13

SIMULTANEOUS EQUATIONS MODELS

13.1 INTRODUCTION

Although most of our work thus far has been in the context of single-equation models, even a cursory look through almost any economics textbook shows that much of the theory is built on sets, or *systems,* of relationships. Familiar examples include market equilibrium, models of the macroeconomy, and sets of factor or commodity demand equations. Whether one's interest is only in a particular part of the system or in the system as a whole, the interaction of the variables in the model will have important implications for both interpretation and estimation of the model's parameters. The implications of simultaneity for econometric estimation were recognized long before the apparatus discussed in this chapter was developed.[1] The subsequent research in the subject, continuing to the present, is among the most extensive in econometrics.

This chapter considers the issues that arise in interpreting and estimating multiple-equations models. Section 13.2 describes the general framework used for analyzing systems of simultaneous equations. Most of the discussion of these models centers on problems of estimation. But before estimation can even be considered, the fundamental question of whether the parameters of interest in the model are even estimable must be resolved. This **problem of identification** is discussed in Section 13.3. Sections 13.4 to 13.7 then discuss methods of estimation. Section 13.8 is concerned with specification tests. In Section 13.9, the special characteristics of dynamic models are examined.

13.2 FUNDAMENTAL ISSUES IN SIMULTANEOUS EQUATIONS MODELS

In this section, we describe the basic terminology and statistical issues in the analysis of simultaneous equations models. We begin with some simple examples and then present a general framework.

13.2.1 ILLUSTRATIVE SYSTEMS OF EQUATIONS

A familiar example of a system of simultaneous equations is a model of market equilibrium, consisting of the following:

$$\begin{aligned} &\text{demand equation:} && q_{d,t} = \alpha_1 p_t + \alpha_2 x_t + \varepsilon_{d,t},\\ &\text{supply equation:} && q_{s,t} = \beta_1 p_t \qquad\quad + \varepsilon_{s,t},\\ &\text{equilibrium condition:} && q_{d,t} = q_{s,t} = q_t. \end{aligned}$$

[1]See, for example, Working (1926) and Haavelmo (1943).

These equations are **structural equations** in that they are derived from theory and each purports to describe a particular aspect of the economy.[2] Because the model is one of the joint determination of price and quantity, they are labeled **jointly dependent** or **endogenous** variables. Income, x, is assumed to be determined outside of the model, which makes it **exogenous.** The disturbances are added to the usual textbook description to obtain an **econometric model.** All three equations are needed to determine the equilibrium price and quantity, so the system is **interdependent.** Finally, because an equilibrium solution for price and quantity in terms of income and the disturbances is, indeed, implied (unless α_1 equals β_1), the system is said to be a **complete system of equations.** *The completeness of the system requires that the number of equations equal the number of endogenous variables.* As a general rule, it is not possible to estimate all the parameters of incomplete systems (although it may be possible to estimate some of them).

Suppose that interest centers on estimating the demand elasticity α_1. For simplicity, assume that ε_d and ε_s are well behaved, classical disturbances with

$$\begin{aligned} E[\varepsilon_{d,t} \mid x_t] &= E[\varepsilon_{s,t} \mid x_t] = 0,\\ E[\varepsilon_{d,t}^2 \mid x_t] &= \sigma_d^2,\\ E[\varepsilon_{s,t}^2 \mid x_t] &= \sigma_s^2,\\ E[\varepsilon_{d,t}\varepsilon_{s,t} \mid x_t] &= 0. \end{aligned}$$

All variables are mutually uncorrelated with observations at different time periods. Price, quantity, and income are measured in logarithms in deviations from their sample means. Solving the equations for p and q in terms of x, ε_d, and ε_s produces the **reduced form** of the model

$$\begin{aligned} p &= \frac{\alpha_2 x}{\beta_1 - \alpha_1} + \frac{\varepsilon_d - \varepsilon_s}{\beta_1 - \alpha_1} = \pi_1 x + v_1,\\ q &= \frac{\beta_1 \alpha_2 x}{\beta_1 - \alpha_1} + \frac{\beta_1 \varepsilon_d - \alpha_1 \varepsilon_s}{\beta_1 - \alpha_1} = \pi_2 x + v_2. \end{aligned} \qquad \textbf{(13-1)}$$

(Note the role of the "completeness" requirement that α_1 not equal β_1.)

