UNU.RAN User Manual

Generating non-uniform random numbers
Version 1.2.1, 19 February 2008

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UNU.RAN – Universal Non-Uniform RANdom number generators

UNU.RAN (Universal Non-Uniform RAndom Number generator) is a collection of algorithms for generating non-uniform pseudorandom variates as a library of C functions designed and implemented by the ARVAG (Automatic Random VAriate Generation) project group in Vienna, and released under the GNU Public License (GPL). It is especially designed for such situations where

- a non-standard distribution or a truncated distribution is needed.
- experiments with different types of distributions are made.
- random variates for variance reduction techniques are used.
- fast generators of predictable quality are necessary.

Of course it is also well suited for standard distributions. However due to its more sophisticated programming interface it might not be as easy to use if you only look for a generator for the standard normal distribution. (Although UNU.RAN provides generators that are superior in many aspects to those found in quite a number of other libraries.)

UNU.RAN implements several methods for generating random numbers. The choice depends primary on the information about the distribution can be provided and – if the user is familar with the different methods – on the preferences of the user.

The design goals of UNU.RAN are to provide *reliable*, *portable* and *robust* (as far as this is possible) functions with a consisent and easy to use interface. It is suitable for all situation where experiments with different distributions including non-standard distributions. For example it is no problem to replace the normal distribution by an empirical distribution in a model.

Since originally designed as a library for so called black-box or universal algorithms its interface is different from other libraries. (Nevertheless it also contains special generators for standard distributions.) It does not provide subroutines for random variate generation for particular distributions. Instead it uses an object-oriented interface. Distributions and generators are treated as independent objects. This approach allows one not only to have different methods for generating non-uniform random variates. It is also possible to choose the method which is optimal for a given situation (e.g. speed, quality of random numbers, using for variance reduction techniques, etc.). It also allows to sample from non-standard distribution or even from distributions that arise in a model and can only be computed in a complicated subroutine.

Sampling from a particular distribution requires the following steps:

1. Create a distribution object. (Objects for standard distributions are available in the library)
2. Choose a method.
3. Initialize the generator, i.e., create the generator object. If the choosen method is not suitable for the given distribution (or if the distribution object contains too little information about the distribution) the initialization routine fails and produces an error message. Thus the generator object does (probably) not produce false results (random variates of a different distribution).
4. Use this generator object to sample from the distribution.

There are four types of objects that can be manipulated independently:

- **Distribution objects:** hold all information about the random variates that should be generated. The following types of distributions are available:
  - Continuous and Discrete distributions
  - Empirical distributions
  - Multivariate distributions

Of course a library of standard distributions is included (and these can be further modified to get, e.g., truncated distributions). Moreover the library provides subroutines to build almost arbitrary distributions.

- **Generator objects:** hold the generators for the given distributions. It is possible to build independent generator objects for the same distribution object which might use the same or different methods for generation. (If the choosen method is not suitable for the given method, a `NULL` pointer is returned in the initialization step).
- **Parameter objects:** Each transformation method requires several parameters to adjust the generator to a given distribution. The parameter object holds all this information. When created it contains all necessary default settings. It is only used to create a generator object and destroyed immediately. Altough there is no need to change these parameters or even know about their existence for "usual distributions", they allow a fine tuning of the generator to work with distributions with some awkward properties. The library provides all necessary functions to change these default parameters.
- **Uniform Random Number Generators:** All generator objects need one (or more) streams of uniform random numbers that are transformed into random variates of the given distribution. These are given as pointers to appropriate functions or structures (objects). Two generator objects may have their own uniform random number generators or share a common one. Any functions that produce uniform (pseudo-) random numbers can be used. We suggest Otmar Lendl's PRNG library.

1 Introduction

1.1 Usage of this document

We designed this document in a way such that one can use UNU.RAN with reading as little as necessary. Read Section 1.2 [Installation], page 3 for the instructions to install the library. Section 1.4 [Concepts of UNU.RAN], page 6, discribes the basics of UNU.RAN. It also has a short guideline for choosing an appropriate method. In Chapter 2 [Examples], page 13 examples are given that can be copied and modified. They also can be found in the directory '`examples`' in the source tree.

Further information are given in consecutive chapters. Chapter 4 [Handling distribution objects], page 55, describes how to create and manipulate distribution objects. Chapter 7 [standard distributions], page 201, describes predefined distribution objects that are ready to use. Chapter 5 [Methods], page 87 describes the various methods in detail. For each of possible distribution classes (continuous, discrete, empirical, multivariate) there exists a short overview section that can be used to choose an appropriate method followed by sections that describe each of the particular methods in detail. These are merely for users with some knowledge about the methods who want to change method-specific parameters and can be ignored by others.

Abbreviations and explanation of some basic terms can be found in Appendix B [Glossary], page 235.

1.2 Installation

UNU.RAN was developed on an Intel architecture under Linux with the GNU C compiler but should compile and run on any computing environment. It requires an ANSI compliant C compiler.

Below find the installation instructions for unices.

Uniform random number generator

UNU.RAN can be used with any uniform random number generator but (at the moment) some features work best with Pierre L'Ecuyer's RngStreams library (see http://statistik.wu-wien.ac.at/software/RngStreams/ for a description and downloading. For details on using uniform random number in UNU.RAN see Chapter 6 [Using uniform random number generators], page 189.

Install the required libraries first.

UNU.RAN

1. First unzip and untar the package and change to the directory:
   ```
   tar zxvf unuran-1.2.1.tar.gz
   cd unuran-1.2.1
   ```
2. Optional: Edit the file '`src/unuran_config.h`'
3. Run a configuration script:
   ```
   sh ./configure --prefix=<prefix>
   ```
   where `<prefix>` is the root of the installation tree. When omitted '`/usr/local`' is used.

   Use `./configure --help` to get a list of other options. In particular the following flags are important:

   - Enable support for some external sources of uniform random number generators (see Chapter 6 [Using uniform random number generators], page 189):

     `--with-urng-rngstream`
     URNG: use Pierre L'Ecuyer's RNGSTREAM library [default=`no`]

`--with-urng-prng`
: URNG: use Otmar Lendl's PRNG library [default=`no`]

`--with-urng-gsl`
: URNG: use random number generators from GNU Scientific Library [default=`no`]

`--with-urng-default`
: URNG: global default URNG (builtin|rngstream) [default=`builtin`]

We strongly recommend to use RngStreams library:

```
sh ./configure --with-urng-rngstream --with-urng-default=rngstream
```

Important: You must install the respective libraries '`RngStreams`', '`PRNG`' and '`GSL`' before `./configure` is executed.

- Also make a shared library:

  `--enable-shared`
  : build shared libraries [default=`no`]

- The library provides the function `unur_gen_info` for information about generator objects. This is intented for using in interactive computing environments. This feature can be enabled / disabled by means of the configure flag

  `--enable-info`
  : INFO: provide function with information about generator objects [default=`yes`]

- Enable support for deprecated UNU.RAN routines if you have some problems with older application after upgrading the library:

  `--enable-deprecated`
  : enable support for deprecated UNU.RAN routines [default=`no`]

- Enable debugging tools:

  `--enable-check-struct`
  : Debug: check validity of pointers to structures [default=`no`]

  `--enable-logging`
  : Debug: print informations about generator into logfile [default=no]

4. Compile and install the libray:

   ```
   make
   make install
   ```

   Obviously `$(prefix)/include` and `$(prefix)/lib` must be in the search path of your compiler. You can use environment variables to add these directories to the search path. If you are using the bash type (or add to your profile):

   ```
   export LIBRARY_PATH="<prefix>/lib"
   export C_INCLURE_PATH="<prefix>/include"
   ```

   If you want to make a shared library, then making such a library can be enabled using

   ```
   sh ./configure --enable-shared
   ```

   If you want to link against the shared library make sure that it can be found when executing the binary that links to the library. If it is not installed in the usual path, then the easiest way is to set the `LD_LIBRARY_PATH` environment variable. See any operating system documentation about shared libraries for more information, such as the ld(1) and ld.so(8) manual pages.

