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Implementing a Class of Structural Change Tests: An Econometric Computing Approach

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  - tests for structural change,
  - econometric computing?

- Generalized M-fluctuation tests
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  - Functionals for testing: `efpFunctional`

- Applications
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Structural change tests

Structural change has been receiving a lot of attention in econometrics and statistics, particularly in time series econometrics.

Aim: to learn if, when and how the structure underlying a set of observations changes.

In a parametric model with parameter θ_i for n totally ordered observations Y_i test the null hypothesis of parameter constancy

$$H_0: \quad \theta_i = \theta_0 \qquad (i = 1, \dots, n).$$

against changes over "time".

Econometric computing

Econometrics & computing:

- Computational econometrics: methods requiring substantial computations (bootstrap or Monte Carlo methods),
- Econometric computing: translating econometric ideas into software.

To transport methodology to the users and apply new methods to data software is needed.

Econometric computing

Desirable features of an implementation:

- easy to use,
- numerically reliable,
- computationally efficient,
- flexible and extensible,
- reusable components,
- open source,
- object oriented,
- reflect features of the conceptual method.

Undesirable: single monolithic functions.

Also important: software delivery.

Econometric computing

All methods implemented in the R system for statistical computing and graphics

http://www.R-project.org/

in the contributed package **strucchange**.

Both are available under the GPL (General Public Licence) from the Comprehensive R Archive Network (CRAN):

http://CRAN.R-project.org/

Guest Survey Austria

Data from the Austrian National Guest Survey about the summer seasons 1994 and 1997.

Here: use logistic regression model

- response: cycling as a vacation activity (done/not done),
- available regressors: age (in years), household income (in ATS/month), gender and year (as a factors/dummies),
- fit model for the subset of male tourists (6256 observations),
- (log-)income is not significant.

```
R> gsa.fm <- glm(cycle ~ poly(Age, 2) + Year, data = gsa,
     family = binomial)
```

But: Maybe there are instabilities in the model for increasing income?

M-fluctuation tests

- fit model
- compute empirical fluctuation process reflecting fluctuation in
  - residuals
  - coefficient estimates
  - *M-scores* (including OLS or ML scores etc.)
- theoretical limiting process is known
- choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability α.
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.

Empirical fluctuation processes

Model fitting: parameters can often be estimated based on a score function or estimating equation ψ with

$$\mathsf{E}[\psi(Y_i, \theta_i)] \quad = \quad 0.$$

Under parameter stability estimate θ_0 by:

$$\sum_{i=1}^{n} \psi(Y_i, \widehat{\theta}) \quad = \quad 0.$$

Includes: OLS, ML, Quasi-ML, robust M-estimation, IV, GMM, GEE.

Available in R: linear models `lm`, GLMs, logit, probit models `glm`, robust regression `rlm`, etc.

Empirical fluctuation processes

Test idea: if θ is not constant the scores ψ should fluctuate and systematically deviate from 0.

Capture fluctuations by partial sums:

$$efp(t) \quad = \quad \hat{J}^{-1/2}\, n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \psi(Y_i, \hat{\theta}).$$

and scale by covariance matrix estimate $\hat{J}$.

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Functional central limit theorem: empirical fluctuation process converges to a Brownian bridge

$$efp(\cdot) \ \xrightarrow{\text{d}} \ W^0(\cdot)$$

Empirical fluctuation processes

Implementation idea:

- don't reinvent the wheel: use existing model fitting functions and just extract the scores or estimating functions,
- also allow plug-in of HC and HAC covariance matrix estimators,
- provide infrastructure for computing processes.