It follows that $\text{Cov}[p, \varepsilon_d] = \sigma_d^2 / (\beta_1 - \alpha_1)$ and $\text{Cov}[p, \varepsilon_s] = -\sigma_s^2 / (\beta_1 - \alpha_1)$ so neither the demand nor the supply equation satisfies the assumptions of the classical regression model. The price elasticity of demand cannot be consistently estimated by least squares regression of q on x and p. This result is characteristic of simultaneous-equations models. Because the endogenous variables are all correlated with the disturbances, the least squares estimators of the parameters of equations with endogenous variables on the right-hand side are inconsistent.[3]

Suppose that we have a sample of T observations on p, q, and x such that

$$\text{plim}(1 / T)\mathbf{x}'\mathbf{x} = \sigma_x^2.$$

[2]The distinction between **structural** and **nonstructural** models is sometimes drawn on this basis. See, for example, Cooley and LeRoy (1985).

[3]This failure of least squares is sometimes labeled **simultaneous equations bias.**

Since least squares is inconsistent, we might instead use an **instrumental variable estimator.**[4] The only variable in the system that is not correlated with the disturbances is x. Consider, then, the IV estimator, $\hat{\beta}_1 = \mathbf{q}'\mathbf{x}/\mathbf{p}'\mathbf{x}$. This estimator has

$$\text{plim}\,\hat{\beta}_1 = \text{plim}\,\frac{\mathbf{q}'\mathbf{x}/T}{\mathbf{p}'\mathbf{x}/T} = \frac{\sigma_x^2\beta_1\alpha_2/(\beta_1-\alpha_1)}{\sigma_x^2\alpha_2/(\beta_1-\alpha_1)} = \beta_1.$$

Evidently, the parameter of the supply curve can be estimated by using an instrumental variable estimator. In the least squares regression of $\mathbf{p}$ on $\mathbf{x}$, the predicted values are $\hat{\mathbf{p}} = (\mathbf{p}'\mathbf{x}/\mathbf{x}'\mathbf{x})\mathbf{x}$. It follows that in the instrumental variable regression the instrument is $\hat{\mathbf{p}}$. That is,

$$\hat{\beta}_1 = \frac{\hat{\mathbf{p}}'\mathbf{q}}{\hat{\mathbf{p}}'\mathbf{p}}.$$

Because $\hat{\mathbf{p}}'\mathbf{p} = \hat{\mathbf{p}}'\hat{\mathbf{p}}$, $\hat{\beta}_1$ is also the slope in a regression of q on these predicted values. This interpretation defines the **two-stage least squares estimator.**

It would be desirable to use a similar device to estimate the parameters of the demand equation, but unfortunately, we have exhausted the information in the sample. Not only does least squares fail to estimate the demand equation, but without some further assumptions, the sample contains no other information that can be used. This example illustrates the **problem of identification** alluded to in the introduction to this chapter.

A second example is the following simple model of income determination.

Example 13.1 A Small Macroeconomic Model

Consider the model

$$\begin{aligned} &\textit{consumption:} && c_t = \alpha_0 + \alpha_1 y_t + \alpha_2 c_{t-1} + \varepsilon_{t1},\\ &\textit{investment:} && i_t = \beta_0 + \beta_1 r_t + \beta_2(y_t - y_{t-1}) + \varepsilon_{t2},\\ &\textit{demand:} && y_t = c_t + i_t + g_t. \end{aligned}$$

The model contains an autoregressive consumption function, an investment equation based on interest and the growth in output, and an equilibrium condition. The model determines the values of the three endogenous variables c_t, i_t, and y_t. This model is a **dynamic model.** In addition to the exogenous variables r_t and g_t, it contains two **predetermined variables,** c_{t-1} and y_{t-1}. These are obviously not exogenous, but with regard to the current values of the endogenous variables, they may be regarded as having already been determined. The deciding factor is whether or not they are uncorrelated with the current disturbances, which we might assume. The reduced form of this model is

$$Ac_t = \alpha_0(1-\beta_2) + \beta_0\alpha_1 + \alpha_1\beta_1 r_t + \alpha_1 g_t + \alpha_2(1-\beta_2)c_{t-1} - \alpha_1\beta_2 y_{t-1} + (1-\beta_2)\varepsilon_{t1} + \alpha_1\varepsilon_{t2},$$

$$Ai_t = \alpha_0\beta_2 + \beta_0(1-\alpha_1) + \beta_1(1-\alpha_1)r_t + \beta_2 g_t + \alpha_2\beta_2 c_{t-1} - \beta_2(1-\alpha_1)y_{t-1} + \beta_2\varepsilon_{t1} + (1-\alpha_1)\varepsilon_{t2},$$

$$Ay_t = \alpha_0 + \beta_0 + \beta_1 r_t + g_t + \alpha_2 c_{t-1} - \beta_2 y_{t-1} + \varepsilon_{t1} + \varepsilon_{t2},$$

where $A = 1 - \alpha_1 - \beta_2$. Note that the reduced form preserves the equilibrium condition.