5. Documentation in various formats (PDF, HTML, info, plain text) can be found in directory '`doc`'.
6. You can run some tests by

```
make check
```

However, some of these tests requires the usage of the PRNG or RngStreams library and are only executed if these are installed enabled by the corresponding configure flag.
An extended set of tests is run by

```
make fullcheck
```

However some of these might fail occasionally due to roundoff errors or the mysteries of floating point arithmetic, since we have used some extreme settings to test the library.

Upgrading

– *Important:*

UNU.RAN now relies on some aspects of IEEE 754 compliant floating point arithmetic. In particular, `1./0.` and `0./0.` must result in `infinity` and `NaN` (not a number), respectively, and must not cause a floating point exception. For allmost all modern compting architecture this is implemented in hardware. For others there should be a special compiler flag to get this feature (e.g., `-MIEEE` on DEC alpha or `-mp` for the Intel C complier).

– Upgrading UNU.RAN from version 0.9.x or earlier:

With UNU.RAN version 1.0.x some of the macro definitions in file '`src/unuran_config.h`' are moved into file '`config.h`' and are set/controlled by the `./configure` script.
Writting logging information into the logfile must now be enabled when running the configure script:

```
sh ./configure --enable-logging
```

– Upgrading UNU.RAN from version 0.7.x or earlier:

With UNU.RAN version 0.8.0 the interface for changing underlying distributions and running a reinitialization routine has been simplified. The old routines can be compiled into the library using the following configure flag:

```
sh ./configure --enable-deprecated
```

Notice: Using these deprecated routines is not supported any more and this strong discouraged.

Wrapper functions for external sources of uniform random numbers are now enabled by configure flags and not by macros defined in file '`src/unuran_config.h`'.

The file '`src/unuran_config.h`' is not installed any more. It is now only included when the library is compiled. It should be removed from the global include path of the compiler.

1.3 Using the library

ANSI C Compliance

The library is written in ANSI C and is intended to conform to the ANSI C standard. It should be portable to any system with a working ANSI C compiler.

The library does not rely on any non-ANSI extensions in the interface it exports to the user. Programs you write using UNU.RAN can be ANSI compliant. Extensions which can be used in a way compatible with pure ANSI C are supported, however, via conditional compilation. This allows the library to take advantage of compiler extensions on those platforms which support them.

To avoid namespace conflicts all exported function names and variables have the prefix `unur_`, while exported macros have the prefix `UNUR_`.

Compiling and Linking

If you want to use the library you must include the UNU.RAN header file

```
#include <unuran.h>
```

If you also need the test routines then also add

```
#include <unuran_tests.h>
```

If wrapper functions for external sources of uniform random number generators are used, the corresponding header files must also be included, e.g.,

```
#include <unuran_urng_rngstream.h>
```

If these header files are not installed on the standard search path of your compiler you will also need to provide its location to the preprocessor as a command line flag. The default location of the '`unuran.h`' is '`/usr/local/include`'. A typical compilation command for a source file '`app.c`' with the GNU C compiler `gcc` is,

```
gcc -I/usr/local/include -c app.c
```

This results in an object file '`app.o`'. The default include path for `gcc` searches '`/usr/local/include`' automatically so the `-I` option can be omitted when UNU.RAN is installed in its default location.

The library is installed as a single file, '`libunuran.a`'. A shared version of the library is also installed on systems that support shared libraries. The default location of these files is '`/usr/local/lib`'. To link against the library you need to specify the main library. The following example shows how to link an application with the library (and the the RNGSTREAMS library if you decide to use this source of uniform pseudo-random numbers),

```
gcc app.o -lunuran -lrngstreams -lm
```

Shared Libraries

To run a program linked with the shared version of the library it may be necessary to define the shell variable `LD_LIBRARY_PATH` to include the directory where the library is installed. For example,

```
LD_LIBRARY_PATH=/usr/local/lib:$LD_LIBRARY_PATH
```

To compile a statically linked version of the program instead, use the `-static` flag in `gcc`,

```
gcc -static app.o -lunuran -lrngstreams -lm
```

Compatibility with C++

The library header files automatically define functions to have `extern "C"` linkage when included in C++ programs.

1.4 Concepts of UNU.RAN

UNU.RAN is a C library for generating non-uniformly distributed random variates. Its emphasis is on the generation of non-standard distribution and on streams of random variates of special purposes. It is designed to provide a consistent tool to sample from distributions with various properties. Since there is no universal method that fits for all situations, various methods for sampling are implemented.

UNU.RAN solves this complex task by means of an object oriented programming interface. Three basic objects are used:

- distribution object `UNUR_DISTR`
  Hold all information about the random variates that should be generated.
- generator object `UNUR_GEN`
  Hold the generators for the given distributions. Two generator objects are completely independent of each other. They may share a common uniform random number generator or have their owns.
- parameter object `UNUR_PAR`
  Hold all information for creating a generator object. It is necessary due to various parameters and switches for each of these generation methods.

> Notice that the parameter objects only hold pointers to arrays but do not have their own copy of such an array. Especially, if a dynamically allocated array is used it *must not* be freed until the generator object has been created!

The idea behind these structures is that creatin distributions, choosing a generation method and draing samples are orthogonal (ie. independent) functions of the library. The parameter object is only introduced due to the necessity to deal with various parameters and switches for each of these generation methods which are required to adjust the algorithms to unusual distributions with extreme properties but have default values that are suitable for most applications. These parameters and the data for distributions are set by various functions.

Once a generator object has been created sampling (from the univariate continuous distribution) can be done by the following command:

```
double x = unur_sample_cont(generator);
```

Analogous commands exist for discrete and multivariate distributions. For detailed examples that can be copied and modified see Chapter 2 [Examples], page 13.

Distribution objects

All information about a distribution are stored in objects (structures) of type `UNUR_DISTR`. UNU.RAN has five different types of distribution objects:

`cont`	Continuous univariate distributions.
`cvec`	Continuous multivariate distributions.
`discr`	Discrete univariate distributions.
`cemp`	Continuous empirical univariate distribution, ie. given by a sample.
`cvemp`	Continuous empirical multivariate distribution, ie. given by a sample.
`matr`	Matrix distributions.

Distribution objects can be created from scratch by the following call

```
distr = unur_distr_<type>_new();
```

where `<type>` is one of the five possible types from the above table. Notice that these commands only create an *empty* object which still must be filled by means of calls for each type of distribution object (see Chapter 4 [Handling distribution objects], page 55). The naming scheme of these functions is designed to indicate the corresponding type of the distribution object and the task to be performed. It is demonstated on the following example.

```
unur_distr_cont_set_pdf(distr, mypdf);
```

This command stores a PDF named `mypdf` in the distribution object `distr` which must have the type `cont`.

Of course UNU.RAN provides an easier way to use standard distributions. Instead of using `unur_distr_<type>_new` calls and fuctions `unur_distr_<type>_set_<...>` for setting data, objects for standard distribution can be created by a single call. Eg. to get an object for the normal distribution with mean 2 and standard deviation 5 use

```
double parameter[2] = {2.0 ,5.0};
UNUR_DISTR *distr = unur_distr_normal(parameter, 2);
```

For a list of standard distributions see Chapter 7 [Standard distributions], page 201.

Generation methods

The information that a distribution object must contain depends heavily on the chosen generation method choosen.

Brackets indicate optional information while a tilde indicates that only an approximation must be provided. See Appendix B [Glossary], page 235, for unfamiliar terms.

Methods for **continuous univariate distributions**
sample with `unur_sample_cont`

method	PDF	dPDF	CDF	mode	area	other
AROU	x	x		[x]		T-concave
HINV	[x]	[x]	x			
HRB						bounded hazard rate
HRD						decreasing hazard rate
HRI						increasing hazard rate
ITDR	x	x		x		monotone with pole
NINV	[x]		x			
NROU	x			[x]		
SROU	x			x	x	T-concave
SSR	x			x	x	T-concave
TABL	x			x	[~]	all local extrema
TDR	x	x				T-concave
TDRGW	x	x				T-concave
UTDR	x			x	~	T-concave
CSTD						build-in standard distribution
CEXT						wrapper for external generator

Methods for **continuous empirical univariate distributions**
sample with `unur_sample_cont`

EMPK: Requires an observed sample.
EMPL: Requires an observed sample.