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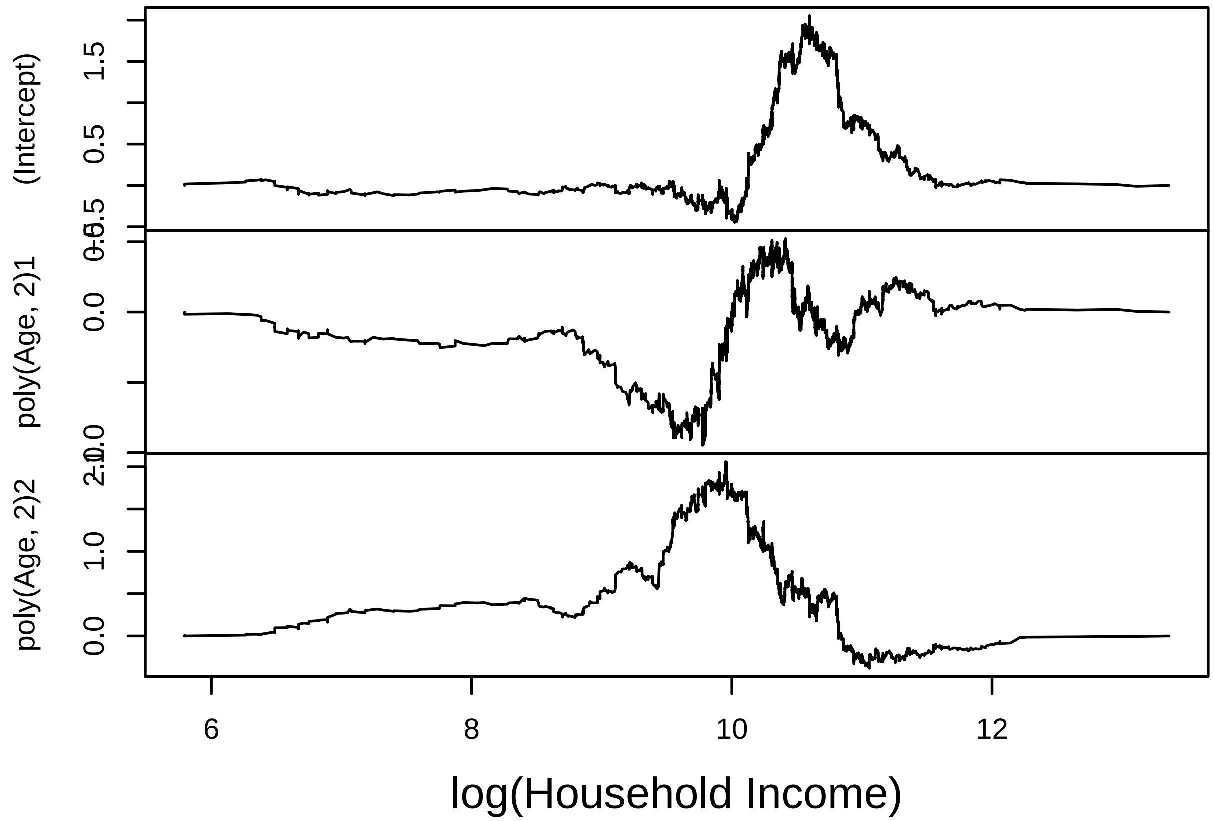
```
gefp(..., fit = glm, scores = estfun,
     vcov = NULL, order.by = NULL)
```

Empirical fluctuation processes

For Austrian guest survey data:

```
R> gsa.efp <- gefp(cycle ~ poly(Age, 2) + Year, family = binomial,
     data = gsa, order.by = ~ log(HHIncome), parm = 1:3)
```

Empirical fluctuation processes



Functionals

The empirical fluctuation process can be aggregated to a scalar test statistic by a functional $\lambda(\cdot)$

$$\lambda\left(efp_j\left(\frac{i}{n}\right)\right),$$

where $j = 1, \dots, k$ and $i = 1, \dots n$.

λ can usually be split into two components: λ_{time} and λ_{comp}.

Typical choices for λ_{time}: L_∞ (absolute maximum), mean, range.

Typical choice for λ_{comp}: L_∞, L_2.

$\Rightarrow$ can identify component and/or timing of shift.

Functionals

Double maximum statistic:

$$\max_{i=1,\dots,n} \max_{j=1,\dots,k} \left| \frac{efp_j(i/n)}{b(i/n)} \right|,$$

typically with $b(t) = 1$.

Cramér-von Mises statistic:

$$n^{-1} \sum_{i=1}^{n} \left\| efp_j(i/n) \right\|_2^2,$$

Critical values can easily be obtained by simulation of $\lambda(W^0)$. In certain special cases, closed form solutions are known.

