3

National and global structural macroeconometric modelling

The discussion of the previous chapter suggested that there is a degree of consensus surrounding the desirable long-run properties of a macroeconomic model and that most recently developed models will be similar in this regard whether they have been developed following the SEM, VAR or DSGE approaches. There is far less agreement about the way in which short-run dynamic adjustment should be tackled, however, and in this chapter we broaden the discussion to consider this aspect of macroeconomic modelling too. To this end, in the following section, we present a canonical dynamic structural model. This allows us to clarify the distinction between short-run and long-run effects and to illustrate the issues involved in identifying these respective effects. Using the model, we can also provide a general description of the modelling strategy involved in constructing a 'structural cointegrating macroeconomic model' as applied to the UK in the subsequent chapters of the book. We believe that this strategy provides a coherent approach to dealing with both short-run and long-run influences in a way that can reflect the strength of conviction with which we believe the underlying economic theory.

The description of the canonical dynamic structural model also helps explain how the identification of short-run dynamics relates to the identification of economically meaningful shocks and the measurement of their dynamic effects. These issues are important since they lie at the heart of the discussion surrounding the identification of monetary policy shocks and the measurement of their effects. In this chapter, we shall review the attempts that have been made in the literature to impose structure on the short-run dynamics of macroeconomic models and to identify the effects of different types of shock, and particularly monetary policy shocks.

Finally in this chapter, we shall elaborate on the context within which a macroeconomic modelling exercise of this sort might be conducted and describe three ways in which the model might be extended. Specifically, we note *first* that, in any national macroeconomic model, there might be influences that are determined exogenously to the model. While much of the discussion is conducted under the (implicit) assumption that all of the variables in the model are determined endogenously, this section demonstrates that the modelling framework can be readily extended to accommodate exogenous shocks. *Next*, we note that any national macroeconomic model might constitute just one element of a broader examination of the economic behaviour of a number of economies. The UK economy within the world economy, for example, or within the European Community, say. With this in mind, we describe how the modelling strategy elaborated and applied to a national model can be extended to place its behaviour within the global context. *Thirdly*, we note that in any modelling exercise, interest might focus on a particular sector of the national economy rather than the whole. A detailed understanding of the macroeconomy might be an essential element of understanding the behaviour of the particular sector, but is not an end in itself. Thus it is worth considering how the analysis of a particular sector might be developed in these circumstances and the final section of the chapter considers this sectoral dimension.

3.1 Identification in a dynamic structural vector error correction model

A very general dynamic linear structural model of the determination of the $m \times 1$ vector of variables $\mathbf{z}_t$ is given by the Vector Error Correcting Model (VECM):

$$\mathbf{A}\Delta\mathbf{z}_t = \tilde{\mathbf{a}} + \tilde{\mathbf{b}}t - \tilde{\mathbf{\Pi}}\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\tilde{\mathbf{\Gamma}}_i\Delta\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_t. \tag{3.1}$$

The m equations in the model embody what are seen as the relevant autonomous economic relationships.[1] $\mathbf{A}$, $\tilde{\mathbf{\Pi}}$ and $\tilde{\mathbf{\Gamma}}_i$ are $m \times m$ matrices, and $\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$ are $m \times 1$ vectors of unknown structural coefficients. The matrix $\mathbf{A}$ contains the contemporaneous structural coefficients, while $\tilde{\mathbf{\Pi}}$ and $\tilde{\mathbf{\Gamma}}_i$

[1] Here, we assume that all variables of interest are determined endogenously within the system. The modelling framework can be readily extended to VEC models with weakly exogenous $I(1)$ variables, as discussed in Section 3.3.1 below.

contain the dynamic coefficients relating $\Delta\mathbf{z}_t$ to past values of $\mathbf{z}_t$. The term $\boldsymbol{\varepsilon}_t$ is an $m \times 1$ vector of disturbances, assumed to be serially uncorrelated, with zero means and a positive definite variance covariance matrix, $\mathbf{\Omega}$. These are the structural shocks relating to the m economic relationships.[2]

The form in (3.1) is particularly useful when the elements of $\mathbf{z}_t$ are stationary in differences, or integrated of order one, $I(1)$. In general, linear combinations of $I(1)$ variables are typically $I(1)$, but when there exist linear combinations of the $\mathbf{z}_t$, say $\boldsymbol{\beta}'\mathbf{z}_t$, that are stationary, or $I(0)$, the $\mathbf{z}_t$ are said to be cointegrated. If $\mathbf{z}_t$ were composed of only stationary variables, then $\tilde{\mathbf{\Pi}}$ would be a matrix of full rank, m. In this case, one cannot meaningfully separate out long-run and short-run effects although a distinction can still be made between level effects and the effects on first differences. Disturbing the system has no long-run impact and the variables eventually return to their unconditional mean (or to a deterministic trend if $\tilde{\mathbf{b}} \neq \mathbf{0}$). If $\mathbf{z}_t$ were $I(1)$ and not cointegrated, $\tilde{\mathbf{\Pi}}$ would be a null matrix. The system is described by a VAR in differences; all shocks have persistent effects, but there are no equilibrium relationships that exist between the levels that impact on these persistent effects. But suppose there are r cointegrating vectors, $0 < r < m$. Then $\boldsymbol{\beta}$ will be an $m \times r$ matrix and $\tilde{\mathbf{\Pi}}$ will be of rank r, with the form

$$\tilde{\mathbf{\Pi}} = \tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}'. \tag{3.2}$$

The $I(0)$ variables $\boldsymbol{\beta}'\mathbf{z}_t$ (appropriately adjusted by demeaning and detrending) are often interpreted as errors or deviations from equilibrium. Thus the $m \times r$ matrix, $\tilde{\boldsymbol{\alpha}}$, has a natural interpretation as a matrix of adjustment coefficients that measure how rapidly deviations from equilibrium feedback onto the variables $\mathbf{z}_t$. Here, the cointegrating relationships act as an attractor for the system and, despite the persistent effect of shocks on the individual variables, shocks to the system have no persistent effect on the equilibrium relations and their effects on such relations will die out eventually.

The reduced form vector error correction model in (3.1) is given by

$$\Delta\mathbf{z}_t = \mathbf{a} + \mathbf{b}t - \mathbf{\Pi}\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\mathbf{\Gamma}_i\Delta\mathbf{z}_{t-i} + \mathbf{v}_t, \tag{3.3}$$

[2] This form appears backward-looking, in the sense that all variables are dated at time t and earlier. But the model can readily accommodate the solution of forward-looking models of the form associated with SEMs incorporating RE or with DSGE models, for example. See Pesaran (1997) for a discussion.

where $\mathbf{a} = \mathbf{A}^{-1}\tilde{\mathbf{a}}$, $\mathbf{b} = \mathbf{A}^{-1}\tilde{\mathbf{b}}$, and $\boldsymbol{\Gamma}_i = \mathbf{A}^{-1}\tilde{\boldsymbol{\Gamma}}_i$. Further, we have $\boldsymbol{\Pi} = \mathbf{A}^{-1}\tilde{\boldsymbol{\Pi}} = \mathbf{A}^{-1}\tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}' = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha} = \mathbf{A}^{-1}\tilde{\boldsymbol{\alpha}}$ and $\mathbf{v}_t = \mathbf{A}^{-1}\boldsymbol{\varepsilon}_t$ are the reduced form errors with variance covariance matrix $\boldsymbol{\Sigma}$, where $\boldsymbol{\Omega} = \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'$. Within this framework, there are two quite distinct questions that we might wish to address. First, how do we identify the r distinct long-run relationships implicit in (3.2)? And second, assuming that the parameters and errors of (3.3) can be estimated, how do we identify the parameters $\mathbf{A}$, $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, $\tilde{\boldsymbol{\Pi}}$, and $\tilde{\boldsymbol{\Gamma}}_i$, $i = 1, \ldots, p-1$, and hence obtain measures of the effects of the economically meaningful structural shocks?

3.1.1 *Identifying long-run relationships*

Consideration of (3.3) shows that, even if an estimate of $\boldsymbol{\Pi}$ was available, without further restrictions, neither $\boldsymbol{\alpha}$ nor $\boldsymbol{\beta}'$ are separately identified. We can choose any non-singular $r \times r$ matrix, $\mathbf{Q}$, and write

$$\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}' = (\boldsymbol{\alpha} Q'^{-1})(\mathbf{Q}'\boldsymbol{\beta}') = \boldsymbol{\alpha}_*\boldsymbol{\beta}_*',$$

so that $\boldsymbol{\alpha}_* = \boldsymbol{\alpha} Q'^{-1}$ and $\boldsymbol{\beta}_* = \boldsymbol{\beta} Q$ constitute observationally equivalent alternative structures. In order to identify the cointegrating vectors, we need to provide r^2 independent pieces of information, formed from r restrictions on each of the r cointegrating relations. Only r restrictions are provided by 'normalisation' conditions and so a further $r^2 - r$ restrictions will be needed to uniquely identify $\boldsymbol{\beta}$ (and hence $\boldsymbol{\alpha}$).[3]

It is important to note that knowledge of the structural coefficients $\mathbf{A}$, $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, $\tilde{\boldsymbol{\Pi}}$, and $\tilde{\boldsymbol{\Gamma}}_i$, $i = 1, \ldots, p-1$, does not resolve the problem of identification of the long-run relations when $\mathbf{z}_t$ is $I(1)$. Reiterating the fact that $\boldsymbol{\Pi} = \mathbf{A}^{-1}\tilde{\boldsymbol{\Pi}} = \mathbf{A}^{-1}\tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}' = \boldsymbol{\alpha}\boldsymbol{\beta}'$, it is clear that knowing the value of $\mathbf{A}$ would not resolve the issue of how to uniquely factor the rank-deficient matrix $\tilde{\boldsymbol{\Pi}}$ into the $m \times r$ matrices $\tilde{\boldsymbol{\alpha}}$ and $\boldsymbol{\beta}$. The structural coefficient matrices determine the short-run responses and on their own do not identify the long-run relations.[4]

Beginning with Johansen (1988), a large and sophisticated literature has developed considering the analysis of cointegrating VAR models of the form in (3.3). Johansen (1988, 1991) provides procedures for testing the rank of $\boldsymbol{\Pi}$ and then estimating $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}'$ using statistically motivated identifying restrictions that assume the columns of $\boldsymbol{\beta}$ form an orthogonal set. While mathematically natural given the statistical structure of the problem, these restrictions have no economic meaning since in general there is no reason to expect economic cointegrating relations to be orthogonal. When $r > 1$, economic interpretation of the Johansen estimates of the cointegrating vectors is almost impossible. Similarly, the identification conditions employed by Phillips (1991), in the context of a triangular VECM, are chosen for their mathematical convenience rather than their suitability for economic interpretation.