Methods for **continuous multivariate distributions**
sample with `unur_sample_vec`

NORTA: Requires rank correlation matrix and marginal distributions.
VNROU: Requires the PDF.
MVSTD: Generator for built-in standard distributions.
MVSTD: Requires PDF and gradiant of PDF.

Methods for **continuous empirical multivariate distributions**
sample with `unur_sample_vec`

VEMPK: Requires an observed sample.

Methods for **discrete univariate distributions**
sample with `unur_sample_discr`

method	PMF	PV	mode	sum	other
DARI	x		x	~	T-concave
DAU	[x]	x			
DGT	[x]	x			
DSROU	x		x	x	T-concave
DSS	[x]	x		x	
DSTD					build-in standard distribution
CEXT					wrapper for external generator

Methods for **matrix distributions**
sample with `unur_sample_matr`

MCORR: Distribution object for random correlation matrix.

Markov Chain Methods for **continuous multivariate distributions**
sample with `unur_sample_vec`

GIBBS: T-concave logPDF and derivatives of logPDF.
HITRO: Requires PDF.

Because of tremendous variety of possible problems, UNU.RAN provides many methods. All information for creating a generator object has to be collected in a parameter object first. For example, if the task is to sample from a continuous distribution the method AROU might be a good choice. Then the call

```
UNUR_PAR *par = unur_arou_new(distribution);
```

creates an parameter object `par` with a pointer to the distribution object and default values for all necessary parameters for method AROU. Other methods can be used by replacing `arou` with the name of the desired methods (in lower case letters):

```
UNUR_PAR *par = unur_<method>_new(distribution);
```

This sets the default values for all necessary parameters for the chosen method. These are suitable for almost all applications. Nevertheless, it is possible to control the behavior of the method using corresponding `set` calls for each method. This might be necessary to adjust the algorithm for an unusual distribution with extreme properties, or just for fine tuning the perforence of the algorithm. The following example demonstrates how to change the maximum number of iterations for method NINV to the value 50:

```
unur_ninv_set_max_iteration(par, 50);
```

All available methods are described in details in Chapter 5 [Methods], page 87.

Creating a generator object

Now it is possible to create a generator object:

```
UNUR_GEN *generator = unur_init(par);
if (generator == NULL) exit(EXIT_FAILURE);
```

Important: You must always check whether `unur_init` has been executed successfully. Otherwise the `NULL` pointer is returned which causes a segmentation fault when used for sampling.

Important: The call of `unur_init` **destroys** the parameter object!
Moreover, it is recommended to call `unur_init` immediately after the parameter object `par` has created and modified.

An existing generator object is a rather static construct. Nevertheless, some of the parameters can still be modified by `chg` calls, e.g.

```
unur_ninv_chg_max_iteration(gen, 30);
```

Notice that it is important *when* parameters are changed because different functions must be used:

The function name includes the term `set` and the first argument must be of type `UNUR_PAR` when the parameters are changed *before* the generator object is created.

The function name includes the term `chg` and the first argument must be of type `UNUR_GEN` when the parameters are changed for an *existing* generator object.

For details see Chapter 5 [Methods], page 87.

Sampling

You can now use your generator object in any place of your program to sample from your distribution. You only have to take care about the type of variates it computes: `double`, `int` or a vector (array of `doubles`). Notice that at this point it does not matter whether you are sampling from a gamma distribution, a truncated normal distribution or even an empirical distribution.

Reinitializing

It is possible for a generator object to change the parameters and the domain of the underlying distribution. This must be done by extracting this object by means of a `unur_get_distr` call and changing the distribution using the correspondig set calls, see Chapter 4 [Handling distribution objects], page 55. The generator object **must** then be reinitialized by means of the `unur_reinit` call.

Important: Currently not all methods allow reinitialization, see the description of the particular method (keyword *Reinit*).

Destroy

When you do not need your generator object any more, you should destroy it:

```
unur_free(generator);
```

Uniform random numbers

Each generator object can have its own uniform random number generator or share one with others. When created a parameter object the pointer for the uniform random number generator is set to the default generator. However, it can be changed at any time to any other generator:

```
unur_set_urng(par, urng);
```

or

```
unur_chg_urng(generator, urng);
```

respectively. See Chapter 6 [Using uniform random number generators], page 189, for details.

1.5 Contact the authors

If you have any problems with UNU.RAN, suggestions how to improve the library, or find a bug, please contact us via email unuran@statistik.wu-wien.ac.at.

For news please visit out homepage at http://statistik.wu-wien.ac.at/unuran/.

2 Examples

The examples in this chapter should compile cleanly and can be found in the directory '`examples`' of the source tree of UNU.RAN. Assuming that UNU.RAN as well as the PRNG libraries have been installed properly (see Section 1.2 [Installation], page 3) each of these can be compiled (using the GCC in this example) with

```
gcc -Wall -O2 -o example example.c -lunuran -lprng -lm
```

Remark: `-lprng` must be omitted when the PRNG library is not installed. Then however some of the examples might not work.

The library uses three objects: `UNUR_DISTR`, `UNUR_PAR` and `UNUR_GEN`. It is not important to understand the details of these objects but it is important not to changed the order of their creation. The distribution object can be destroyed *after* the generator object has been made. (The parameter object is freed automatically by the `unur_init` call.) It is also important to check the result of the `unur_init` call. If it has failed the `NULL` pointer is returned and causes a segmentation fault when used for sampling.

We give all examples with the UNU.RAN standard API and the more convenient string API.

2.1 As short as possible

Select a distribution and let UNU.RAN do all necessary steps.

```
/* ---------------------------------------------------------- */
/* File: example0.c                                           */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                              */
  double x;     /* will hold the random number                */

  /* Declare the three UNURAN objects.                        */
  UNUR_DISTR *distr;    /* distribution object                */
  UNUR_PAR   *par;      /* parameter object                   */
  UNUR_GEN   *gen;      /* generator object                   */

  /* Use a predefined standard distribution:                  */
  /*   Gaussian with mean zero and standard deviation 1.      */
  /*   Since this is the standard form of the distribution,   */
  /*   we need not give these parameters.                     */
  distr = unur_distr_normal(NULL, 0);

  /* Use method AUTO:                                         */
  /*   Let UNURAN select a suitable method for you.           */
  par = unur_auto_new(distr);

  /* Now you can change some of the default settings for the  */
  /* parameters of the chosen method. We don't do it here.    */

  /* Create the generator object.                             */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter   */
  /* object 'par' as a side effect.                           */

  /* It is important to check if the creation of the generator */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create */
  /* another generator object. If you do not need it any more, */
  /* it should be destroyed to free memory.                   */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from */
  /* the standard Gaussian distribution.                      */
  /* Eg.:                                                     */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you  */
  /* can destroy it.                                          */
  unur_free(gen);
```

```
  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

2.2 As short as possible (String API)

Select a distribution and let UNU.RAN do all necessary steps.

```
/* ------------------------------------------------------------ */
/* File: example0_str.c                                         */
/* ------------------------------------------------------------ */
/* String API.                                                  */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;     /* loop variable                                */
  double x;     /* will hold the random number                  */

  /* Declare UNURAN generator object.                           */
  UNUR_GEN   *gen;      /* generator object                     */

  /* Create the generator object.                               */
  /* Use a predefined standard distribution:                    */
  /*   Standard Gaussian distribution.                          */
  /* Use method AUTO:                                           */
  /*   Let UNURAN select a suitable method for you.             */
  gen = unur_str2gen("normal()");

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from  */
  /* the standard Gaussian distribution.                        */
  /* Eg.:                                                       */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

2.3 Select a method

Select method AROU and use it with default parameters.

```
/* ------------------------------------------------------------ */
/* File: example1.c                                             */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;     /* loop variable                                */
  double x;     /* will hold the random number                  */

  /* Declare the three UNURAN objects.                          */
  UNUR_DISTR *distr;    /* distribution object                  */
  UNUR_PAR   *par;      /* parameter object                     */
  UNUR_GEN   *gen;      /* generator object                     */

  /* Use a predefined standard distribution:                    */
  /*   Gaussian with mean zero and standard deviation 1.        */
  /*   Since this is the standard form of the distribution,     */
  /*   we need not give these parameters.                       */
  distr = unur_distr_normal(NULL, 0);

  /* Choose a method: AROU.                                     */
  /* For other (suitable) methods replace "arou" with the       */
  /* respective name (in lower case letters).                   */
  par = unur_arou_new(distr);

  /* Now you can change some of the default settings for the    */
  /* parameters of the chosen method. We don't do it here.      */

  /* Create the generator object.                               */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter     */
  /* object 'par' as a side effect.                             */

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create  */
  /* another generator object. If you do not need it any more,  */
  /* it should be destroyed to free memory.                     */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from  */
  /* the standard Gaussian distribution.                        */
  /* Eg.:                                                       */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
```

```
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------- */
```