The preceding two examples illustrate systems in which there are **behavioral equations** and **equilibrium conditions.** The latter are distinct in that even in an econometric model, they have no disturbances. Another model, which illustrates nearly all the concepts to be discussed in this chapter, is shown in the next example.

[4]See Section 12.1.

Example 13.2 Klein's Model I

A widely used example of a simultaneous equations model of the economy is Klein's (1950) *Model I.* The model may be written

$$\begin{aligned} C_t &= \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_3(W_t^p + W_t^g) + \varepsilon_{1t} && \text{(consumption)},\\ I_t &= \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 K_{t-1} + \varepsilon_{2t} && \text{(investment)},\\ W_t^p &= \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 A_t + \varepsilon_{3t} && \text{(private wages)},\\ X_t &= C_t + I_t + G_t && \text{(equilibrium demand)},\\ P_t &= X_t - T_t - W_t^p && \text{(private profits)},\\ K_t &= K_{t-1} + I_t && \text{(capital stock)}. \end{aligned}$$

The endogenous variables are each on the left-hand side of an equation and are labeled on the right. The exogenous variables are G_t = government nonwage spending, T_t = indirect business taxes plus net exports, W_t^g = government wage bill, A_t = time trend measured as years from 1931, and the constant term. There are also three predetermined variables: the lagged values of the capital stock, private profits, and total demand. The model contains three behavioral equations, an equilibrium condition and two accounting identities. This model provides an excellent example of a small, dynamic model of the economy. It has also been widely used as a test ground for simultaneous equations estimators. Klein estimated the parameters using yearly data for 1921 to 1941. The data are listed in Appendix Table F13.1.

13.2.2 ENDOGENEITY AND CAUSALITY

The distinction between "exogenous" and "endogenous" variables in a model is a subtle and sometimes controversial complication. It is the subject of a long literature.[5] We have drawn the distinction in a useful economic fashion at a few points in terms of whether a variable in the model could reasonably be expected to vary "autonomously," independently of the other variables in the model. Thus, in a model of supply and demand, the weather variable in a supply equation seems obviously to be exogenous in a pure sense to the determination of price and quantity, whereas the current price clearly is "endogenous" by any reasonable construction. Unfortunately, this neat classification is of fairly limited use in macroeconomics, where almost no variable can be said to be truly exogenous in the fashion that most observers would understand the term. To take a common example, the estimation of consumption functions by ordinary least squares, as we did in some earlier examples, is usually treated as a respectable enterprise, even though most macroeconomic models (including the examples given here) depart from a consumption function in which income is exogenous. This departure has led analysts, for better or worse, to draw the distinction largely on statistical grounds.

The methodological development in the literature has produced some consensus on this subject. As we shall see, the definitions formalize the economic characterization we drew earlier. We will loosely sketch a few results here for purposes of our derivations to follow. The interested reader is referred to the literature (and forewarned of some challenging reading).

[5]See, for example, Zellner (1979), Sims (1977), Granger (1969), and especially Engle, Hendry, and Richard (1983).

Engle, Hendry, and Richard (1983) define a set of variables $\mathbf{x}_t$ in a parameterized model to be **weakly exogenous** if the full model can be written in terms of a marginal probability distribution for $\mathbf{x}_t$ and a conditional distribution for $\mathbf{y}_t \mid \mathbf{x}_t$ such that estimation of the parameters of the conditional distribution is no less efficient than estimation of the full set of parameters of the joint distribution. This case will be true if none of the parameters in the conditional distribution appears in the marginal distribution for $\mathbf{x}_t$. In the present context, we will need this sort of construction to derive reduced forms the way we did previously.