[3] Wickens (1996) also considers the interpretation of cointegrating vectors, raising the possibility of imposing restrictions on the loading vector $\boldsymbol{\alpha}$ and also concluding that prior, structural information is essential for identification of meaningful cointegrating vectors.

[4] Similarly, identification of the long run relations do not generally help with identification of short-run coefficients.

The obvious alternative means of obtaining the $r^2 - r$ required restrictions is to draw on economic theory and other *a priori* information. Since the restrictions are to be imposed on the r cointegrating vectors, the relevant economic theory is that of the long run. This avoids the criticisms of many that economic theory is insufficiently well-defined to impose restrictions based on the short-run dynamics (*cf.* the Sims critique cited previously); indeed, as the discussion of the previous chapter made clear, in the context of macroeconometric models, there exists a broad consensus on the nature of the restrictions that might be imposed on a macroeconometric model in the long run. Pesaran and Shin (2002) describe precisely this approach to identifying the long-run relationships embedded within the cointegrating VAR. The modelling strategy implied by this approach is elaborated upon at the end of the subsection (and the econometric methods are described in detail in Chapter 6). First, however, we consider the identification issues relating to the short-run parameters and the structural shocks of (3.1).

3.1.2 *Identifying short-run structural parameters and shocks*

Abstracting from the issues relating to the long-run relationships, exact identification of the structural coefficients in (3.1) from the estimated parameters of the reduced form model in (3.3) requires m^2 restrictions to be imposed on the structural parameters. These are typically imposed on $\mathbf{A}$ and/or $\boldsymbol{\Omega}$. The traditional econometric approach to restricting short-run dynamics was to impose a particular shape (such as a geometrically declining or bell-shaped form), deemed plausible on *a priori* grounds, on the distributed lag functions relating to different components of $\mathbf{z}_t$. Early contributors to this approach include Nerlove (1958), Griliches (1967), and Jorgenson (1966). Dryhmes (1971) provides a comprehensive review of this early literature. A more recent literature on dynamic economic theory has attempted to provide restrictions on the short-run dynamics that are more plausible theoretically. An important example is the short-run restrictions involved in Real Business Cycle models due to the intertemporal nature of decision-making in these models and to the particular specification adopted in characterising technological progress.[5] Another example is given by the restrictions implied by the rational expectations hypothesis in the context of the Linear–Quadratic (LQ) optimisation models involving adjustment costs. Dynamic adjustment cost models have been applied in a number of important areas in applied econometrics with some success and could be an important source of *a priori* restrictions on the short-run dynamics of macroeconometric models. Some recent versions of the DSGE model, based on New Keynesian sticky price models, attempt to integrate intertemporal optimisation and adjustment cost models.

A third approach taken to identify the structural parameters is that promoted by the 'Structural VAR' approach to macroeconometric modelling, discussed in the previous chapter. The identifying restrictions imposed here are typically motivated with reference to some 'tentative' theory on macroeconomic dynamics, expressed with reference to contemporaneous relationships or with reference to the long-run impulse responses. Perhaps the most prominent example of this approach is the familiar recursive structure pioneered by Sims (1980) which requires $\mathbf{A}$ to be a lower or an upper triangular matrix and $\boldsymbol{\Omega}$ to be a diagonal matrix. This imposes a recursive, Wold-causal ordering on the contemporaneous relationships among the variables in $\mathbf{z}_t$ and can be motivated by 'tentative' economic theory on the timing of decisions and the detailed arrangements of the decision-making context. Such theories can also be used to motivate non-recursive structures on the structural parameters. This approach is particularly prevalent in the literature concerned with identifying monetary policy shocks. We shall discuss these three approaches to imposing structure on the short-run dynamics to identify short-run structural parameters and the structural shocks in more detail below.

Although there is a range of possible sources of theoretical restrictions on the short-run dynamics, in many cases, theory is either silent on the nature of the adjustment process or the theoretical restrictions are overly strong (making all the dynamics a function of a few deep parameters) with the consequence that they are invariably rejected by the data. In such cases, inferences about the long-run parameters and the dynamic properties of

[5] Other types of short-run restrictions could be obtained in the context of learning models.

the macroeconomy need to be conducted using unrestricted short-run parameters.

3.1.3 *A modelling strategy*

The canonical model described above emphasises the distinct contributions of economic theory as it relates to identification of the short-run and long-run relationships. In developing a modelling strategy, we need to consider the different characteristics of these contributions.

Typically, theories relating to the short run are concerned with relationships between variables motivated as the outcome of specific decisions made at a particular moment in time. Each row of the structural model of (3.1) describes the determination of one of the variables in the system and the restrictions imposed on the contemporaneous parameter matrix $\mathbf{A}$ reflect the assumed behaviour of the agent or group of agents setting the variable. Each equation in the model shows the factors taken into account by the decision-makers when they determine the value of a particular variable through their actions. The factors are either included explicitly in the model in the form of the contemporaneous values of the other variables in $\mathbf{z}_t$ or the lagged values of $\mathbf{z}_t$, or implicitly as part of the economically meaningful structural shocks, $\boldsymbol{\varepsilon}_t$.

The long-run relationships identified by economic theory typically do not relate to a specific time period or to particular events or decisions, but reflect the outcome of (potentially numerous) equilibrating pressures exerted over a (typically unspecified) period of time. Economic theory might provide little insight on the means by which a particular disequilibrium feeds back on the system and might involve concepts that are inherently unobservable or are difficult to measure accurately, such as expectations, natural rates or potential output. For example, economic theory might suggest that, in the long run, supply and demand of a good will be equal but might not elaborate on the tatonnement process involved other than to observe that price and quantity will react in an unspecified way to eliminate excess demand or supply (where the excess is itself unobserved). At any time, the system might be out of equilibrium along any of the dimensions suggested by economic theory, but neither these disequilibria nor the corresponding equilibrating pressures will be observable. On the other hand, theory also suggests the existence of long-run equilibrium relationships between the observed variables and the deviations from these will be observed. These deviations from equilibria are more useful for modelling purposes and might be termed *'long-run errors'*, which we denote by ξ_t.[6]

The modelling strategy involved in constructing a structural cointegrating model of the macroeconomy is based on the idea that the long-run errors, ξ_t, can be expressed as a linear combination of the variables in the system, possibly supplemented by appropriate deterministic intercepts and trends; that is,

$$\xi_t = \boldsymbol{\beta}'\mathbf{z}_t - \mathbf{b}_0 - \mathbf{b}_1 t, \tag{3.4}$$

for appropriate parameter vectors $\mathbf{b}_0$ and $\mathbf{b}_1$ and where $\boldsymbol{\beta}'$ is the $r \times m$ matrix of parameters that describes the r equilibrium relationships expected to hold between the m variables in $\mathbf{z}_t$ in the long run. In modelling the short-run dynamics of the variables in $\mathbf{z}_t$, we follow the standard VAR approach established by Sims (1980) and others, and assume that changes in the $\mathbf{z}_t$ can be well-approximated by a linear function of a finite number of past changes in $\mathbf{z}_t$. Assuming that the variables in $\mathbf{z}_t$ are difference-stationary, our modelling strategy is to embody the ξ_t in an otherwise unrestricted VAR(p) model in $\mathbf{z}_t$; that is, we consider the $(p-1)$th order VEC model

$$\Delta\mathbf{z}_t = \mathbf{a}_0 - \boldsymbol{\alpha}\xi_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{z}_{t-i} + \mathbf{v}_t. \tag{3.5}$$

Given the definition of the long-run errors in (3.4), the model in (3.5) can be rewritten as

$$\Delta\mathbf{z}_t = \mathbf{a}_0 - \boldsymbol{\alpha}\left[\boldsymbol{\beta}'\mathbf{z}_{t-1} - \mathbf{b}_0 - \mathbf{b}_1(t-1)\right] + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{z}_{t-i} + \mathbf{v}_t, \tag{3.6}$$

or

$$\Delta\mathbf{z}_t = \mathbf{a} + \mathbf{b}t - \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{z}_{t-i} + \mathbf{v}_t,$$

which is of the form of (3.3), with $\mathbf{a} = \mathbf{a}_0 + \boldsymbol{\alpha}(\mathbf{b}_0 - \mathbf{b}_1)$ and $\mathbf{b} = \boldsymbol{\alpha}\mathbf{b}_1$. This model embodies directly the predictions of economic theory, as it relates to the long run. This is in contrast to some cointegrating VAR analysis which starts with an unrestricted VAR and investigates some vague priors about the nature of the long-run relations. Estimation of a model of the

[6] The relationship between economically meaningful but unobservable disequilibria and observable long-run errors is illustrated in the economic theory of the long run elaborated in Chapter 4.

form in (3.6) can be carried out using the long-run structural modelling approach described in Pesaran and Shin (2002) and Pesaran, Shin and Smith (2000). A complete description of the econometric methods is postponed until Chapter 6. However, it is worth noting here that this approach not only provides estimates of the parameters in (3.6), but it can also provide a straightforward test of the long-run theory. Specifically, estimation of a VECM $(p-1)$ of the form in (3.6) can be carried out first imposing just r^2 exact identifying restrictions on the cointegrating relations. This will ensure that there are r cointegrating relations among the series but is not likely to impose the full structure suggested by the economic theory; *i.e.* these are likely to be a subset of the restrictions suggested by economic theory and embedded directly within (3.6). Estimation of the model subject to the full set of restrictions suggested by economic theory provides over-identifying restrictions that can be tested.

3.2 Specifying the dynamic structure of a macroeconomic model

The discussion above makes it clear that estimation of the structural parameters and the structural shocks to the model in (3.1) requires m^2 restrictions to be imposed, typically on $\mathbf{A}$ and/or $\boldsymbol{\Omega}$. In this section, we elaborate on some of the approaches taken in the literature to motivate such restrictions. We focus on three approaches that are pervasive in the literature. The first approach is that associated with the Dynamic Stochastic General Equilibrium models; the second is a broad class of 'Adjustment Cost' models; and the third class, associated with the Structural VAR approach to macroeconomic modelling, relies on miscellaneous assumptions on the contemporaneous and long-run interactions among variables, including recursiveness and exclusion assumptions, based on 'tentative' economic theory.