2.4 Select a method (String API)

Select method AROU and use it with default parameters.

```
/* ------------------------------------------------------------ */
/* File: example1_str.c                                         */
/* ------------------------------------------------------------ */
/* String API.                                                  */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;     /* loop variable                                */
  double x;     /* will hold the random number                  */

  /* Declare UNURAN generator object.                           */
  UNUR_GEN   *gen;      /* generator object                     */

  /* Create the generator object.                               */
  /* Use a predefined standard distribution:                    */
  /*   Standard Gaussian distribution.                          */
  /* Choose a method: AROU.                                     */
  /*   For other (suitable) methods replace "arou" with the     */
  /*   respective name.                                         */
  gen = unur_str2gen("normal() & method=arou");

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from  */
  /* the standard Gaussian distribution.                        */
  /* Eg.:                                                       */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

2.5 Arbitrary distributions

If you want to sample from a non-standard distribution, UNU.RAN might be exactly what you need. Depending on the information is available, a method must be choosen for sampling, see Section 1.4 [Concepts], page 6 for an overview and Chapter 5 [Methods], page 87 for details.

```
/* ---------------------------------------------------------- */
/* File: example2.c                                           */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

/* In this example we build a distribution object from scratch   */
/* and sample from this distribution.                         */
/*                                                            */
/* We use method TDR (Transformed Density Rejection) which       */
/* required a PDF and the derivative of the PDF.              */

/* ---------------------------------------------------------- */

/* Define the PDF and dPDF of our distribution.               */
/*                                                            */
/* Our distribution has the PDF                               */
/*                                                            */
/*          /  1 - x*x  if |x| <= 1                           */
/*  f(x) = <                                                  */
/*          \  0        otherwise                             */
/*                                                            */

/* The PDF of our distribution:                               */
double mypdf( double x, const UNUR_DISTR *distr )
     /* The second argument (‘distr’) can be used for parameters */
     /* for the PDF. (We do not use parameters in our example.)  */
{
  if (fabs(x) >= 1.)
    return 0.;
  else
    return (1.-x*x);
} /* end of mypdf() */

/* The derivative of the PDF of our distribution:             */
double mydpdf( double x, const UNUR_DISTR *distr )
{
  if (fabs(x) >= 1.)
    return 0.;
  else
    return (-2.*x);
} /* end of mydpdf() */

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                              */
  double x;     /* will hold the random number                */

  /* Declare the three UNURAN objects.                        */
  UNUR_DISTR *distr;    /* distribution object                */
  UNUR_PAR   *par;      /* parameter object                   */
  UNUR_GEN   *gen;      /* generator object                   */

  /* Create a new distribution object from scratch.           */
```

```
  /* It is a continuous distribution, and we need a PDF and the  */
  /* derivative of the PDF. Moreover we set the domain.          */

  /* Get empty distribution object for a continuous distribution */
  distr = unur_distr_cont_new();

  /* Assign the PDF and dPDF (defined above).                    */
  unur_distr_cont_set_pdf( distr, mypdf );
  unur_distr_cont_set_dpdf( distr, mydpdf );

  /* Set the domain of the distribution (optional for TDR).      */
  unur_distr_cont_set_domain( distr, -1., 1. );

  /* Choose a method: TDR.                                       */
  par = unur_tdr_new(distr);

  /* Now you can change some of the default settings for the     */
  /* parameters of the chosen method. We don't do it here.       */

  /* Create the generator object.                                */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter      */
  /* object 'par' as a side effect.                              */

  /* It is important to check if the creation of the generator   */
  /* object was successful. Otherwise 'gen' is the NULL pointer  */
  /* and would cause a segmentation fault if used for sampling.  */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create   */
  /* another generator object. If you do not need it any more,   */
  /* it should be destroyed to free memory.                      */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

2.6 Arbitrary distributions (String API)

If you want to sample from a non-standard distribution, UNU.RAN might be exactly what you need. Depending on the information is available, a method must be choosen for sampling, see Section 1.4 [Concepts], page 6 for an overview and Chapter 5 [Methods], page 87 for details.

```
/* ------------------------------------------------------------ */
/* File: example2_str.c                                         */
/* ------------------------------------------------------------ */
/* String API.                                                  */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

/* In this example we use a generic distribution object         */
/* and sample from this distribution.                           */
/*                                                              */
/* The PDF of our distribution is given by                      */
/*                                                              */
/*          /  1 - x*x  if |x| <= 1                             */
/*  f(x) = <                                                    */
/*          \  0        otherwise                               */
/*                                                              */
/* We use method TDR (Transformed Density Rejection) which      */
/* required a PDF and the derivative of the PDF.                */

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;     /* loop variable                                */
  double x;     /* will hold the random number                  */

  /* Declare UNURAN generator object.                           */
  UNUR_GEN   *gen;      /* generator object                     */

  /* Create the generator object.                               */
  /* Use a generic continuous distribution.                     */
  /* Choose a method: TDR.                                      */
  gen = unur_str2gen(
        "distr = cont; pdf=\"1-x*x\"; domain=(-1,1) & method=tdr");

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from  */
  /* the distribution. Eg.:                                     */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
  unur_free(gen);

  exit (EXIT_SUCCESS);
```

```
} /* end of main() */

/* ------------------------------------------------------------ */
```

2.7 Change parameters of the method

Each method for generating random numbers allows several parameters to be modified. If you do not want to use default values, it is possible to change them. The following example illustrates how to change parameters. For details see Chapter 5 [Methods], page 87.

```
/* ------------------------------------------------------------ */
/* File: example3.c                                             */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;          /* loop variable                          */
  double x;          /* will hold the random number            */

  double fparams[2]; /* array for parameters for distribution  */

  /* Declare the three UNURAN objects.                          */
  UNUR_DISTR *distr;    /* distribution object                 */
  UNUR_PAR   *par;      /* parameter object                    */
  UNUR_GEN   *gen;      /* generator object                    */

  /* Use a predefined standard distribution:                    */
  /*   Gaussian with mean 2. and standard deviation 0.5.        */
  fparams[0] = 2.;
  fparams[1] = 0.5;
  distr = unur_distr_normal( fparams, 2 );

  /* Choose a method: TDR.                                      */
  par = unur_tdr_new(distr);

  /* Change some of the default parameters.                     */

  /* We want to use T(x)=log(x) for the transformation.         */
  unur_tdr_set_c( par, 0. );

  /* We want to have the variant with immediate acceptance.     */
  unur_tdr_set_variant_ia( par );

  /* We want to use 10 construction points for the setup        */
  unur_tdr_set_cpoints ( par, 10, NULL );

  /* Create the generator object.                               */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter     */
  /* object 'par' as a side effect.                             */

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create  */
  /* another generator object. If you do not need it any more,  */
  /* it should be destroyed to free memory.                     */
  unur_distr_free(distr);
```

```
  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* It is possible with method TDR to truncate the distribution */
  /* for an existing generator object ...                        */
  unur_tdr_chg_truncated( gen, -1., 0. );

  /* ... and sample again.                                       */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------------- */
```

2.8 Change parameters of the method (String API)

Each method for generating random numbers allows several parameters to be modified. If you do not want to use default values, it is possible to change them. The following example illustrates how to change parameters. For details see Chapter 5 [Methods], page 87.

```
/* ------------------------------------------------------------ */
/* File: example3_str.c                                            */
/* ------------------------------------------------------------ */
/* String API.                                                  */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;          /* loop variable                          */
  double x;          /* will hold the random number            */

  /* Declare UNURAN generator object.                           */
  UNUR_GEN   *gen;      /* generator object                    */

  /* Create the generator object.                               */
  /* Use a predefined standard distribution:                    */
  /*   Gaussian with mean 2. and standard deviation 0.5.        */
  /* Choose a method: TDR with parameters                       */
  /*   c = 0: use T(x)=log(x) for the transformation;           */
  /*   variant "immediate acceptance";                          */
  /*   number of construction points = 10.                      */
  gen = unur_str2gen(
       "normal(2,0.5) & method=tdr; c=0.; variant_ia; cpoints=10");

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from  */
  /* the distribution. Eg.:                                     */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* It is possible with method TDR to truncate the distribution */
  /* for an existing generator object ...                       */
  unur_tdr_chg_truncated( gen, -1., 0. );

  /* ... and sample again.                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
  unur_free(gen);

  exit (EXIT_SUCCESS);
```

```
} /* end of main() */

/* ------------------------------------------------------------ */
```

2.9 Change uniform random generator

All generator object use the same default uniform random number generator by default. This can be changed to any generator of your choice such that each generator object has its own random number generator or can share it with some other objects. It is also possible to change the default generator at any time. See Chapter 6 [Using uniform random number generators], page 189, for details.