Functionals

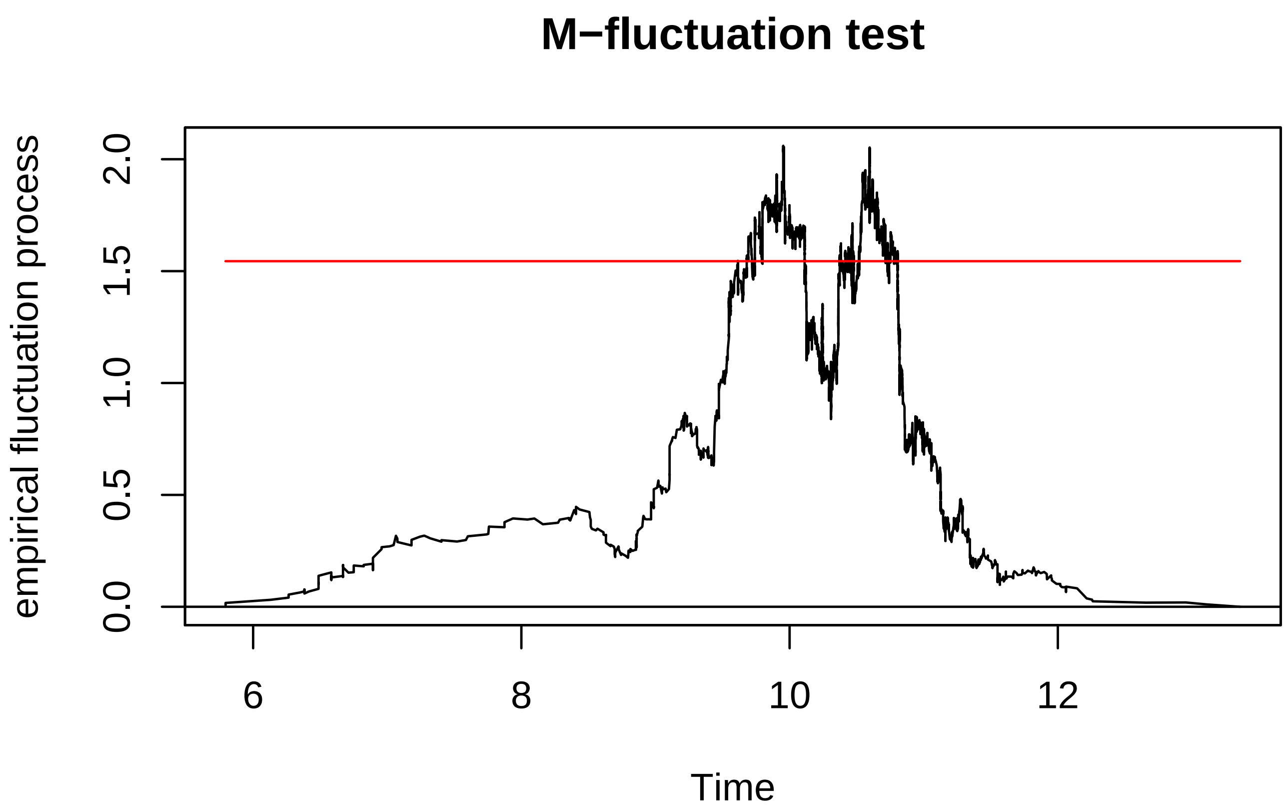
Implementation idea:

- specify functional (and boundary function)
- simulate critical values (or use closed form solution)
- combine all information about a functional in a single object: process visualization, computation of test statistic, computation of p values,
- provide infrastructure which can be used by the methods of the generic functions `plot` for visualization and `sctest` for significance testing.

For the double maximum and the Cramér-von Mises functionals such objects are available in `strucchange`: `maxBB`, `meanL2BB`.