3.2.1 *Dynamics of DSGE models*

The strength of the DSGE approach to modelling the macroeconomy is that, in principle at least, it is based on the decisions of all agents in the economy. In this approach, macroeconomic phenomena are the outcome of the decisions made by these agents, driven by individual preferences, subject to constraints and relating to a whole range of variables simultaneously. The different decisions are reconciled to form a general equilibrium through the economy-wide market system. One result of this is that the models designed following this approach have clearly defined steady-state properties built into them automatically. This is the feature of the models discussed at length in the previous chapter. A second important feature of DSGE models arises because the decisions made by agents are usually intertemporal, relating to choices and constraints on economic magnitudes both today and in the future. This means that the dynamic structure of the DSGE model is also specified explicitly.

The derivation of an explicit dynamic structure for DSGE models is both a strength and a weakness. On the positive side, the approach provides very clear predictions on the dynamic responses of variables to different types of innovation impacting on the macroeconomy and these predictions can be tested. The downside is the difficulty of matching these predictions with the data; see, for example, the discussion of Kim and Pagan (1995) below. The DSGE modellers have responded to this challenge by supplementing the 'intrinsic dynamics' generated by the intertemporal optimisation of the DSGE models' agents with 'extrinsic dynamics' of various forms (motivated by the presence of adjustment costs, learning, aggregation issues, and so on).[7]

To illustrate these points, we consider below the DSGE model presented in Christiano and Eichenbaum (1992b) and analysed in some detail in Binder and Pesaran (1995). This model is in the spirit of the 'first phase' of DSGE models which focused on real magnitudes. But the ideas would carry over to the 'New Keynesian DSGE' models discussed in the previous chapter that are based on an IS curve, a Phillips curve and a policy rule (either in their closed economy form or in the open economy version derived in Gali and Monacelli (2005), for example).[8] In particular, the explicit reliance on forward-looking behaviour and rational expectations can be readily accommodated by our VARX modelling structure.

The model we consider here assumes an aggregate constant returns to scale Cobb–Douglas production function in which labour augmenting technology A_t, labour N_t and capital K_t are used to produce output Y_t. This can be used for consumption C_t, investment I_t or government spending G_t. The representative household has a time endowment of N. Capital is assumed to depreciate at rate δ so that

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{3.7}$$

[7] See the citations provided in Chapter 2.
[8] Pesaran and Smith (2005) provide a formal account of the relationship between the New Keynesian DSGE models and the VARX models that will be discussed below in Section 3.3.1.

Given the Cobb–Douglas production function, the production constraint is

$$C_t + K_{t+1} - (1-\delta)K_t + G_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}. \tag{3.8}$$

The forces outside the control of the representative household are technology and government spending and it is assumed that the laws of motion of these variables can be represented as follows:

$$\Delta \ln(A_t) = \gamma_t = \gamma + \varepsilon_{at}, \qquad \varepsilon_{at} \sim N(0, \sigma_a^2), \tag{3.9}$$

and

$$\ln(G_t/A_t) = \ln(g_t) = \tau_0 + \tau_1 \ln(g_{t-1}) + \varepsilon_{gt}, \qquad \varepsilon_{gt} \sim N(0, \sigma_g^2), \tag{3.10}$$

$|\tau_1| < 1$, so that the logarithm of A_t is represented by a random walk with drift and government expenditure expressed relative to technology, $g_t = G_t/A_t$, follows a simple AR(1) process. As is well known, the competitive equilibrium outcome is the same as that of a Social Planner who acts to maximise the utility of the representative household, which is assumed to be given by

$$\sum_{t=1}^{\infty} \rho^t \left[\ln(C_t) + \theta(N - N_t)\right], \tag{3.11}$$

where ρ is the discount factor and θ reflects the weight given to leisure in the utility function.

The Social Planner maximises the representative household's utility through the choice of C_t, K_{t+1} and N_t for $t = 0, 1, 2, \ldots$, demonstrating the simultaneous and intertemporal nature of the model solution. The source of the dynamics in the solution to this model are also apparent, arising from the forward-looking properties of the utility function and the intertemporal nature of the production constraint arising from capital accumulation. These dynamics are supplemented by the dynamics introduced through the processes assumed to drive technology and government expenditure.

To be more precise, the solution of the model is obtained with the Social Planner maximising the following Lagrangian

$$L_0 = E_0\left\{\sum_{t=1}^{\infty}\rho^t\left[\ln(c_t)+\theta(N-N_t)+\ln(A_t)\right]\right\}$$
$$+E_0\left\{\sum_{t=1}^{\infty}\lambda_t\rho^t\left[k_t^{\alpha}N_t^{1-\alpha}\exp(-\alpha\gamma_t)-c_t\right.\right.$$
$$\left.\left.-k_{t+1}+(1-\delta)k_t\exp(-\gamma_t)-g_t\right]\right\},$$

through choice of c_t, k_{t+1} and N_t for $t = 0, 1, 2, \ldots$, where E_0 indicates the expectation formed on the basis of information at time 0 and lower case letters indicate that the variable is expressed as a ratio relative to A_t; *i.e.* $y_t = Y_t/A_t$, $c_t = C_t/A_t$, $k_t = K_t/A_t$ and $i_t = I_t/A_t$. The first-order conditions for this optimisation are non-linear and in general do not lend themselves to an exact solution. However, the non-stochastic steady-state values of y_t, c_t, k_t, i_t and N_t, denoted by $\bar{y}$, $\bar{c}$, $\bar{k}$, $\bar{i}$ and $\bar{N}$, respectively, are given by non-linear equations derived from the first-order conditions as follows:

$$\bar{y} = \bar{k}^{\alpha}\bar{N}^{1-\alpha}\exp(-\alpha\gamma),$$
$$\bar{c} = \bar{y}-\bar{i}-\bar{g},$$
$$\bar{k} = [\exp(-\alpha\gamma)\bar{s}_k]^{1/(1-\alpha)}\bar{N},$$
$$\bar{i} = \bar{k}-(1-\delta)\exp(-\gamma)\bar{k},$$
$$\bar{N} = \left[\frac{1-\alpha}{\theta}+\frac{\bar{g}}{\bar{s}_k^{\alpha/(1-\alpha)}\exp(-\alpha\gamma/(1-\alpha))}\right]\div[(1-\delta)\exp(-\gamma)\bar{s}_k],$$

where $\bar{g}$ is the mean of g_t and

$$\bar{s}_k = \frac{\alpha\rho}{1-\rho(1-\delta)\exp(-\gamma)} = \bar{k}/\bar{y}.$$

These non-linear equations illustrate the steady-state properties automatically built into the model, with steady-state labour inputs explained by the 'deep' parameters of the model, α, ρ, δ, γ, θ and $\bar{g}$, and the steady-state growth path of the remaining variables driven by technological progress (recalling from the specification of the labour augmenting technological progress in (3.9) that steady-state growth is γ).

Further, expanding the first-order conditions obtained from the Social Planner's optimisation around the non-stochastic steady-state outcomes

above, we obtain a log-linear approximation for the model solution with which to characterise the model dynamics. Specifically, eliminating the shadow prices from the log-linearised first-order conditions, we obtain the following:

$$\mathbf{A}\mathbf{z}_t = \mathbf{B}\mathbf{z}_{t-1}+\mathbf{C}E_t(\mathbf{z}_{t+1})+\mathbf{u}_t, \tag{3.12}$$

where

$$\mathbf{z}_t = \begin{pmatrix}\tilde{k}_{t+1}\\ \tilde{N}_t\\ \tilde{c}_t\\ \tilde{y}_t\\ \tilde{i}_t\end{pmatrix}, \qquad \mathbf{u}_t = \begin{pmatrix}\bar{s}_g\tilde{g}_t\\ \theta_1\tilde{\gamma}_t\\ \alpha\tilde{\gamma}_t\\ \alpha\tilde{\gamma}_t\\ \theta_3\tilde{\gamma}_t\end{pmatrix}$$

and

$$\mathbf{A} = \begin{pmatrix}0 & 0 & -\bar{s}_c & 1 & -\bar{s}_i\\ 1 & 0 & 0 & 0 & -(1-\theta_1)\\ 0 & -(1-\alpha) & 0 & 1 & 0\\ 0 & \alpha & 1 & 0 & 0\\ \theta_2 & 0 & -1 & 0 & 0\end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix}0 & 0 & 0 & 0 & 0\\ \theta_1 & 0 & 0 & 0 & 0\\ \alpha & 0 & 0 & 0 & 0\\ \alpha & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & \theta_2 & -1 & 0 & 0\end{pmatrix},$$

where a variable written with an '~' overstrike means that it is measured relative to its non-stochastic steady-state value (*i.e.* $\tilde{k}_t = \ln(k_t/\bar{k}_t)$, for example), where $\bar{s}_c = \bar{c}/\bar{y}$, $\bar{s}_i = \bar{i}/\bar{y}$ and $\bar{s}_g = \bar{g}/\bar{y}$, and where $\theta_1 = (1-\delta)\exp(-\gamma)$, $\theta_2 = \rho\alpha(1-\alpha)\bar{s}_k^{-1}$ and $\theta_3 = \rho\alpha^2\bar{s}_k^{-1}+\rho\theta_1$. Hence, the dynamic specification derived for the DSGE model in (3.7)–(3.11) is summarised by the multivariate linear rational expectations model given in (3.12) which is driven by the exogenous shock to technology and fiscal policy.