The following example shows how the uniform random number generator can be set or changed for a generator object. It requires the RNGSTREAMS library to be installed and used. Otherwise the example must be modified accordingly.

```
/* ------------------------------------------------------------ */
/* File: example_rngstreams.c                                   */
/* ------------------------------------------------------------ */
#ifdef UNURAN_SUPPORTS_RNGSTREAM
/* ------------------------------------------------------------ */
/* This example makes use of the RNGSTREAM library for          */
/* for generating uniform random numbers.                       */
/* (see http://statmath.wu-wien.ac.at/software/RngStreams/)     */
/* To compile this example you must have set                    */
/*   ./configure --with-urng-rngstream                          */
/* (Of course the executable has to be linked against the       */
/* RNGSTREAM library.)                                          */
/* ------------------------------------------------------------ */

/* Include UNURAN header files.                                 */
#include <unuran.h>
#include <unuran_urng_rngstreams.h>

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;          /* loop variable                          */
  double x;          /* will hold the random number            */
  double fparams[2]; /* array for parameters for distribution  */

  /* Declare the three UNURAN objects.                          */
  UNUR_DISTR *distr;    /* distribution object                 */
  UNUR_PAR   *par;      /* parameter object                    */
  UNUR_GEN   *gen;      /* generator object                    */

  /* Declare objects for uniform random number generators.      */
  UNUR_URNG  *urng1, *urng2;    /* uniform generator objects    */

  /* The RNGSTREAMS library sets a package seed.                */
  unsigned long seed[] = {111u, 222u, 333u, 444u, 555u, 666u};
  RngStream_SetPackageSeed(seed);

  /* RngStreams only:                                           */
  /* Make a object for uniform random number generator.         */
  /* For details see                                            */
  /* http://statmath.wu-wien.ac.at/software/RngStreams/         */
  urng1 = unur_urng_rngstream_new("urng-1");
  if (urng1 == NULL) exit (EXIT_FAILURE);

  /* Use a predefined standard distribution:                    */
  /*   Beta with parameters 2 and 3.                            */
  fparams[0] = 2.;
  fparams[1] = 3.;
  distr = unur_distr_beta( fparams, 2 );

  /* Choose a method: TDR.                                      */
  par = unur_tdr_new(distr);
```

```
  /* Set uniform generator in parameter object                  */
  unur_set_urng( par, urng1 );

  /* Create the generator object.                               */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter     */
  /* object 'par' as a side effect.                             */

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create  */
  /* another generator object. If you do not need it any more,  */
  /* it should be destroyed to free memory.                     */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from  */
  /* the distribution. Eg.:                                     */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* Now we want to switch to a different (independent) stream  */
  /* of uniform random numbers.                                 */
  urng2 = unur_urng_rngstream_new("urng-2");
  if (urng2 == NULL) exit (EXIT_FAILURE);
  unur_chg_urng( gen, urng2 );

  /* ... and sample again.                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you    */
  /* can destroy it.                                            */
  unur_free(gen);

  /* We also should destroy the uniform random number generators.*/
  unur_urng_free(urng1);
  unur_urng_free(urng2);

  exit (EXIT_SUCCESS);
} /* end of main() */

/* ---------------------------------------------------------- */
#else
#include <stdio.h>
#include <stdlib.h>
int main(void) {
  printf("You must enable the RNGSTREAM library to run this example!\n\n");
  exit (77);    /* exit code for automake check routines */
}
#endif
/* ---------------------------------------------------------- */
```

2.10 Sample pairs of antithetic random variates

Using Method TDR it is easy to sample pairs of antithetic random variates.

```
/* ---------------------------------------------------------- */
/* File: example_anti.c                                       */
/* ---------------------------------------------------------- */
#ifdef UNURAN_SUPPORTS_PRNG
/* ---------------------------------------------------------- */
/* This example makes use of the PRNG library for generating  */
/* uniform random numbers.                                    */
/* (see http://statistik.wu-wien.ac.at/prng/)                 */
/* To compile this example you must have set                  */
/*   ./configure --with-urng-prng                             */
/* (Of course the executable has to be linked against the     */
/* PRNG library.)                                             */
/* ---------------------------------------------------------- */

/* Example how to sample from two streams of antithetic random   */
/* variates from Gaussian N(2,5) and Gamma(4) distribution, resp.*/

/* ---------------------------------------------------------- */

/* Include UNURAN header files.                               */
#include <unuran.h>
#include <unuran_urng_prng.h>

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;          /* loop variable                        */
  double xn, xg;     /* will hold the random number          */
  double fparams[2]; /* array for parameters for distribution    */

  /* Declare the three UNURAN objects.                        */
  UNUR_DISTR *distr;    /* distribution object               */
  UNUR_PAR   *par;      /* parameter object                  */
  UNUR_GEN   *gen_normal, *gen_gamma;
                        /* generator objects                 */

  /* Declare objects for uniform random number generators.    */
  UNUR_URNG  *urng1, *urng2;    /* uniform generator objects  */

  /* PRNG only:                                               */
  /* Make a object for uniform random number generator.       */
  /* For details see http://statistik.wu-wien.ac.at/prng/.    */


  /* The first generator: Gaussian N(2,5) */

  /* uniform generator: We use the Mersenne Twister.          */
  urng1 = unur_urng_prng_new("mt19937(1237)");
  if (urng1 == NULL) exit (EXIT_FAILURE);

  /* UNURAN generator object for N(2,5) */
  fparams[0] = 2.;
  fparams[1] = 5.;
  distr = unur_distr_normal( fparams, 2 );

  /* Choose method TDR with variant PS.                       */
  par = unur_tdr_new( distr );
  unur_tdr_set_variant_ps( par );

  /* Set uniform generator in parameter object.               */
  unur_set_urng( par, urng1 );
```

```
/* Set auxilliary uniform random number generator.             */
/* We use the default generator.                               */
unur_use_urng_aux_default( par );

/* Alternatively you can create and use your own auxilliary    */
/* uniform random number generator:                            */
/*    UNUR_URNG  *urng_aux;                                    */
/*    urng_aux = unur_urng_prng_new("tt800");                  */
/*    if (urng_aux == NULL) exit (EXIT_FAILURE);               */
/*    unur_set_urng_aux( par, urng_aux );                      */

/* Create the generator object.                                */
gen_normal = unur_init(par);
if (gen_normal == NULL) {
   fprintf(stderr, "ERROR: cannot create generator object\n");
   exit (EXIT_FAILURE);
}

/* Destroy distribution object (gen_normal has its own copy).  */
unur_distr_free(distr);


/* The second generator: Gamma(4) with antithetic variates.    */

/* uniform generator: We use the Mersenne Twister.             */
urng2 = unur_urng_prng_new("anti(mt19937(1237))");
if (urng2 == NULL) exit (EXIT_FAILURE);

/* UNURAN generator object for gamma(4) */
fparams[0] = 4.;
distr = unur_distr_gamma( fparams, 1 );

/* Choose method TDR with variant PS.                          */
par = unur_tdr_new( distr );
unur_tdr_set_variant_ps( par );

/* Set uniform generator in parameter object.                  */
unur_set_urng( par, urng2 );

/* Set auxilliary uniform random number generator.             */
/* We use the default generator.                               */
unur_use_urng_aux_default( par );

/* Alternatively you can create and use your own auxilliary    */
/* uniform random number generator (see above).                */
/* Notice that both generator objects gen_normal and           */
/* gen_gamma can share the same auxilliary URNG.               */

/* Create the generator object.                                */
gen_gamma = unur_init(par);
if (gen_gamma == NULL) {
   fprintf(stderr, "ERROR: cannot create generator object\n");
   exit (EXIT_FAILURE);
}

/* Destroy distribution object (gen_normal has its own copy).  */
unur_distr_free(distr);


/* Now we can sample pairs of negatively correlated random     */
/* variates. E.g.:                                             */
for (i=0; i<10; i++) {
  xn = unur_sample_cont(gen_normal);
  xg = unur_sample_cont(gen_gamma);
```

```
    printf("%g, %g\n",xn,xg);
  }


  /* When you do not need the generator objects any more, you    */
  /* can destroy it.                                             */
  unur_free(gen_normal);
  unur_free(gen_gamma);

  /* We also should destroy the uniform random number generators.*/
  unur_urng_free(urng1);
  unur_urng_free(urng2);

  exit (EXIT_SUCCESS);
} /* end of main() */

/* ---------------------------------------------------------------- */
#else
#include <stdio.h>
#include <stdlib.h>
int main(void) {
  printf("You must enable the PRNG library to run this example!\n\n");
  exit (77);    /* exit code for automake check routines */
}
#endif
/* ---------------------------------------------------------------- */
```

