mean L2 norm

data:  scus
f(efp) = 2.796, p-value = 0.02212
```