There have been many methods proposed for the solution of such models, and these are reviewed in Binder and Pesaran (1995). Clearly, the system is capable of generating very sophisticated dynamics, and the solution depends on whether the quadratic determinantal equation,

$$\det(\mathbf{C}\lambda^2 - \mathbf{A}\lambda + \mathbf{B}) = 0,$$

has pairs of solutions which satisfy the regularity conditions; namely whether, for each pair, one root will fall inside the unit circle and the other outside it. Assuming these conditions are satisfied, then the model has a unique stable solution given by

$$\mathbf{z}_t = \mathbf{b} + \boldsymbol{\lambda}\mathbf{z}_{t-1} + \mathbf{v}_t,$$

where $\boldsymbol{\lambda}$ is the solution with all its roots on or inside the unit circle and $(\mathbf{A} - \mathbf{C}\boldsymbol{\lambda})\mathbf{v}_t = \mathbf{u}_t$, so the DSGE model fits readily into a VARX structure. Denoting the steady-state values with an overbar once more, the long-run structural relations associated with (3.12) will be given by

$$(\mathbf{A} - \mathbf{B} - \mathbf{C})\bar{\mathbf{z}}_t = \boldsymbol{\eta}_t,$$

where the $\boldsymbol{\eta}_t$ are the long-run errors and

$$(\mathbf{A} - \mathbf{B} - \mathbf{C}) = \begin{pmatrix} 0 & 0 & -\bar{s}_c & 1 & -\bar{s}_i \\ 1-\theta_1 & 0 & 0 & 0 & -(1-\theta_1) \\ -\alpha & -(1-\alpha) & 0 & 1 & 0 \\ -\alpha & \alpha & 1 & 0 & 0 \\ \theta_2 & -\theta_2 & 0 & 0 & 0 \end{pmatrix}.$$

It is also worth emphasising that it is not only the matrix $(\mathbf{A} - \mathbf{B} - \mathbf{C})$ that is subject to restrictions. The elements of $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are themselves subject to a number of restrictions. Given the number of coefficients of zero and unity in the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, it is also clear that the solution imposes a large number of restrictions on the system dynamics. These are precisely the restrictions that are tested in Kim and Pagan (1995), for example, and which are easily rejected by the data.

3.2.2 *Dynamics of adjustment cost models*

A second approach in which the dynamic structure of a model is specified is one where there is an explicit intertemporal optimisation problem involving adjustment costs. There are many examples of models of this type found in the applied econometrics literature. Nickell (1985) and Breeson *et al.* (1992) focus on models of labour demand, for example; Blundell *et al.* (1992) focus on capital investment; West (1995) focuses

on inventory models; and so on. Hansen and Sargent (1995) also provide a useful overview of linear–quadratic general equilibrium models with adjustment costs. The latter reference works with a familiar but restrictive class of objective function, having the advantage that the optimisation produces a linear decision rule. But the approach is similar if the analysis starts with a general non-linear specification of an objective function and linearises around the resultant first-order conditions.

To briefly review the nature of the dynamic specification arising from these models, consider the following quadratic optimisation problem

$$\min_{(z_{t+s})} E_t \left\{ \sum_{s=0}^{\infty} \rho^s \left[\left(\mathbf{z}_{t+s} - \mathbf{z}^{\dagger}_{t+s}\right)' \mathbf{H} \left(\mathbf{z}_{t+s} - \mathbf{z}^{\dagger}_{t+s}\right) \right.\right.$$
$$\left.\left. + \Delta\mathbf{z}_{t+s}\mathbf{G}\Delta\mathbf{z}_{t+s} + \Delta^2\mathbf{z}_{t+s}\mathbf{K}\Delta^2\mathbf{z}_{t+s} \right] \right\} \tag{3.13}$$

for given values of $\mathbf{z}^{\dagger}_t, \mathbf{z}^{\dagger}_{t+1}, \mathbf{z}^{\dagger}_{t+2}, \ldots$, where $\mathbf{z}_t$ is the vector of decision variables, $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{K}$ are symmetric matrices of structural parameters and ρ is a discount factor in $(0, 1)$. $\mathbf{z}^{\dagger}_{t+s}$ represents the corresponding vector of targets, which may be fixed or evolving stochastically. The problem in (3.13) indicates that costs are quadratic and strictly convex in the arguments. Costs are incurred if the decision variables deviate from their targets and if the decision variables are changed from their previous value. The third term allows for the possibility that the *rate* at which the variables are changed also generates an independent cost. The relative importance of the three elements of costs are captured by the parameters in $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{K}$. Assuming that the targets are indeed varying stochastically, differentiating (3.13) with respect to $\mathbf{z}_t$ and rearranging the resultant conditions yields the following stochastic Euler equation system:

$$\begin{aligned} \mathbf{z}_t = {} & \mathbf{M}^{-1}[\mathbf{G} + 2(1+\rho)\mathbf{K}]\mathbf{z}_{t-1} - \mathbf{M}^{-1}\mathbf{K}\mathbf{z}_{t-2} \\ & + \rho^{-1}\mathbf{M}^{-1}[\mathbf{G} + 2(1+\rho)\mathbf{K})]\, E_t(\mathbf{z}_{t+1}) \\ & - \rho^2\mathbf{M}^{-1}\mathbf{K}\, E_t(\mathbf{z}_{t+2}) + \mathbf{M}^{-1}\mathbf{H}\mathbf{z}^{\dagger}_t, \end{aligned} \tag{3.14}$$

where $\mathbf{M} = \mathbf{H} + (1+\rho)\mathbf{G} + (1 + 4\rho + \rho^2)\mathbf{K}$. The solution of (3.14) can be obtained in a number of different ways, some of which are reviewed in Binder and Pesaran (1995). However, it is intuitively clear that the solution for $\mathbf{z}_t$ will depend, in general, on $\mathbf{z}_{t-1}$, $\mathbf{z}_{t-2}$ and expressions for $E_t(\mathbf{z}_{t+i})$, $i = 0, 1, 2, \ldots$, so that an explicit solution requires a statement on the process driving the targets. For this purpose, we might describe the process explaining the targets by the vector ARMA specification

$$\theta(L)z^{\dagger}_{t+s} \equiv \phi(L)\epsilon_{t+s}, \qquad \epsilon_{t+s} \sim i.i.d.\,(0, \Sigma_{\epsilon}),$$

where Σ_{ϵ} is an $m \times m$ covariance matrix for ϵ_t. The solution for z_t will therefore depend on z_{t-1}, z_{t-2}, current and lagged values of $z^{\dagger}_t$ and ϵ_t. The dynamic structure of the solution will be a complicated function of the parameters in $\mathbf{H}$, $\mathbf{G}$, $\mathbf{K}$, θ and ϕ but, given the linearity of the problem, will be readily written in the form of a VAR.[9] Moreover, despite its complexity, the solution will be explicitly derived and the restrictions suggested in the model solution can usually be tested in the context of a straightforward regression exercise.[10]

3.2.3 *Identification of short-run dynamics based on 'tentative' theory on contemporaneous relations*

The Structural VAR approach to macroeconometric modelling described in the previous chapter aims to provide economic meaning to the estimated shocks and associated impulse responses by suggesting restrictions on the contemporaneous relations between variables based on a 'tentative' economic theory. Perhaps the most frequently used form of restriction imposed on the short-run dynamics of a VAR model of the form in (3.1) is that suggested by Sims (1980). This approach assumes a recursive structure among the variables whereby the first variable in the VAR is assumed to be contemporaneously independent of the other variables, the second is contemporaneously influenced by the first but no other, the third by the first two but not the rest, and so on. It is also assumed that the structural shocks are independent of each other. This identification scheme imposes a triangular structure on $\mathbf{A}$ and assumes that $\mathbf{\Omega}$ is a diagonal matrix, namely:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ a_{21} & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & 1 \end{pmatrix}, \quad \text{and} \quad \mathbf{\Omega} = \begin{pmatrix} \omega_{11} & 0 & \dots & 0 \\ 0 & \omega_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \omega_{mm} \end{pmatrix}. \tag{3.15}$$

9 This point was illustrated in Nickell (1985), who showed that adjustment costs models can frequently be represented by a simple VECM, depending on the nature of the stochastic process characterising the determination of the target variables.

10 Pesaran (1991) considers these issues explicitly in the context of a univariate model. The paper describes the order conditions necessary for the identification of the structural parameters and the cross-equation restrictions that can be tested (in either the structural equations or the reduced form) in this case.

The zeros on the top right-hand of $\mathbf{A}$, imposed by the recursiveness, provides $\frac{1}{2}m(m-1)$ restrictions; the 'normalising' unit coefficients on the diagonal of $\mathbf{A}$ provide a further m restrictions; and the zero off-diagonals in $\mathbf{\Omega}$, imposed by the orthogonality assumption, provide a further $\frac{1}{2}m(m-1)$ restrictions. This provides the m^2 restrictions required in total to exactly identify the structural parameters of (3.1).

In fact, the identifying structure of (3.15) described in Sims (1980) was initially suggested as part of a more mechanical, statistical exercise in which the dynamics of an estimated reduced form system could be illustrated by tracing the impact of specific 'orthogonalised' shocks. Starting from an estimated reduced form VECM of the type given in (3.3), orthogonalised impulses can be derived from the estimated reduced form errors, $\mathbf{v}_t$, and associated estimated variance–covariance matrix, $\mathbf{\Sigma}$, using the Choleski decomposition $\mathbf{A}\Sigma\mathbf{A}' = \mathbf{\Omega}$ where $\mathbf{A}$ is lower triangular and $\mathbf{\Omega}$ is a diagonal matrix. Since the errors $\boldsymbol{\varepsilon}_t = \mathbf{A}\mathbf{v}_t$ are orthogonal by construction, it is straightforward to trace out the dynamic effects on the variables in $\mathbf{z}_t$ of an impulse in one of the elements of $\boldsymbol{\varepsilon}_t$. Sims acknowledged that the choice of Choleski decomposition, and associated orthogonalised shocks, was arbitrary and depended on the ordering of the variables and suggested trying various different Choleski decompositions to characterise the dynamic properties of the system (see Sims (1980, 1981) for example). However, Cooley and Leroy (1985) noted that, despite the apparent atheoretic content of the orthogonalisation, the choice of any Choleski decomposition is equivalent to imposing a clearly defined recursive structure on the contemporaneous relationships across the variables in the system.

There are many applied economists who would find a recursive structure of the sort given in (3.15) no more persuasive than the exclusion restrictions used to distinguish between exogenous and endogenous variables in the early Cowles Commission work and described by Sims as 'incredible'. But there are others who embrace this sort of structure, motivating the restrictions imposed through 'tentative' theory on the timing and sequencing of decisions. The theory is 'tentative' in the sense that it is typically not derived from any form of optimising behaviour on the part of agents or given any explicit microeconomic foundations. Rather, the theory reflects the investigator's *a priori* beliefs on the costs involved in making particular decisions, on the consequent frequency of decisions and on the sequencing of these decisions, based on the investigator's understanding of the institutional background and the decision-making context. The difficulty in translating such beliefs into an identifying structure is that there is a degree of judgement involved and it is difficult to formalise the nature of any disagreements that arise between investigators on such restrictions. The approach also relies to a considerable extent on the frequency with which the main variables in the VAR are observed. For example, the degree of contemporaneous dependence in variables measured at an annual frequency might be large as compared to the case when the frequency of observations is monthly or weekly.[11] The challenge facing this approach is, therefore, to choose an identification scheme that is sufficiently well-grounded in theory and sufficiently loosely defined that the restrictions are relatively uncontentious for the dataset available.