With reference to time-series applications (although the notion extends to cross sections as well), variables $\mathbf{x}_t$ are said to be **predetermined** in the model if $\mathbf{x}_t$ is independent of all *subsequent* structural disturbances ε_{t+s} for $s \geq 0$. Variables that are predetermined in a model can be treated, at least asymptotically, as if they were exogenous in the sense that consistent estimators can be derived when they appear as regressors. We used this result in Section 12.8.2 as well, when we derived the properties of regressions containing lagged values of the dependent variable.

A related concept is **Granger causality.** Granger causality (a kind of statistical feedback) is absent when $f(\mathbf{x}_t \mid \mathbf{x}_{t-1}, \mathbf{y}_{t-1})$ equals $f(\mathbf{x}_t \mid \mathbf{x}_{t-1})$. The definition states that in the conditional distribution, lagged values of $\mathbf{y}_t$ add no information to explanation of movements of $\mathbf{x}_t$ beyond that provided by lagged values of $\mathbf{x}_t$ itself. This concept is useful in the construction of forecasting models. Finally, if $\mathbf{x}_t$ is weakly exogenous and if $\mathbf{y}_{t-1}$ does not Granger cause $\mathbf{x}_t$, then $\mathbf{x}_t$ is **strongly exogenous.**

13.2.3 A GENERAL NOTATION FOR LINEAR SIMULTANEOUS EQUATIONS MODELS[6]

The **structural form** of the model is[7]

$$
\begin{aligned}
\gamma_{11}y_{t1} + \gamma_{21}y_{t2} + \cdots + \gamma_{M1}y_{tM} + \beta_{11}x_{t1} + \cdots + \beta_{K1}x_{tK} &= \varepsilon_{t1},\\
\gamma_{12}y_{t1} + \gamma_{22}y_{t2} + \cdots + \gamma_{M2}y_{tM} + \beta_{12}x_{t1} + \cdots + \beta_{K2}x_{tK} &= \varepsilon_{t2},\\
&\vdots\\
\gamma_{1M}y_{t1} + \gamma_{2M}y_{t2} + \cdots + \gamma_{MM}y_{tM} + \beta_{1M}x_{t1} + \cdots + \beta_{KM}x_{tK} &= \varepsilon_{tM}.
\end{aligned}
\tag{13-2}
$$

There are M equations and M endogenous variables, denoted $y_1, \ldots, y_M$. There are K exogenous variables, $x_1, \ldots, x_K$, that may include predetermined values of $y_1, \ldots, y_M$ as well. The first element of $\mathbf{x}_t$ will usually be the constant, 1. Finally, $\varepsilon_{t1}, \ldots, \varepsilon_{tM}$ are the **structural disturbances.** The subscript t will be used to index observations, $t = 1, \ldots, T$.

[6] We will be restricting our attention to linear models in this chapter. **Nonlinear systems** occupy another strand of literature in this area. Nonlinear systems bring forth numerous complications beyond those discussed here and are beyond the scope of this text. Gallant (1987), Gallant and Holly (1980), Gallant and White (1988), Davidson and MacKinnon (2004), and Wooldridge (2002a) provide further discussion.

[7] For the present, it is convenient to ignore the special nature of lagged endogenous variables and treat them the same as the strictly exogenous variables.

In matrix terms, the system may be written

$$
[y_1 \;\; y_2 \;\; \cdots \;\; y_M]_t \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1M} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2M} \\ & & \vdots & \\ \gamma_{M1} & \gamma_{M2} & \cdots & \gamma_{MM} \end{bmatrix}
+ [x_1 \;\; x_2 \;\; \cdots \;\; x_K]_t \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1M} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2M} \\ & & \vdots & \\ \beta_{K1} & \beta_{K2} & \cdots & \beta_{KM} \end{bmatrix} = [\varepsilon_1 \;\; \varepsilon_2 \;\; \cdots \;\; \varepsilon_M]_t,
$$

or

$$
\mathbf{y}_t'\boldsymbol{\Gamma} + \mathbf{x}_t'\mathbf{B} = \boldsymbol{\varepsilon}_t'.
$$

Each column of the parameter matrices is the vector of coefficients in a particular equation, whereas each row applies to a specific endogenous variable.