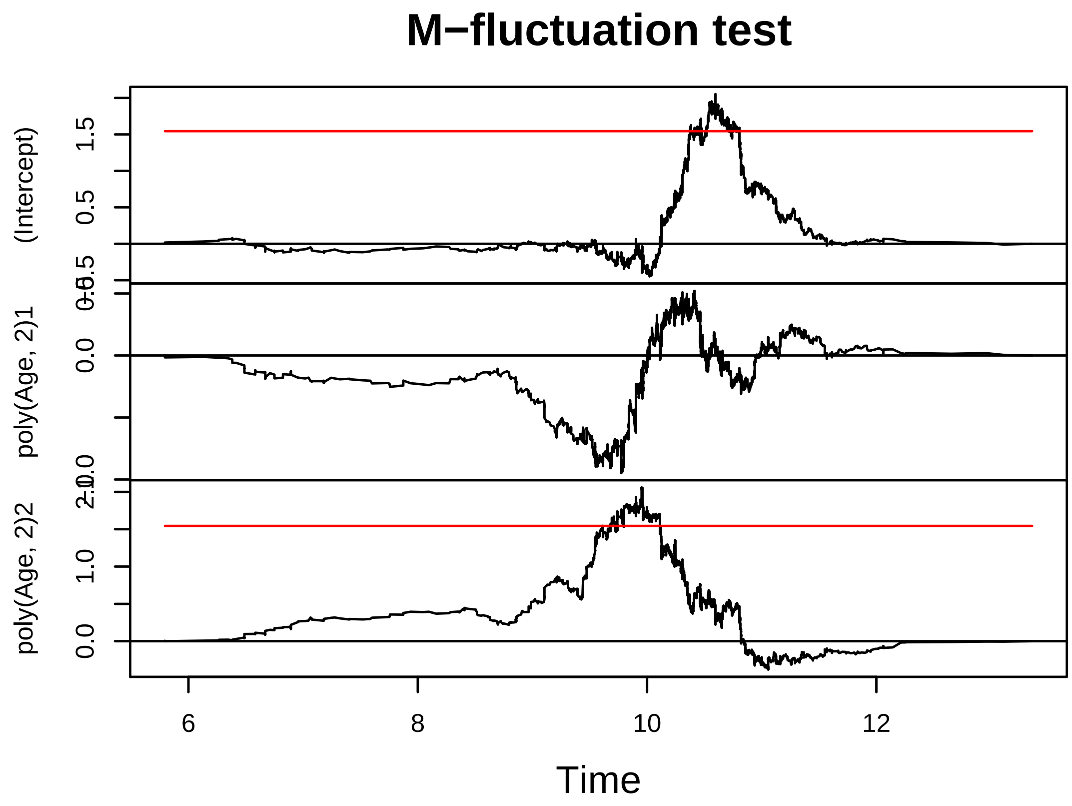
Functionals

```
R> plot(gsa.efp, functional = maxBB)
```



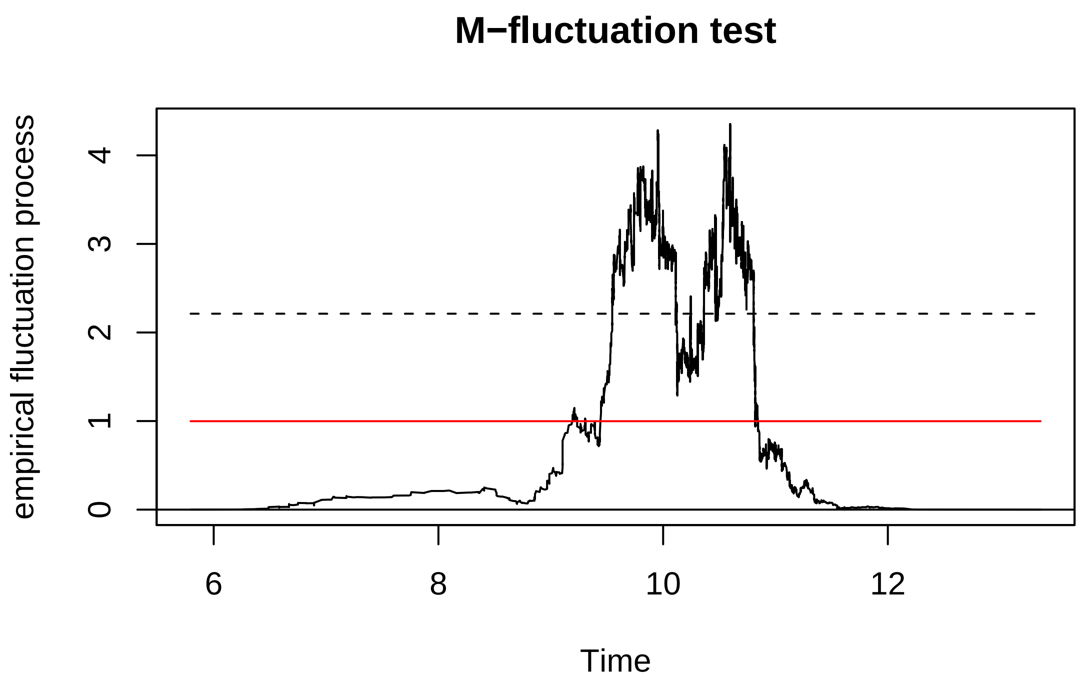
Functionals

```
R> plot(gsa.efp, functional = maxBB, aggregate = FALSE)
```