2.11 Sample pairs of antithetic random variates (String API)

Using Method TDR it is easy to sample pairs of antithetic random variates.

```
/* ---------------------------------------------------------- */
/* File: example_anti_str.c                                   */
/* ---------------------------------------------------------- */
/* String API.                                                */
/* ---------------------------------------------------------- */
#ifdef UNURAN_SUPPORTS_PRNG
/* ---------------------------------------------------------- */
/* This example makes use of the PRNG library for generating  */
/* uniform random numbers.                                    */
/* (see http://statistik.wu-wien.ac.at/prng/)                 */
/* To compile this example you must have set                  */
/*   ./configure --with-urng-prng                             */
/* (Of course the executable has to be linked against the     */
/* PRNG library.)                                             */
/* ---------------------------------------------------------- */

/* Example how to sample from two streams of antithetic random   */
/* variates from Gaussian N(2,5) and Gamma(4) distribution, resp.*/

/* ---------------------------------------------------------- */

/* Include UNURAN header files.                               */
#include <unuran.h>
#include <unuran_urng_prng.h>

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;          /* loop variable                         */
  double xn, xg;     /* will hold the random number           */

  /* Declare UNURAN generator objects.                        */
  UNUR_GEN   *gen_normal, *gen_gamma;

  /* PRNG only:                                               */
  /* Make a object for uniform random number generator.       */
  /* For details see http://statistik.wu-wien.ac.at/prng/.    */

  /* Create the first generator: Gaussian N(2,5)              */
  gen_normal = unur_str2gen("normal(2,5) & method=tdr; variant_ps & urng=mt19937(1237)");
  if (gen_normal == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }
  /* Set auxilliary uniform random number generator.          */
  /* We use the default generator.                            */
  unur_chgto_urng_aux_default(gen_normal);

  /* The second generator: Gamma(4) with antithetic variates.    */
  gen_gamma = unur_str2gen("gamma(4) & method=tdr; variant_ps & urng=anti(mt19937(1237))");
  if (gen_gamma == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }
  unur_chgto_urng_aux_default(gen_gamma);


  /* Now we can sample pairs of negatively correlated random  */
  /* variates. E.g.:                                          */
  for (i=0; i<10; i++) {
    xn = unur_sample_cont(gen_normal);
```

```
    xg = unur_sample_cont(gen_gamma);
    printf("%g, %g\n",xn,xg);
  }


  /* When you do not need the generator objects any more, you    */
  /* can destroy it.                                             */

  /* But first we have to destroy the uniform random number      */
  /* generators.                                                 */
  unur_urng_free(unur_get_urng(gen_normal));
  unur_urng_free(unur_get_urng(gen_gamma));

  unur_free(gen_normal);
  unur_free(gen_gamma);


  exit (EXIT_SUCCESS);
} /* end of main() */

/* ---------------------------------------------------------------- */
#else
#include <stdio.h>
#include <stdlib.h>
int main(void) {
  printf("You must enable the PRNG library to run this example!\n\n");
  exit (77);    /* exit code for automake check routines */
}
#endif
/* ---------------------------------------------------------------- */
```

2.12 More examples

See Section 5.3 [Methods for continuous univariate distributions], page 91.

See Section 5.4 [Methods for continuous empirical univariate distributions], page 139.

See Section 5.7 [Methods for continuous empirical multivariate distributions], page 163.

See Section 5.8 [Methods for discrete univariate distributions], page 166.

3 String Interface

The string interface (string API) provided by the `unur_str2gen` call is the easiest way to use UNU.RAN. This function takes a character string as its argument. The string is parsed and the information obtained is used to create a generator object. It returns `NULL` if this fails, either due to a syntax error, or due to invalid data. In both cases `unur_error` is set to the corresponding error codes (see Section 8.3 [Error reporting], page 217). Additionally there exists the call `unur_str2distr` that only produces a distribution object.

Notice that the string interface does not implement all features of the UNU.RAN library. For trickier tasks it might be necessary to use the UNU.RAN calls.

In Chapter 2 [Examples], page 13, all examples are given using both the UNU.RAN standard API and this convenient string API. The corresponding programm codes are equivalent.

Function reference

`UNUR_GEN* unur_str2gen` (*const* `char* string`)
: Get a generator object for the distribution, method and uniform random number generator as described in the given *string*. See Section 3.1 [Syntax of String Interface], page 37, for details.

`UNUR_DISTR* unur_str2distr` (*const* `char* string`)
: Get a distribution object for the distribution described in *string*. See Section 3.1 [Syntax of String Interface], page 37, and Section 3.2 [Distribution String], page 40, for details. However, only the block for the distribution object is allowed.

`UNUR_GEN* unur_makegen_ssu` (*const* `char* distrstr`, *const* `char* methodstr`, *UNUR_URNG** `urng`)
`UNUR_GEN* unur_makegen_dsu` (*const* `UNUR_DISTR* distribution`, *const* `char* methodstr`, *UNUR_URNG** `urng`)
: Make a generator object for the distribution, method and uniform random number generator. The distribution can be given either as string *distrstr* or as a distribution object *distr*. The method must be given as a string *methodstr*. For the syntax of these strings see Section 3.1 [Syntax of String Interface], page 37. However, the `method` keyword is optional for these calls and can be omitted. If *methodstr* is the empty (blank) string or `NULL` method AUTO is used. The uniform random number generator is optional. If *urng* is `NULL` then the default uniform random number generator is used.

3.1 Syntax of String Interface

The given string holds information about the requested distribution and (optional) about the sampling method and the uniform random number generator invoked. The interpretation of the string is not case-sensitive, all white spaces are ignored.

The string consists of up to three blocks, separated by ampersands `&`.

Each block consists of `<key>=<value>` pairs, separated by semicolons `;`.

The first key in each block is used to indicate each block. We have three different blocks with the following (first) keys:

`distr` definition of the distribution (see Section 3.2 [Distribution String], page 40).

`method` description of the transformation method (see Section 3.4 [Method String], page 46).

`urng` uniform random number generation (see Section 3.5 [Uniform RNG String], page 54).

The `distr` block must be the very first block and is obligatory. All the other blocks are optional and can be arranged in arbitrary order.

For details see the following description of each block.

In the following example

```
distr = normal(3.,0.75); domain = (0,inf) & method = tdr; c = 0
```

we have a distribution block for the truncated normal distribution with mean 3 and standard deviation 0.75 on domain (0,infinity); and block for choosing method TDR with parameter c set to 0.