The underlying theory will imply a number of restrictions on $\boldsymbol{\Gamma}$ and $\mathbf{B}$. One of the variables in each equation is labeled the *dependent* variable so that its coefficient in the model will be 1. Thus, there will be at least one "1" in each column of $\boldsymbol{\Gamma}$. This **normalization** is not a substantive restriction. The relationship defined for a given equation will be unchanged if every coefficient in the equation is multiplied by the same constant. Choosing a "dependent variable" simply removes this indeterminacy. If there are any identities, then the corresponding columns of $\boldsymbol{\Gamma}$ and $\mathbf{B}$ will be completely known, and there will be no disturbance for that equation. Because not all variables appear in all equations, some of the parameters will be zero. The theory may also impose other types of restrictions on the parameter matrices.

If $\boldsymbol{\Gamma}$ is an upper triangular matrix, then the system is said to be **triangular.** In this case, the model is of the form

$$
\begin{aligned}
y_{t1} &= f_1(\mathbf{x}_t) + \varepsilon_{t1},\\
y_{t2} &= f_2(y_{t1}, \mathbf{x}_t) + \varepsilon_{t2},\\
&\vdots\\
y_{tM} &= f_M(y_{t1}, y_{t2}, \ldots, y_{t,M-1}, \mathbf{x}_t) + \varepsilon_{tM}.
\end{aligned}
$$

The joint determination of the variables in this model is **recursive.** The first is completely determined by the exogenous factors. Then, given the first, the second is likewise determined, and so on.

The solution of the system of equations determining $\mathbf{y}_t$ in terms of $\mathbf{x}_t$ and $\boldsymbol{\varepsilon}_t$ is the **reduced form** of the model,

$$\mathbf{y}_t' = [x_1 \quad x_2 \quad \cdots \quad x_K]_t \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1M} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2M} \\ & & \vdots & \\ \pi_{K1} & \pi_{K2} & \cdots & \pi_{KM} \end{bmatrix} + [\nu_1 \quad \cdots \quad \nu_M]_t$$
$$= -\mathbf{x}_t'\mathbf{B}\boldsymbol{\Gamma}^{-1} + \boldsymbol{\varepsilon}_t'\boldsymbol{\Gamma}^{-1}$$
$$= \mathbf{x}_t'\boldsymbol{\Pi} + \mathbf{v}_t'.$$

For this solution to exist, the model must satisfy the **completeness condition** for simultaneous equations systems: $\boldsymbol{\Gamma}$ must be nonsingular.

Example 13.3 Structure and Reduced Form
For the small model in Example 15.1, $\mathbf{y}' = [c, i, y]$, $\mathbf{x}' = [1, r, g, c_{-1}, y_{-1}]$, and

$$\boldsymbol{\Gamma} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -\alpha_1 & -\beta_2 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -\alpha_0 & -\beta_0 & 0 \\ 0 & -\beta_1 & 0 \\ 0 & 0 & -1 \\ -\alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 \end{bmatrix}, \quad \boldsymbol{\Gamma}^{-1} = \frac{1}{\Delta}\begin{bmatrix} 1-\beta_2 & \beta_2 & 1 \\ \alpha_1 & 1-\alpha_1 & 1 \\ \alpha_1 & \beta_2 & 1 \end{bmatrix},$$

$$\boldsymbol{\Pi}' = \frac{1}{\Delta}\begin{bmatrix} \alpha_0(1-\beta_2+\beta_0\alpha_1) & \alpha_1\beta_1 & \alpha_1 & \alpha_2(1-\beta_2) & -\beta_2\alpha_1 \\ \alpha_0\beta_2+\beta_0(1-\alpha_1) & \beta_1(1-\alpha_1) & \beta_2 & \alpha_2\beta_2 & -\beta_2(1-\alpha_1) \\ \alpha_0+\beta_0 & \beta_1 & 1 & \alpha_2 & -\beta_2 \end{bmatrix},$$

where $\Delta = 1 - \alpha_1 - \beta_2$. The completeness condition is that α_1 and β_2 do not sum to one.

The structural disturbances are assumed to be randomly drawn from an M-variate distribution with

$$E[\boldsymbol{\varepsilon}_t \,|\, \mathbf{x}_t] = \mathbf{0} \quad \text{and} \quad E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' \,|\, \mathbf{x}_t] = \boldsymbol{\Sigma}.$$

For the present, we assume that

$$E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_s' \,|\, \mathbf{x}_t, \mathbf{x}_s] = \mathbf{0}, \quad \forall t, s.$$