Two further examples of the identification of structural shocks through the application of 'tentative' theory are provided by Blanchard (1989) and Gali (1992). Both papers describe a broadly 'Keynesian' IS-LM aggregate demand and aggregate supply model that places restrictions on $\mathbf{A}$ and $\mathbf{\Omega}$. Blanchard considers a five equation system involving output, unemployment, prices, wages and money explained, in turn, by relationships based on aggregate demand, Okun's (1962) Law, wage-setting behaviour, price-setting behaviour and a money rule. These relationships motivate restrictions on the contemporaneous short-run effects of the structural disturbances. For example, given contemporaneous output, (un)employment is determined solely by productivity shocks in the Okun's Law relationship; given wages and output, price-setters decisions are influenced by (only) price-setting innovations and productivity shocks in the price-setting relationship; and so on. The structural innovations are themselves considered to be independent of each other. Gali's (1992) paper is in a similar vein. Restricting attention to output, interest rates, money and prices, Gali describes a four equation system relating to the IS, LM, money supply rule and Phillips curve relationships. The associated structural innovations are 'spending', 'money demand', 'money supply' and 'supply' shocks. Identifying restrictions considered here include that neither money supply nor money demand shocks have a contemporaneous effect on output, and that neither output nor prices enter the money supply rule.

Both Blanchard and Gali provide time series results, analyses of impulse responses and discussion of US macroeconomic fluctuations on the basis of their identifying schemes, arguing that, broadly speaking, the results confirm the usefulness of the Keynesian modelling framework. However, while the results are interesting and informative, their conclusions are

[11] Even in the case of observations sampled at daily frequencies, the presence of common factors can bring about a substantial degree of correlated behaviour and contemporaneous dependence.

only as reliable as the identifying assumptions on which the analysis is based. The exclusion restrictions imposed are very similar in form to those employed in the traditional Cowles Commission work which have been subject to so much scepticism over the past three decades. Moreover, the systems described by Blanchard and Gali work at a level of abstraction that makes it difficult to relate the economic relationships of their systems to specific decision-making by particular agents. This renders the definition of the relationships, and the associated shocks, almost tautological. For example, output in Blanchard's system depends on demand shocks and productivity shocks only. If productivity shocks could be identified from the remaining supply-side relationships of the system, then there is a one-to-one correspondence between output movements and 'demand shocks'. The denomination of these innovations as 'demand' shocks conveys an economic meaning to them, but a description of these as 'output shocks' seems equally justified. In the absence of a more elaborate theory, it is difficult to envisage which agents take the decision to set output and what information is at their disposal when they do so. Seen in this light, this identifying assumption appears rather vacuous and difficult to justify.

3.2.4 *Measuring the effects of monetary policy*

The use of tentative theory on the contemporaneous relations among variables is perhaps most widely used in the study of monetary policy shocks. Christiano, Eichenbaum and Evans (1999, CEE) provide a very useful review of this literature. In their paper, CEE describe three alternative identifying structures in which variables are grouped into three sets: a set including variables for which the contemporaneous values are known when policy is set; a set of policy instruments; and a set of variables for which the contemporaneous values are observed only after the policy decisions are made. The frequency of the observations on the variables is taken to be monthly and the assumed structure imposes a block recursive structure on the matrix $\mathbf{A}$.

In the first identifying structure, (s1), the monetary policy instrument (measured by the federal funds rate) is set taking into account information on the contemporaneous values of output, domestic prices and commodity prices, but considering only lagged values of total reserves, of non-borrowed reserves and of the money supply. In the second identifying scheme, (s2), the instrument is the non-borrowed reserves, and the federal funds rate is included only as one of the variables observed after policy is set. And in the third scheme, (s3), the non-borrowed reserves remains the instrument, but total reserves is switched into the set of contemporaneously observed variables. CEE point out that identifying structures of the sort outlined in (s1) and (s2) are widely used; for example, in papers by Christiano and Eichenbaum (1992a), Christiano *et al.* (1996, 1998), Eichenbaum and Evans (1995), Bernanke and Blinder (1992), Bernanke and Mihov (1998) and Gertler and Gilchrist (1994). But the very fact that CEE include in their review three alternative identification schemes is immediate evidence of the lack of consensus that exists on theories of contemporaneous dependence.

The motivation for the three identifying structures considered in CEE is based on two complementary and interrelated arguments: the first argument is concerned with the information available to decision-makers and the second is concerned with the operational procedures followed by the monetary authorities. On the first of these, the motivation given by CEE for the assumption that policy-makers observe the contemporaneous values of output, domestic prices and commodity prices when setting policy is provided as follows:

> The Fed does have at its disposal monthly data on aggregate employment, industrial output and other indicators of real activity. It also has substantial amounts of information regarding the price level. In our view, the assumption that the Fed sees output and prices when they choose the policy instrument seems at least as plausible as assuming that they don't [CEE, p. 83][12]

CEE recognise the relative frailty of this assumption, acknowledging that quarterly measures of output and prices, namely real GDP and the GDP deflator, are actually only known with a lag. This point is elaborated in Brunner (2000) and Rotemberg and Woodford (1999). Brunner notes that information on most broad measures of time-t economic activity and on time-t domestic prices is simply not available until one month after the time-t monetary policy is set (and is indeed subject to considerable subsequent data revision).[13] Rotemberg and Woodford (1999) make the point that the political process of responding to data takes time even if the data is, in principle, reported concurrently. CEE also make reference to Sims and Zha (1998), for example, in which an entirely different sequencing

12 The argument is further developed in footnote 64 of CEE where the available indicators of prices and output are listed in a little more detail.

13 Brunner also makes the point that the information available to policy-makers is almost certainly considerably wider than that represented by the variables included in these VAR analyses and persuasively makes the case for the inclusion of direct measures of market participants' expectations in the VAR as a parsimonious means of including additional relevant information.

is adopted, assuming that only contemporaneous commodity prices and money supply are known to policy-makers when setting interest rates. Moreover, this structure is assumed in the context of a model estimated using quarterly data.

Kim and Roubini (2000) also make use of an identification scheme based on information flows. They note that, while price and activity data is published with (at least) a one month lag, monetary data is available within the month and financial data is available daily. This reasoning is used to motivate an identification scheme that is broadly based on the sequencing of decisions, but which is more complex than the usual block recursive structure.

The motivation for the identifying assumptions considered by CEE, Kim and Roubini and others are also based on the perceived institutional arrangements and operational procedures implemented by the US monetary policy-makers. These provide an explanation for the different treatment of the federal funds rate, total reserves, non-borrowed reserves and money supply under the identification schemes (s1)–(s3). This aspect of the tentative theory underlying the identification of the short-run relations is emphasised in Bernanke and Blinder (1992), Bernanke and Mihov (1998), Strongin (1995) and Gordon and Leeper (1994). The latter paper, for example, focuses attention directly on the actions of the Fed in the federal funds market via open market operations and discount window operations. They argue that, at least in the case of the United States, much work in the area incorrectly associates innovations in monetary policy either with movements in the funds rate or with movements in reserves because it fails to fully specify the underlying behavioural relationships in the federal funds market. They argue that identification of monetary policy requires a fully specified model of the market relationships and, to this end, they argue that the demand for reserves will depend on the federal funds rate, prices and output only, while the Fed's decisions on the fund rate will depend on reserves, long-term interest rates and commodity prices only.[14] Imposing the further recursive assumption that financial and goods markets respond to money market disturbances only with a lag, this identifying structure on the supply and demand for reserves allows the short-run structural parameters of the relations of the federal funds market, and hence monetary policy shocks, to be identified (the latter defined as unexplained movements in the rate-setting equation).

14 There is also an explicit informational assumption here that the Fed's interest rate decisions are based on money and financial market data released at high frequencies but not based on current innovations in goods market variables which are observed only with a lag.

Gordon and Leeper (1994) make the further important point that many empirical studies use broad monetary aggregates as measures of policy variables, as though the separation of policy behaviour from financial sector behaviour is of only secondary importance (failing to distinguish, for example, between the federal funds rate and the Treasury bill rate). Clearly, when the identification of shocks relies on the fine distinctions of timing in decision-making, it is very likely that monetary policy shocks derived from the federal funds market in which the monetary authorities operate will be distinct from money supply shocks derived from the money markets in which financial institutions operate. The authors demonstrate empirically that this is indeed the case.

The literature on identifying monetary policy shocks has produced some important and innovative contributions. But, as the discussion surrounding CEE makes clear, there is little consensus in the literature on the identifying restrictions motivated by the sequencing of decisions or the information flows faced by decision-makers. And, as the work of Gordon and Leeper illustrates, identifying structures motivated by the operational procedures implemented by the monetary authorities requires very detailed knowledge of the money market and financial markets and is also unlikely to deliver uncontentious restrictions. Both of these points illustrate well the general difficulties involved in the identification of contemporaneous dependencies in macroeconomic models.

3.2.5 *Identification using 'tentative' theory on long-run relations*

An alternative approach to identification is to follow the Structural VAR approach and impose restrictions on the model parameters based on 'tentative' theory relating to the long-run properties of the model. This approach was popularised by Blanchard and Quah (1989) who provide a structural interpretation to the shocks by imposing *a priori* restrictions on the covariance matrix of the structural errors and on the long-run responses of variables to the shocks. This approach has been widely employed, including more recent uses of the approach in Gali (1992), Clarida and Gali (1994), Lastrapes and Selgin (1994, 1995), Bullard and Keating (1995), Astley and Garratt (1996), Crowder *et al.* (1999), Gonzalo and Ng (2001). Blanchard and Quah (1989) identify 'demand' and 'supply' shocks in a bivariate VAR in output growth and unemployment on the usual assumption that the two shocks are contemporaneously orthogonal plus the further assumption that 'demand' shocks have no permanent

effect on output whilst 'supply' shocks do. In the moving average representation for output growth, this assumption ensures that the *sum* of the coefficients on the current and lagged observations of the demand shock is equal to zero. Inverting the moving average representation to obtain the (approximate) VAR representation, the restriction translates to the imposition of restrictions involving *all* the reduced form parameters. The implications of the transitory/permanent decomposition of the shocks for the structural parameters varies across the structural equations and are discussed in Pagan and Pesaran (2005). Given that these restrictions relate to the long run, this approach does not rely on the precise timing and sequencing of decisions and is less sensitive to the frequency of the observations of the data.