Functionals

```
R> plot(gsa.efp, functional = meanL2BB)
```



Functionals

```
R> sctest(gsa.efp, functional = maxBB)

        M-fluctuation test

data:  gsa.efp
f(efp) = 2.0594, p-value = 0.001242


R> sctest(gsa.efp, functional = meanL2BB)

        M-fluctuation test

data:  gsa.efp
f(efp) = 2.2119, p-value = 0.005
```

Functionals

New functionals can be easily generated with

```
efpFunctional(
  functional = list(comp = function(x) max(abs(x)), time = max),
  boundary = function(x) rep(1, length(x)),
  computePval = NULL, computeCritval = NULL,
  nobs = 10000, nrep = 50000, nproc = 1:20)
```

An object created by `efpFunctional` has slots with functions

- `plotProcess`
- `computeStatistic`
- `computePval`

that are defined based on lexical scoping.

Functionals

Use functional similar to double max functional, but with boundary function

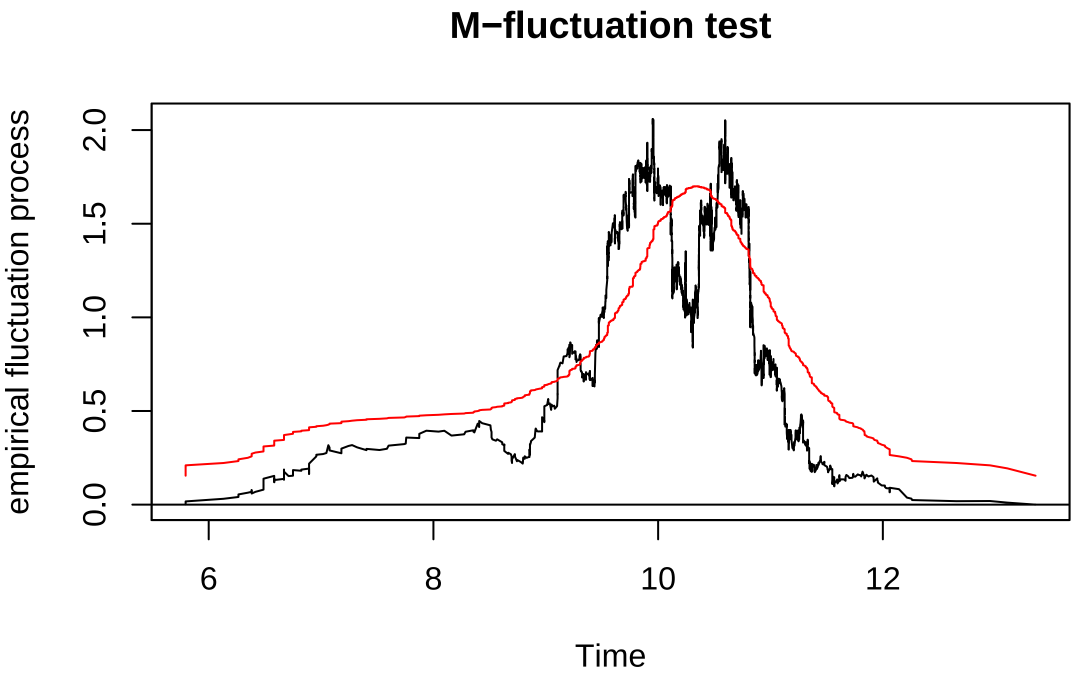
$$b(t) \quad = \quad \sqrt{t \cdot (1-t)} + 0.05,$$

which is proportional to the standard deviation of the process plus an offset.

```
myFun1 <- efpFunctional(
  functional = list(comp = function(x) max(abs(x)), time = max),
  boundary = function(x) sqrt(x * (1-x)) + 0.05,
  nobs = 10000, nrep = 50000, nproc = NULL)
```