The `<key>=<value>` pairs that follow the first (initial) pair in each block are used to set parameters. The name of the parameter is given by the `<key>` string. It is deduced from the UNU.RAN set calls by taking the part after `..._set_`. The `<value>` string holds the parameters to be set, separated by commata `,`. There are three types of parameters:

string `"..."`
: i.e. any sequence of characters enclosed by double quotes `"..."`,

list `(...,...)`
: i.e. list of *numbers*, separated by commata `,`, enclosed in parenthesis `(...)`, and

number
: a sequence of characters that is not enclosed by quotes `"..."` or parenthesis `(...)`. It is interpreted as float or integer depending on the type of the corresponding parameter.

The `<value>` string (including the character `=`) can be omitted when no argument is required.

At the moment not all `set` calls are supported. The syntax for the `<value>` can be directly derived from the corresponding `set` calls. To simplify the syntax additional shortcuts are possible. The following table lists the parameters for the `set` calls that are supported by the string interface; the entry in parenthesis gives the type of the argument as `<value>` string:

`int` *(number)*:
: The *number* is interpreted as an integer. `true` and `on` are transformed to `1`, `false` and `off` are transformed to `0`. A missing argument is interpreted as `1`.

`int, int` *(number, number* or *list)*:
: The two numbers or the first two entries in the list are interpreted as a integers. `inf` and `-inf` are transformed to `INT_MAX` and `INT_MIN` respectively, i.e. the largest and smallest integers that can be represented by the computer.

`unsigned` *(number)*:
: The *number* is interpreted as an unsigned hexadecimal integer.

`double` *(number)*:
: The number is interpreted as a floating point number. `inf` is transformed to `UNUR_INFINITY`.

`double, double` *(number, number* or *list)*:
: The two numbers or the first two entries in the list are interpreted as a floating point numbers. `inf` is transformed to `UNUR_INFINITY`. However using `inf` in the list might not work for all versions of C. Then it is recommended to use two single numbers instead of a list.

`int, double*` *([number,] list* or *number)*:
: – The list is interpreted as a double array. The (first) number as its length. If it is less than the actual size of the array only the first entries of the array are used.

- If only the list is given (i.e., if the first number is omitted), the first number is set to the actual size of the array.
- If only the number is given (i.e., if the list is omitted), the `NULL` pointer is used instead an array as argument.

`double*, int (list [,number])`:
: The list is interpreted as a double array. The (second) number as its length. If the length is omitted, it is replaced by the actual size of the array. (Only in the `distribution` block!)

`char* (string)`:
: The character string is passed as is to the corresponding set call.

Notice that missing entries in a list of numbers are interpreted as `0`. E.g, a the list `(1,,3)` is read as `(1,0,3)`, the list `(1,2,)` as `(1,2,0)`.

The the list of `key` strings in Section 3.2.1 [Keys for Distribution String], page 40, and Section 3.4.1 [Keys for Method String], page 46, for further details.

3.2 Distribution String

The `distr` block must be the very first block and is obligatory. For that reason the keyword `distr` is optional and can be omitted (together with the `=` character). Moreover it is ignored while parsing the string. However, to avoid some possible confusion it has to start with the letter `d` (if it is given at all).

The value of the `distr` key is used to get the distribution object, either via a `unur_distr_<value>` call for a standard distribution via a `unur_distr_<value>_new` call to get an object of a generic distribution. However not all generic distributions are supported yet.

The parameters for the standard distribution are given as a list. There must not be any character (other than white space) between the name of the standard distribution and the opening parenthesis `(` of this list. E.g., to get a beta distribution, use

```
distr = beta(2,4)
```

To get an object for a discrete distribution with probability vector (0.5,0.2,0.3), use

```
distr = discr; pv = (0.5,0.2,0.3)
```

It is also possible to set a PDF, PMF, or CDF using a string. E.g., to create a continuous distribution with PDF proportional to `exp(-sqrt(2+(x-1)^2) + (x-1))` and domain (0,inf) use

```
distr = cont; pdf = "exp(-sqrt(2+(x-1)^2) + (x-1))"
```

Notice: If this string is used in an `unur_str2distr` or `unur_str2gen` call the double quotes `"` must be protected by `\"`. Alternatively, single quotes may be used instead

```
distr = cont; pdf = 'exp(-sqrt(2+(x-1)^2) + (x-1))'
```

For the details of function strings see Section 3.3 [Function String], page 43.

3.2.1 Keys for Distribution String

List of standard distributions see Chapter 7 [Standard distributions], page 201

- `[distr =] beta(...)` ⇒ see Section 7.1.2 [beta], page 203
- `[distr =] binomial(...)` ⇒ see Section 7.3.1 [binomial], page 211
- `[distr =] cauchy(...)` ⇒ see Section 7.1.3 [cauchy], page 203
- `[distr =] chi(...)` ⇒ see Section 7.1.4 [chi], page 203
- `[distr =] chisquare(...)` ⇒ see Section 7.1.5 [chisquare], page 204
- `[distr =] exponential(...)` ⇒ see Section 7.1.6 [exponential], page 204
- `[distr =] extremeI(...)` ⇒ see Section 7.1.7 [extremeI], page 204
- `[distr =] extremeII(...)` ⇒ see Section 7.1.8 [extremeII], page 205
- `[distr =] F(...)` ⇒ see Section 7.1.1 [F], page 203
- `[distr =] gamma(...)` ⇒ see Section 7.1.9 [gamma], page 205
- `[distr =] geometric(...)` ⇒ see Section 7.3.2 [geometric], page 211
- `[distr =] hypergeometric(...)` ⇒ see Section 7.3.3 [hypergeometric], page 211
- `[distr =] laplace(...)` ⇒ see Section 7.1.10 [laplace], page 205
- `[distr =] logarithmic(...)` ⇒ see Section 7.3.4 [logarithmic], page 212
- `[distr =] logistic(...)` ⇒ see Section 7.1.11 [logistic], page 206
- `[distr =] lomax(...)` ⇒ see Section 7.1.12 [lomax], page 206
- `[distr =] negativebinomial(...)` ⇒ see Section 7.3.5 [negativebinomial], page 212
- `[distr =] normal(...)` ⇒ see Section 7.1.13 [normal], page 206
- `[distr =] pareto(...)` ⇒ see Section 7.1.14 [pareto], page 207
- `[distr =] poisson(...)` ⇒ see Section 7.3.6 [poisson], page 212
- `[distr =] powerexponential(...)` ⇒ see Section 7.1.15 [powerexponential], page 207
- `[distr =] rayleigh(...)` ⇒ see Section 7.1.16 [rayleigh], page 207

– `[distr =] student(...)` ⇒ see Section 7.1.17 [student], page 207
– `[distr =] triangular(...)` ⇒ see Section 7.1.18 [triangular], page 208
– `[distr =] uniform(...)` ⇒ see Section 7.1.19 [uniform], page 208
– `[distr =] weibull(...)` ⇒ see Section 7.1.20 [weibull], page 208

List of generic distributions see Chapter 4 [Handling Distribution Objects], page 55
– `[distr =] cemp` ⇒ see Section 4.4 [CEMP], page 69
– `[distr =] cont` ⇒ see Section 4.2 [CONT], page 59
– `[distr =] discr` ⇒ see Section 4.9 [DISCR], page 83

Notice: Order statistics for continuous distributions (see Section 4.3 [CORDER], page 66) are supported by using the key `orderstatistics` for distributions of type `CONT`.

List of keys that are available via the String API. For description see the corresponding UNU.RAN set calls.