The two-variable example considered by Blanchard and Quah is a rather special case. In more general VAR models, where there are $m - r$ unit roots and r cointegrating relations, the assumption that there are r transitory structural shocks and $m - r$ permanent structural shocks, orthogonal to each other, is equivalent to assuming that $m - r$ of the variables evolve independently of the long-run relations that exist between the variables. This imposes $(m - r) \times r$ restrictions on the matrix of adjustment coefficients, $\tilde{\boldsymbol{\alpha}}$, in the structural VECM of (3.1).[15] Even if the cointegrating vector $\boldsymbol{\beta}$ is identified, the Blanchard and Quah assumptions on the permanent/transitory nature and orthogonality of the shocks provide only $(m-r)r + \frac{1}{2}(m+1)m$ restrictions (including m normalisation restrictions), fewer than the m^2 restrictions required to fully identify the shocks to the system. The assumption that shocks can be split into those that are permanent and those that are transitory will provide sufficient restrictions to identify the shocks only in the special case where $m = 2$ and $r = 1$; of course, this is the case in the bivariate model used by Blanchard and Quah (1989) where there is one transitory (demand) shock and one permanent (supply) shock.[16] But generally, further restrictions are required. For example, a further $\frac{1}{2}(m - r)(m - r - 1) + \frac{1}{2}(r - 1)r$ restrictions, giving m^2 in total, would be provided by using Sims' recursive identification approach applied to the two types of shock (permanent and temporary) separately. This approach is described in Gonzalo and Ng (2001). But such an identification strategy is subject to the same criticisms already levelled against the recursive schemes discussed above.

[15] See Pagan and Pesaran (2005) for details.

[16] Blanchard and Quah trivially identify $\boldsymbol{\beta}$ by assuming unemployment (u) is stationary and output (y) is $I(1)$. Including u as the first variable in their bivariate VAR identifies $\boldsymbol{\beta}$ as $(1, 0)$ since u is $I(0)$ by assumption.

3.3 National macroeconomic modelling in a global context

The modelling strategy reviewed in Section 3.1.3 treats all the core variables included in the VAR symmetrically and no distinction has been drawn between endogenous and exogenous variables. However, this modelling strategy, and the associated econometric methods, is not efficient in the case of small open economies, or if one wishes to develop satellite models that are influenced by the core variables but they themselves have little feedback into the core variables (to be made precise below). In the context of most macroeconometric modelling exercises, a natural example of variables that we might choose to treat as exogenous to the domestic economy is the international price of oil which is largely set outside the UK economy.

For most small open economies whose decisions do not significantly influence the rest of the world, including the UK, one might consider macroeconomic events abroad to be exogenously determined. Having said this, however, there may be occasions when movements in macroeconomic variables of a small economy like the UK provide important contemporaneous indicators of movements in world-wide economic variables. For example, news on the threat of war is likely to impact on demand and output across all of the world's economies. In these circumstances, treating domestic output as though it can have no power for explaining contemporaneous movements in foreign output will incorrectly omit these important feedbacks from unobserved external events. Hence, the decision on how to treat a foreign variable can involve a judgement between, on the one hand, treating the variable as exogenous to capture the (obvious) characteristic that foreigners do not look to the domestic variable in their decision-making and, on the other hand, the less direct gain of capturing influences from outside the model which impact jointly and contemporaneously on domestic and foreign variables.

This judgement needs to take into account the statistical implications of the modelling decision. These will typically encourage the treatment of variables as endogenous, as the treatment of an exogenous variable as endogenous involves a loss of efficiency in estimation, which is usually relatively harmless. In contrast, the treatment of an endogenous variable as exogenous will introduce biases in the estimation which can be considerably more damaging. In what follows we consider an intermediate case where the foreign $I(1)$ variables are treated as weakly exogenous, in the sense that they affect the domestic variables contemporaneously (and could be affected by lagged changes of domestic and foreign variables)

but are not affected by disequilibria in the domestic economy. In other words, in error correcting regressions of changes in foreign variables none of the lagged error correction terms associated with the domestic economy should be statistically significant. Note that this is not the same as the notion of 'Granger Causality' under which none of the domestic variables are allowed to enter the model for the foreign variables. A weakly exogenous $I(1)$ variable is also referred to as 'long-run' forcing by Granger and Lin (1995).

3.3.1 *VARX models: VAR models with weakly exogenous variables*

To elaborate on the treatment of endogenous and exogenous variables in our modelling framework, denote the m_y variables in $\mathbf{z}_t$ that are endogenously determined by $\mathbf{y}_t$, and denote the m_x variables that are exogenously determined by $\mathbf{x}_t$. In this case, $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{x}_t')'$, and the structural model of (3.1) can be rewritten as

$$\begin{pmatrix} \mathbf{A}_{yy} & \mathbf{A}_{yx} \\ \mathbf{0} & \mathbf{A}_{xx} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{y}_t \\ \Delta \mathbf{x}_t \end{pmatrix} = \tilde{\mathbf{a}} + \tilde{\mathbf{b}}t - \tilde{\mathbf{\Pi}} \begin{pmatrix} \mathbf{y}_{t-1} \\ \mathbf{x}_{t-1} \end{pmatrix} + \sum_{i=1}^{p-1} \tilde{\mathbf{\Gamma}}_i \begin{pmatrix} \Delta \mathbf{y}_{t-i} \\ \Delta \mathbf{x}_{t-i} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{yt} \\ \boldsymbol{\varepsilon}_{xt} \end{pmatrix}, \tag{3.16}$$

where

$$\tilde{\mathbf{\Pi}} = \begin{pmatrix} \underset{(m_y \times m)}{\tilde{\mathbf{\Pi}}_y} \\ \underset{(m_x \times m)}{\mathbf{0}} \end{pmatrix} = \begin{pmatrix} \underset{(m_y \times r)}{\tilde{\boldsymbol{\alpha}}_y} \\ \underset{(m_x \times r)}{\mathbf{0}} \end{pmatrix} \underset{(r \times m)}{\boldsymbol{\beta}'}.$$

The first m_y equations in the system in (3.16) provide the decision-rules explaining the determination of the endogenous variables and, hence, the disturbances $\boldsymbol{\varepsilon}_{yt}$ continue to have a clear structural interpretation. Non-zero values of $\mathbf{A}_{yy}$ and $\mathbf{A}_{yx}$ allow for contemporaneous influences on the variables in $\mathbf{y}_t$ from the other variables in $\mathbf{y}_t$ and from the variables in $\mathbf{x}_t$. Non-zero values of $\tilde{\mathbf{\Pi}}_y$ allow for feedback from long-run reduced form disturbances, $\boldsymbol{\xi}_t$. These are linear combinations of variables which may be endogenously or exogenously determined; *i.e.* we continue to define $\boldsymbol{\xi}_t = \boldsymbol{\beta}' \mathbf{z}_{t-1}$. Given the exogeneity of the variables in $\mathbf{x}_t$ and given that they continue to exert an influence on the long-run outcomes of $\mathbf{y}_t$ via $\boldsymbol{\xi}_t$, the $\mathbf{x}_t$ are often termed 'long-run forcing' variables.

The remaining m_x equations in (3.16) characterise the determination of the exogenously determined variables. A zero matrix in the lower triangle of $\mathbf{A}$ shows that there are no direct contemporaneous feedbacks from the variables in $\mathbf{y}_t$ to those in $\mathbf{x}_t$, and the $m_x \times m$ matrix of zeros in $\widetilde{\mathbf{\Pi}}$ shows that there are no feedbacks from the long-run reduced form disturbances, ξ_t, to $\mathbf{x}_t$ either. The structural disturbances, $\boldsymbol{\varepsilon}_{xt}$, have a clear economic meaning in the sense that they relate to unanticipated movements in the exogenous variables but, given that these variables are considered to be determined outside the system under consideration, they do not have the same behavioural content as the $\boldsymbol{\varepsilon}_{yt}$ structural shocks.

Strict exogeneity in the $\mathbf{x}_t$ requires the $\boldsymbol{\varepsilon}_{xt}$ shocks to be uncorrelated with the $\boldsymbol{\varepsilon}_{yt}$ shocks. In the case where they are not, the asymptotically innocuous assumption that $(\boldsymbol{\varepsilon}_{yt}, \boldsymbol{\varepsilon}_{xt})$ are jointly normally distributed provides the linear relationship:

$$\boldsymbol{\varepsilon}_{yt} = \mathbf{\Omega}_{yx}\mathbf{\Omega}_{xx}^{-1}\boldsymbol{\varepsilon}_{xt} + \boldsymbol{\eta}_{yt},$$

where the structural errors have variance covariance matrix

$$\mathbf{\Omega} = \begin{pmatrix} \mathbf{\Omega}_{yy} & \mathbf{\Omega}_{yx} \\ \mathbf{\Omega}_{xy} & \mathbf{\Omega}_{xx} \end{pmatrix}$$

and, by construction, $\boldsymbol{\varepsilon}_{xt}$ and $\boldsymbol{\eta}_{yt}$ are uncorrelated. The first m_y equations of (3.16) can then be rewritten as

$$\mathbf{A}_{yy}\Delta\mathbf{y}_t + \mathbf{A}_{yx}^{*}\Delta\mathbf{x}_t = \widetilde{\mathbf{a}}_y^{*} + \widetilde{\mathbf{b}}_y^{*}t - \widetilde{\mathbf{\Pi}}_y\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\widetilde{\mathbf{\Gamma}}_{yi}^{*}\Delta\mathbf{z}_{t-i} + \boldsymbol{\eta}_{yt}, \tag{3.17}$$

where $\widetilde{\mathbf{a}}_y^{*} = \widetilde{\mathbf{a}}_y - \mathbf{\Omega}_{yx}\mathbf{\Omega}_{xx}^{-1}\widetilde{\mathbf{a}}_x$, $\widetilde{\mathbf{b}}_y^{*} = \widetilde{\mathbf{b}}_y - \mathbf{\Omega}_{yx}\mathbf{\Omega}_{xx}^{-1}\widetilde{\mathbf{b}}_x$, $\widetilde{\mathbf{\Gamma}}_{yi}^{*} = \widetilde{\mathbf{\Gamma}}_{yi} - \mathbf{\Omega}_{yx}\mathbf{\Omega}_{xx}^{-1}\widetilde{\mathbf{\Gamma}}_{xi}$, and $\mathbf{A}_{yx}^{*} = \mathbf{A}_{yx} - \mathbf{\Omega}_{yx}\mathbf{\Omega}_{xx}^{-1}\mathbf{A}_{xx}$ and where we have used the decomposition, $\widetilde{\mathbf{a}} = \left(\widetilde{\mathbf{a}}_y', \widetilde{\mathbf{a}}_x'\right)'$, $\widetilde{\mathbf{b}} = \left(\widetilde{\mathbf{b}}_y', \widetilde{\mathbf{b}}_x'\right)'$ and $\widetilde{\mathbf{\Gamma}}_{yi} = \left(\widetilde{\mathbf{\Gamma}}_{yi}', \widetilde{\mathbf{\Gamma}}_{xi}'\right)'$. This formulation of the structural equations explaining $\mathbf{y}_t$ allows for the indirect effects of changes in $\mathbf{x}_t$ on $\mathbf{y}_t$, experienced through the contemporaneous dependence between the structural shocks, as well as the direct effects captured through the elements of $\mathbf{A}$.