Functionals

```
R> plot(gsa.efp, functional = myFun1)
```



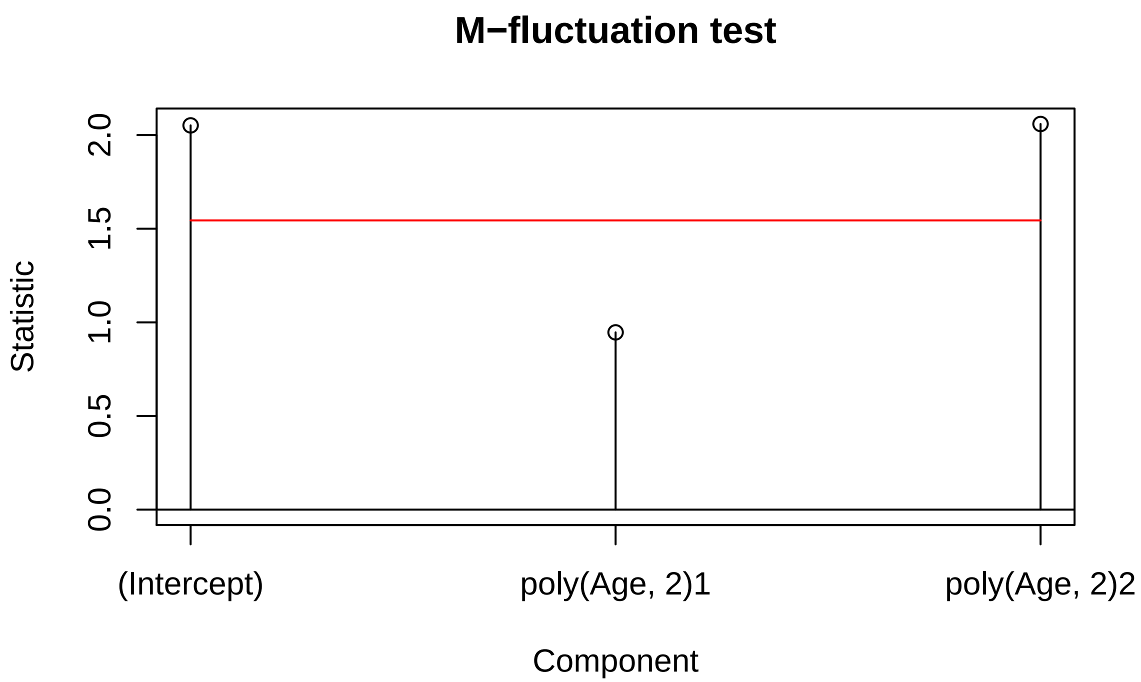
Functionals

Use standard double max functional but aggregate over "time" first. Leads to the same test statistic and p value, but the aggregated process looks different.

```
myFun2 <- efpFunctional(
  functional = list(time = function(x) max(abs(x)), comp = max),
  computePval = maxBB$computePval)
```

Functionals

```
R> plot(gsa.efp, functional = myFun2)
```



Functionals

```
R> sctest(gsa.efp, functional = myFun1)

        M-fluctuation test

data:  gsa.efp
f(efp) = 4.7947, p-value = < 2.2e-16


R> sctest(gsa.efp, functional = myFun2)

        M-fluctuation test

data:  gsa.efp
f(efp) = 2.0594, p-value = 0.001242
```

Conclusions

The general class of M-fluctuation tests is implemented in **strucchange**:

- **gefp** – computation of empirical fluctuation processes from (possibly user-defined) estimation functions,
- **efpFunctional** – aggregation of empirical fluctuation processes to test statistics, automatic tabulation of critical values,
- **plot** and **sctest** – methods for visualization and significance testing based on empirical fluctuation processes and corresponding functionals.

See more at ...

The 1st R user conference

Vienna, May 20–22, 2004

http://www.ci.tuwien.ac.at/Conferences/useR-2004/