Later, we will drop this assumption to allow for heteroscedasticity and autocorrelation. It will occasionally be useful to assume that $\boldsymbol{\varepsilon}_t$ has a multivariate normal distribution, but we shall postpone this assumption until it becomes necessary. It may be convenient to retain the identities without disturbances as separate equations. If so, then one way to proceed with the stochastic specification is to place rows and columns of zeros in the appropriate places in $\boldsymbol{\Sigma}$. It follows that the **reduced-form disturbances, $\mathbf{v}_t' = \boldsymbol{\varepsilon}_t'\boldsymbol{\Gamma}^{-1}$** have

$$E[\mathbf{v}_t \,|\, \mathbf{x}_t] = (\boldsymbol{\Gamma}^{-1})'\mathbf{0} = \mathbf{0},$$
$$E[\mathbf{v}_t\mathbf{v}_t' \,|\, \mathbf{x}_t] = (\boldsymbol{\Gamma}^{-1})'\boldsymbol{\Sigma}\boldsymbol{\Gamma}^{-1} = \boldsymbol{\Omega}.$$

This implies that

$$\boldsymbol{\Sigma} = \boldsymbol{\Gamma}'\boldsymbol{\Omega}\boldsymbol{\Gamma}.$$

The preceding formulation describes the model as it applies to an observation $[\mathbf{y}', \mathbf{x}', \boldsymbol{\varepsilon}']_t$ at a particular point in time or in a cross section. In a sample of data, each joint observation will be one row in a data matrix,

$$[\mathbf{Y} \quad \mathbf{X} \quad \mathbf{E}] = \begin{bmatrix} \mathbf{y}_1' & \mathbf{x}_1' & \boldsymbol{\varepsilon}_1' \\ \mathbf{y}_2' & \mathbf{x}_2' & \boldsymbol{\varepsilon}_2' \\ & \vdots & \\ \mathbf{y}_T' & \mathbf{x}_T' & \boldsymbol{\varepsilon}_T' \end{bmatrix}.$$

In terms of the full set of T observations, the structure is

$$\mathbf{Y}\boldsymbol{\Gamma} + \mathbf{X}\mathbf{B} = \mathbf{E},$$

with

$$E[\mathbf{E} \,|\, \mathbf{X}] = \mathbf{0} \quad \text{and} \quad E[(1/T)\mathbf{E}'\mathbf{E} \,|\, \mathbf{X}] = \boldsymbol{\Sigma}.$$

Under general conditions, we can strengthen this structure to

$$\text{plim}[(1/T)\mathbf{E}'\mathbf{E}] = \boldsymbol{\Sigma}.$$

An important assumption, comparable with the one made in Chapter 4 for the classical regression model, is

$$\text{plim}(1/T)\mathbf{X}'\mathbf{X} = \mathbf{Q}, \quad \text{a finite positive definite matrix.} \tag{13-3}$$

We also assume that

$$\text{plim}(1/T)\mathbf{X}'\mathbf{E} = \mathbf{0}. \tag{13-4}$$

This assumption is what distinguishes the predetermined variables from the endogenous variables. The reduced form is

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\Pi} + \mathbf{V}, \quad \text{where } \mathbf{V} = \mathbf{E}\boldsymbol{\Gamma}^{-1}.$$

Combining the earlier results, we have

$$\text{plim}\,\frac{1}{T}\begin{bmatrix} \mathbf{Y}' \\ \mathbf{X}' \\ \mathbf{V}' \end{bmatrix}[\mathbf{Y} \quad \mathbf{X} \quad \mathbf{V}] = \begin{bmatrix} \boldsymbol{\Pi}'\mathbf{Q}\boldsymbol{\Pi} + \boldsymbol{\Omega} & \boldsymbol{\Pi}'\mathbf{Q} & \boldsymbol{\Omega} \\ \mathbf{Q}\boldsymbol{\Pi} & \mathbf{Q} & \mathbf{0}' \\ \boldsymbol{\Omega} & \mathbf{0} & \boldsymbol{\Omega} \end{bmatrix}. \tag{13-5}$$

13.3 THE PROBLEM OF IDENTIFICATION

Solving the problem to be considered here, the identification problem, logically precedes estimation. We ask at this point whether there is *any* way to obtain estimates of the parameters of the model. We have in hand a certain amount of information upon which to base any inference about its underlying structure. If more than one theory is consistent with the same "data," then the theories are said to be **observationally equivalent** and there is no way of distinguishing them. The structure is said to be *unidentified*.[8]

[8] A useful survey of this issue is Hsiao (1983).

SIXTH EDITION

ECONOMETRIC ANALYSIS

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