The above representation decomposes the modelling task into the specification of a 'conditional model' for $\Delta\mathbf{y}_t$ given by (3.17), and a 'marginal model' for $\Delta\mathbf{x}_t$ which under weak exogeneity can be written more generally as

$$\mathbf{A}_{xx}\Delta\mathbf{x}_t = \widetilde{\mathbf{a}}_x + \widetilde{\mathbf{b}}_x t - \mathbf{\Pi}_{xx}\mathbf{x}_{t-1} + \sum_{i=1}^{p-1}\widetilde{\mathbf{\Gamma}}_{xi}\Delta\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_{xt}. \tag{3.18}$$

Note the absence of error correction terms from the conditional model in the marginal model. In the case where $\mathbf{x}_t$'s are $I(1)$ and not cointegrated amongst themselves we have the further restrictions, $\mathbf{\Pi}_{xx} = \mathbf{0}$. In this set-up $\mathbf{x}_t$ is said to be long-run forcing for the error correcting model in $\Delta\,\mathbf{y}_t$.

The combined model of the structural equations explaining $\Delta\mathbf{y}_t$ in (3.17) and the structural equations explaining $\Delta\mathbf{x}_t$ in (3.18) can now be written as:

$$\mathbf{A}^{*}\Delta\mathbf{z}_t = \widetilde{\mathbf{a}}^{*} + \widetilde{\mathbf{b}}^{*}t - \widetilde{\mathbf{\Pi}}\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\widetilde{\mathbf{\Gamma}}_i^{*}\Delta\mathbf{z}_{t-i} + \boldsymbol{\varepsilon}_t^{*}, \tag{3.19}$$

where

$$\mathbf{A}^{*} = \begin{pmatrix} \mathbf{A}_{yy} & \mathbf{A}_{yx}^{*} \\ \mathbf{0} & \mathbf{A}_{xx} \end{pmatrix}, \quad \widetilde{\mathbf{\Pi}} = \begin{pmatrix} \mathbf{\Pi}_{yy} & \mathbf{\Pi}_{yx} \\ \mathbf{0} & \mathbf{\Pi}_{xx} \end{pmatrix} \quad \widetilde{\mathbf{a}}^{*} = \begin{pmatrix} \widetilde{\mathbf{a}}_y^{*} \\ \widetilde{\mathbf{a}}_x \end{pmatrix},$$

$$\widetilde{\mathbf{b}}^{*} = \begin{pmatrix} \widetilde{\mathbf{b}}_y^{*} \\ \widetilde{\mathbf{b}}_x \end{pmatrix}, \quad \widetilde{\mathbf{\Gamma}}_i^{*} = \begin{pmatrix} \widetilde{\mathbf{\Gamma}}_{yi}^{*} \\ \widetilde{\mathbf{\Gamma}}_{xi} \end{pmatrix} \text{ and } \quad \boldsymbol{\varepsilon}_t^{*} = \begin{pmatrix} \boldsymbol{\eta}_{yt} \\ \boldsymbol{\varepsilon}_{xt} \end{pmatrix}.$$

The associated reduced form system is readily obtained, following the arguments surrounding (3.1) and (3.3) but using $\mathbf{A}^{*}$ in place of $\mathbf{A}$. Estimation of the reduced form system can proceed by the maximum likelihood (ML) method, taking account of the long-run restrictions implied by the economic theory on the elements in $\widetilde{\mathbf{\Pi}}$, as described in Chapter 6; see Pesaran and Shin (2002) and Pesaran, Shin and Smith (2000) for further details. Identification of the structural parameters of (3.19), and the structural errors, requires quite separate identifying restrictions on the short-run dynamics of the sort discussed in the subsections above. These restrictions will clearly need to take account of the structure incorporated into (3.19) to reflect the exogeneity of the $\mathbf{x}_t$ but, otherwise, the modelling framework is unaffected.

3.3.2 *Developing satellite or sectoral models*

The structural modelling strategy advanced here can also be adapted to account for sectoral effects. This can be done by linking the national macroeconomic models to other 'sectoral' models assuming a block recursive structure. The identifying structure imposed on the system assumes that the variables of the sectoral models are influenced by the variables of the national macroeconomic model, but do not exert a corresponding influence on the variables of the national model. Such a set-up would allow analysis to focus on issues of particular interest in the labour market, international trade, or particular sectors or regions in the national economy.

To illustrate the idea, consider a set of sectoral variables, $\mathbf{w}_t$, which are influenced by the variables in the national model, $\mathbf{z}_t$, but have no feedback to the 'core' macroempirical variables. As above, the elements of $\mathbf{z}_t$ can themselves be distinguished according to whether they are endogenously determined within the national model, $\mathbf{y}_t$, or determined exogenously, $\mathbf{x}_t$. For expositional purposes, assume that a simple first-order model is appropriate and that there are only contemporaneous interactions between the different types of variables. To broaden the possible relevance of the model, assume also that anticipated and unanticipated values of the explanatory variables can have different effects. The structure is then

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{D}_1\mathbf{x}_{t-1} + \mathbf{u}_{1t},\\
\mathbf{y}_t &= \mathbf{F}_1\mathbf{y}_{t-1} + \mathbf{F}_2\mathbf{x}_t + \mathbf{F}_3\left[\mathbf{x}_t - E_{t-1}(\mathbf{x}_t)\right] + \mathbf{u}_{2t},\\
\mathbf{w}_t &= \mathbf{G}_1\mathbf{w}_{t-1} + \mathbf{G}_2\mathbf{y}_t + \mathbf{G}_3\left[\mathbf{y}_t - E_{t-1}(\mathbf{y}_t)\right]\\
&\quad + \mathbf{G}_4\mathbf{x}_t + \mathbf{G}_5\left[\mathbf{x}_t - E_{t-1}(\mathbf{x}_t)\right] + \mathbf{u}_{3t}.
\end{aligned}$$

Solving for the terms involving expectations, and stacking the relationships in vector form, we have

$$\begin{aligned}
&\begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -(\mathbf{F}_2+\mathbf{F}_3) & \mathbf{I} & \mathbf{0} \\ -(\mathbf{G}_4+\mathbf{G}_5) & -(\mathbf{G}_2+\mathbf{G}_3) & \mathbf{I} \end{pmatrix}\begin{pmatrix} \mathbf{x}_t \\ \mathbf{y}_t \\ \mathbf{w}_t \end{pmatrix}\\
&= \begin{pmatrix} \mathbf{D}_1 & \mathbf{0} & \mathbf{0} \\ -\mathbf{F}_3\mathbf{D}_1 & \mathbf{F}_1 & \mathbf{0} \\ -(\mathbf{G}_3\mathbf{F}_2\mathbf{D}_1+\mathbf{G}_5\mathbf{D}_1) & -\mathbf{G}_3\mathbf{F}_1 & \mathbf{G}_1 \end{pmatrix}\begin{pmatrix} \mathbf{x}_{t-1} \\ \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_{1t} \\ \mathbf{u}_{2t} \\ \mathbf{u}_{3t} \end{pmatrix}.
\end{aligned} \tag{3.20}$$

The structure of the model is block triangular, as is immediately apparent from the matrix premultiplying the vector of variables on the LHS of (3.20). There is clearly a causal structure from $\mathbf{x}_t$ to $\mathbf{y}_t$; and from $\mathbf{x}_t$ and $\mathbf{y}_t$ to $\mathbf{w}_t$. If we assume that $E(\mathbf{u}_{it}\mathbf{u}_{jt}) = \mathbf{0}$, for $i \neq j$, then the model is block recursive. This structure has the advantage that the analysis of the variables in $\mathbf{z}_t = (\mathbf{x}_t', \mathbf{y}_t')'$ can be carried out without reference to those in $\mathbf{w}_t$, while the variables in $\mathbf{w}_t$ can be studied by means of 'sectoral models', estimated independently of the model for $\mathbf{z}_t$ and taking values of $\mathbf{x}_t$, $\mathbf{y}_t$, $\mathbf{u}_{1t}$ and $\mathbf{u}_{2t}$ as given.

The development of satellite or sectoral models opens up many opportunities for modelling decision-making in the real world. The number of parameters that are estimated in a VAR could confine the approach to the analysis of a relatively small number of variables. This would be a potentially significant limitation of the modelling approach. Further, longer time series do not necessarily resolve the problem because the structural stability of the model might be called into question with very long spans of data. The assumptions described above provide a means of circumventing these problems so long as the block recursive structure of (3.20) corresponds to the decision-making context.

The purpose of this chapter has been to set out a canonical dynamic structural model which demonstrates the distinction between short-run and long-run effects in a model and to highlight the important role of economic theory in identifying these respective effects. Economic theory was shown to be central to the identification of long-run relations and of economically meaningful shocks, and we argued that some of the identifying assumptions used in the literature, especially those relating to the short run and based on the sequencing of decisions or release of data, seem to be relatively frail. On the other hand, our approach to macroeconometric modelling is a pragmatic one; we recognise that it will never be possible to model the economy in its entirety and all models are imperfect and potentially contentious, therefore. For this reason, we emphasise the criterion of model relevance, urging modellers to construct models that are useful for policy analysis and decision-making. We believe our approach to modelling will allow the modeller to capture the properties of the data well while informing the model with economic theory. Our emphasis is on the economic theory of the long run, based on the strength of conviction with which we believe theory as it relates to the long run and the short run, but our modelling approach can readily accommodate the restrictions suggested by theories of the short run as well as the long run. In deciding on the theory to be embedded within the model, the model should consider what is the *minimal structure required for its purpose*. This will include a clear view on the part of the modeller of what can be taken as exogenous and what needs to be modelled endogenously. Hence, for the purposes of monetary or macroeconomic decisions in the UK, for example, it seems entirely reasonable that the world economy might be taken as long-run forcing (weakly exogenous) to the core macroeconomic model of the UK, and that the variables of the core macromodel might be taken as long-run forcing in particular sectors of the economy. The modelling choice on what constitutes the minimal structure is, of course, itself informed by economic theory and, in what follows, we describe in more detail particular models of the long run, in Chapter 4, and of the short run, in Chapter 5, that will inform our modelling choices as they relate to the UK macroeconomy.

3.4 Global vector autoregressive (GVAR) models

The discussion so far has considered the modelling of a single national economy, possibly containing exogenous variables to take into account the effects of variables determined outside the national economy. In many instances, however, one might be interested to model more explicitly the source of the foreign influences on the domestic economy and the contributions of the national economy to the broad global changes that are, in turn, influencing the other economies of the world. One possible example of such an analysis is a model of the UK economy and its interactions with the economies in the euro area. Such an analysis might be used to establish the impact of shocks to the euro area economies on the UK, and *vice versa*, to quantify the likely effects of the UK's entry into the monetary union. Pesaran, Smith and Smith (2005) provide such an exercise. Or, given the increasing globalisation of world financial markets, a second application could be the analysis of the effect of shocks to financial markets on business cycles both within and across economies.

These analyses require the development of a 'global' modelling framework within which the national model can be incorporated, along with equivalent models of the other economies in the rest of the world. The straight application of the modelling framework outlined in Section 3.1.3 is an attractive approach, but is almost certainly constrained by computational limitations. Hence, while it is possible in principle to extend the modelling strategy to cover the same m variables in each of, say, $N+1$ separate economies, in practice this would involve the estimation of a cointegrated VAR involving around $mp(N+1)$ parameters in each equation of the model to be estimated (where p is the order of the VAR). If there are five variables modelled in each economy, 20 economies and a lag-order of 2 is used, this generates at least 210 parameters to be estimated in each equation, which is clearly infeasible for the data series that are available.

The issue of how to overcome this problem is pursued in Pesaran, Schuermann and Weiner (2004, PSW), with further development in Dees, di Mauro, Pesaran and Smith (2005, DdPS), in which a Global VAR (GVAR) model is developed to investigate global interactions and the analysis

of regional shocks on the world economy in general. The problem of modelling many economies in a coherent and consistent manner is solved through the careful construction of separate measures of 'foreign' variables for use in each of the separate national models. The country-specific foreign variables are then treated as weakly exogenous (in the sense discussed above) when estimating each of the country models. Specifically, individual country (or region) VEC models are estimated using a range of domestic macroeconomic variables plus corresponding foreign variables constructed from other economies' data using weights to match the international trade pattern of the country under consideration. The individual country models are then combined in a consistent and cohesive manner to generate forecasts for *all* the variables in the world economy simultaneously.

To illustrate these ideas in a little more detail, assume that there are $N+1$ economies in the world, indexed by $i = 0, 1, \ldots, N$, and denote the country-specific variables by the $m \times 1$ vector $\mathbf{y}_{it}$ and the associated country-specific foreign variables by $\mathbf{y}_{it}^*$, then a first-order country-specific model can be written[17]

$$\mathbf{y}_{it} = \mathbf{a}_{i0} + \mathbf{\Phi}_i \mathbf{y}_{i,t-1} + \mathbf{\Lambda}_{i0} \mathbf{y}_{it}^* + \mathbf{\Lambda}_{i1} \mathbf{y}_{i,t-1}^* + \mathbf{u}_{it}, \tag{3.21}$$

$$\mathbf{y}_{it}^* = \sum_{j=0}^{N} w_{ij} \mathbf{y}_{jt}, \tag{3.22}$$

where $w_{ij} \geq 0$ are the weights attached to the foreign variables such that $\sum_{j=0}^{N} w_{ij} = 1$, and $w_{ii} = 0$ for all i. These weights could be based on trade shares, for example (*i.e.* the share of country j in the total trade of country i). The country-specific errors $\mathbf{u}_{it}$ are assumed to be serially uncorrelated with mean zero and a non-singular covariance matrix $\mathbf{\Omega}_{ii}$. For the purpose of estimation and inference, it is worth noting that the model in (3.21) can be readily recast as a cointegrating VARX of the form that we have considered above. Moreover, the country-specific foreign variables $\mathbf{y}_{it}^*$ can be treated as weakly exogenous for most countries so this construction will introduce no extra difficulties in terms of the econometric estimation.[18]

Although the model is estimated on a country by country basis, the shocks are allowed to be weakly correlated across countries. In particular,

[17] The number of variables in the different country models need not be the same. Here we assume all country models are based on the same set of variables to simplify the exposition.
[18] The econometric issues involved are explored in detail in Pesaran (2004b).

it is assumed that

$$E\left(\mathbf{u}_{it}\mathbf{u}_{jt}'\right) = \mathbf{\Omega}_{ij} \text{ for } t = t',$$
$$= 0 \text{ for } t \neq t'.$$

Global interactions take place through three distinct, but interrelated channels:

1. Direct dependence of $\mathbf{y}_{it}$ on $\mathbf{y}_{it}^*$ and its lagged values.
2. Dependence of the region-specific variables on common global exogenous variables such as oil prices.
3. Non-zero contemporaneous dependence of shocks in region i on the shocks in region j, measured via the cross-country covariances, $\mathbf{\Omega}_{ij}$.

The individual models are estimated allowing for unit roots and cointegration assuming that region-specific foreign variables are weakly exogenous, with the exception of the model for the US economy which is treated as a closed economy model. The US model is linked to the outside world through exchange rates themselves being determined in rest of the region-specific models. While models of the form in equation (3.21) are relatively standard, PSW show that the careful construction of the global variables as weighted averages of the other regional variables leads to a simultaneous system of regional equations that may be solved to form a global system. They also provide theoretical arguments as well as empirical evidence in support of the weak exogeneity assumption that allows the region-specific models to be estimated consistently.

To obtain the global VAR (GVAR) model, define the $(m+m^*) \times 1$ vector as

$$\mathbf{z}_{it} = \begin{pmatrix} \mathbf{y}_{it} \\ \mathbf{y}_{it}^* \end{pmatrix}$$

and rewrite (3.21) as

$$\mathbf{A}_i \mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{B}_i \mathbf{z}_{i,t-1} + \mathbf{u}_{it}, \tag{3.23}$$

where $\mathbf{A}_i = (\mathbf{I}_m, -\mathbf{\Lambda}_{i0})$ and $\mathbf{B}_i = (\mathbf{\Phi}_i, \mathbf{\Lambda}_{i1})$. Collecting all the country-specific variables together in a $(N+1)\,m \times 1$ vector $\mathbf{y}_t = (\mathbf{y}_{0t}', \mathbf{y}_{1t}', \mathbf{y}_{2t}', \ldots, \mathbf{y}_{Nt}')'$, it is easily seen that the country-specific variables can all be written in terms of $\mathbf{y}_t$:

$$\mathbf{z}_{it} = \mathbf{W}_i \mathbf{y}_t, \qquad i = 0, 1, \ldots, N, \tag{3.24}$$

where $\mathbf{W}_i$ is a matrix of known weights such that

$$\begin{pmatrix} \mathbf{y}_{it} \\ \mathbf{y}_{it}^* \end{pmatrix} = \mathbf{W}_i \mathbf{y}_t.$$

For example, for country $i = 0$ we have

$$\mathbf{W}_0 = \begin{pmatrix} \mathbf{I}_m & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & w_{01}\mathbf{I}_m & w_{02}\mathbf{I}_m & \cdots & w_{0N}\mathbf{I}_m \end{pmatrix},$$

where $w_{01} + w_{02} + \ldots. + w_{0N} = 1$. Using (3.24) in (3.23), we have

$$\mathbf{A}_i \mathbf{W}_i \mathbf{y}_t = \mathbf{a}_{i0} + \mathbf{B}_i \mathbf{W}_i \mathbf{y}_{t-1} + \mathbf{u}_{it} \tag{3.25}$$

and stacking these equations yields the model

$$\mathbf{G}\mathbf{y}_t = \mathbf{a}_0 + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{u}_t, \tag{3.26}$$

where

$$\mathbf{a}_0 = \begin{pmatrix} \mathbf{a}_{00} \\ \mathbf{a}_{10} \\ \vdots \\ \mathbf{a}_{N0} \end{pmatrix}, \ \mathbf{u}_t = \begin{pmatrix} \mathbf{u}_{0t} \\ \mathbf{u}_{1t} \\ \vdots \\ \mathbf{u}_{Nt} \end{pmatrix}, \ \mathbf{G} = \begin{pmatrix} \mathbf{A}_0\mathbf{W}_0 \\ \mathbf{A}_1\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_N\mathbf{W}_N \end{pmatrix}, \ \mathbf{H} = \begin{pmatrix} \mathbf{B}_0\mathbf{W}_0 \\ \mathbf{B}_1\mathbf{W}_1 \\ \vdots \\ \mathbf{B}_N\mathbf{W}_N \end{pmatrix}.$$

Finally, the GVAR model simplifies to a large dimensional VAR model

$$\mathbf{y}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{H}\mathbf{y}_{t-1} + \mathbf{G}^{-1}\mathbf{u}_t. \tag{3.27}$$

Hence, having estimated the separate national models in the form of (3.21), the global model in (3.27) can be solved recursively forward to obtain future values of all the endogenous variables in the global model, $\mathbf{y}_t$, for forecasting multi-step ahead to investigate the dynamic response of the global economy to shocks and in the analysis of international interactions.

The GVAR model allows for cross-country as well as inter-country cointegration. For example, within each country model one could have the Fisher parity that relates domestic nominal short term interest rate to the domestic inflation rate, as well as relating domestic prices and output to their foreign counter parts and exchange rates. As shown in DdPS, the GVAR can also be derived from global factor models where there might be one or more unobserved common factors with differential effects across countries. Finally, it is worth noting that cointegration properties of the individual country models are preserved in the GVAR model, and in general mean-reverting features of the individual economies carry over to the world economy.