- All distribution types

  `name = "<string>"`
  ⇒ see [unur_distr_set_name], page 57

- `cemp` *(Distribution Type)* (see Section 4.4 [CEMP], page 69)

  `data = (<list>) [, <int>]`
  ⇒ see [unur_distr_cemp_set_data], page 69

  `hist_bins = (<list>) [, <int>]`
  ⇒ see [unur_distr_cemp_set_hist_bins], page 70

  `hist_domain = <double>, <double> | (<list>)`
  ⇒ see [unur_distr_cemp_set_hist_domain], page 70

  `hist_prob = (<list>) [, <int>]`
  ⇒ see [unur_distr_cemp_set_hist_prob], page 69

- `cont` *(Distribution Type)* (see Section 4.2 [CONT], page 59)

  `cdf = "<string>"`
  ⇒ see [unur_distr_cont_set_cdfstr], page 61

  `center = <double>`
  ⇒ see [unur_distr_cont_set_center], page 64

  `domain = <double>, <double> | (<list>)`
  ⇒ see [unur_distr_cont_set_domain], page 62

  `hr = "<string>"`
  ⇒ see [unur_distr_cont_set_hrstr], page 63

  `logcdf = "<string>"`
  ⇒ see [unur_distr_cont_set_logcdfstr], page 62

  `logpdf = "<string>"`
  ⇒ see [unur_distr_cont_set_logpdfstr], page 62

  `mode = <double>`
  ⇒ see [unur_distr_cont_set_mode], page 64

`pdf = "<string>"`
⇒ see [unur_distr_cont_set_pdfstr], page 61

`pdfarea = <double>`
⇒ see [unur_distr_cont_set_pdfarea], page 64

`pdfparams = (<list>) [, <int>]`
⇒ see [unur_distr_cont_set_pdfparams], page 61

`orderstatistics = <int>, <int> | (<list>)`
Make order statistics for given distribution. The first parameter gives the sample size, the second parameter its rank. (see see [unur_distr_corder_new], page 66)

- `discr` *(Distribution Type)* (see Section 4.9 [DISCR], page 83)

`cdf = "<string>"`
⇒ see [unur_distr_discr_set_cdfstr], page 85

`domain = <int>, <int> | (<list>)`
⇒ see [unur_distr_discr_set_domain], page 85

`mode [= <int>]`
⇒ see [unur_distr_discr_set_mode], page 86

`pmf = "<string>"`
⇒ see [unur_distr_discr_set_pmfstr], page 84

`pmfparams = (<list>) [, <int>]`
⇒ see [unur_distr_discr_set_pmfparams], page 85

`pmfsum = <double>`
⇒ see [unur_distr_discr_set_pmfsum], page 86

`pv = (<list>) [, <int>]`
⇒ see [unur_distr_discr_set_pv], page 83

3.3 Function String

In unuran it is also possible to define functions (e.g. CDF or PDF) as strings. As you can see in Example 2 (Section 2.6 [Example_2_str], page 22) it is very easy to define the PDF of a distribution object by means of a string. The possibilities using this string interface are more restricted than using a pointer to a routine coded in C (Section 2.5 [Example_2], page 20). But the differences in evaluation time is small. When a distribution object is defined using this string interface then of course the same conditions on the given density or CDF must be satisfied for a chosen method as for the standard API. This string interface can be used for both within the UNU.RAN string API using the `unur_str2gen` call, and for calls that define the density or CDF for a particular distribution object as done with (e.g.) the call `unur_distr_cont_set_pdfstr`. Here is an example for the latter case:

```
unur_distr_cont_set_pdfstr(distr,"1-x*x");
```

Syntax

The syntax for the function string is case insensitive, white spaces are ingnored. The expressions are similar to most programming languages and mathematical programs (see also the examples below). It is especially influenced by C. The usual preceedence rules are used (from highest to lowest preceedence: functions, power, multiplication, addition, relation operators). Use parentheses in case of doubt or when these preceedences should be changed.

Relation operators can be used as indicator functions, i.e. the term `(x>1)` is evaluted as `1` if this relation is satisfied, and as `0` otherwise.

The first unknown symbol (letter or word) is interpreted as the variable of the function. It is recommended to use `x`. Only one variable can be used.

Important: The symbol `e` is used twice, for Euler's constant (= 2.7182...) and as exponent. The multiplication operator `*` must not be omitted, i.e. `2 x` is interpreted as the string `2x` (which will result in a syntax error).

List of symbols

Numbers

Numbers are composed using digits and, optionally, a sign, a decimal point, and an exponent indicated by `e`.

Symbol	Explanation	Examples
`0...9`	*digits*	`2343`
`.`	*decimal point*	`165.567`
`-`	*negative sign*	`-465.223`
`e`	*exponet*	`13.2e-4` (=0.00132)

Constants

`pi`	*pi = 3.1415...*	`3*pi+2`
`e`	*Euler's constant*	`3*e+2` (= 10.15...; do not cofuse with `3e2` = 300)
`inf`	*infinity*	(used for domains)

Special symbols

`(`	*opening parenthesis*	`2*(3+x)`
`)`	*closing parenthesis*	`2*(3+x)`
`,`	*(argument) list separator*	`mod(13,2)`

Relation operators (Indicator functions)

`<`	*less than*	`(x<1)`
`=`	*equal*	`(2=x)`
`==`	*same as* `=`	`(x==3)`
`>`	*greater than*	`(x>0)`
`<=`	*less than or equal*	`(x<=1)`
`!=`	*not equal*	`(x!0)`
`<>`	*same as* `!=`	`(x<>pi)`
`>=`	*greater or equal*	`(x>=1)`

Arithmetic operators

`+`	*addition*	`2+x`
`-`	*subtraction*	`2-x`
`*`	*multiplication*	`2*x`
`/`	*division*	`x/2`
`^`	*power*	`x^2`

Functions

`mod`	`mod(m,n)` *remainder of devision m over n*	mod(x,2)
`exp`	*exponential function (same as* `e^x`*)*	`exp(-x^2)` (same as `e^(-x^2)`)
`log`	*natural logarithm*	`log(x)`
`sin`	*sine*	`sin(x)`
`cos`	*cosine*	`cos(x)`
`tan`	*tangent*	`tan(x)`
`sec`	*secant*	`sec(x*2)`
`sqrt`	*square root*	`sqrt(2*x)`
`abs`	*absolute value*	`abs(x)`
`sgn`	*sign function*	`sign(x)*3`

Variable		
`x`	*variable*	`3*x^2`

Examples

```
1.231+7.9876*x-1.234e-3*x^2+3.335e-5*x^3
```

```
sin(2*pi*x)+x^2
```

```
exp(-((x-3)/2.1)^2)
```

It is also possible to define functions using different terms on separate domains. However, instead of constructs using `if ... then ... else ...` indicator functions are available.

For example to define the density of triangular distribution with domain (-1,1) and mode 0 use

```
(x>-1)*(x<0)*(1+x) + (x>=0)*(x<1)*(1-x)
```

3.4 Method String

The key `method` is obligatory, it must be the first key and its value is the name of a method suitable for the choosen standard distribution. E.g., if method AROU is chosen, use

```
method = arou
```

Of course the all following keys dependend on the method choosen at first. All corresponding `set` calls of UNU.RAN are available and the key is the string after the `unur_<methodname>_set_` part of the command. E.g., UNU.RAN provides the command `unur_arou_set_max_sqhratio` to set a parameter of method AROU. To call this function via the string-interface, the key `max_sqhratio` can be used:

```
max_sqhratio = 0.9
```

Additionally the keyword `debug` can be used to set debugging flags (see Section 8.2 [Debugging], page 215, for details).

If this block is omitted, a suitable default method is used. Notice however that the default method may change in future versions of UNU.RAN.

3.4.1 Keys for Method String

List of methods and keys that are available via the String API. For description see the corresponding UNU.RAN set calls.

- `method = arou` ⇒ `unur_arou_new` (see Section 5.3.1 [AROU], page 95)

  `cpoints = <int> [, (<list>)] | (<list>)`
  ⇒ see [unur_arou_set_cpoints], page 96

  `darsfactor = <double>`
  ⇒ see [unur_arou_set_darsfactor], page 95

  `guidefactor = <double>`
  ⇒ see [unur_arou_set_guidefactor], page 96

  `max_segments [= <int>]`
  ⇒ see [unur_arou_set_max_segments], page 96

  `max_sqhratio = <double>`
  ⇒ see [unur_arou_set_max_sqhratio], page 95

  `pedantic [= <int>]`
  ⇒ see [unur_arou_set_pedantic], page 96

  `usecenter [= <int>]`
  ⇒ see [unur_arou_set_usecenter], page 96

  `usedars [= <int>]`
  ⇒ see [unur_arou_set_usedars], page 95

  `verify [= <int>]`
  ⇒ see [unur_arou_set_verify], page 96

- `method = ars` ⇒ `unur_ars_new` (see Section 5.3.2 [ARS], page 98)

  `cpoints = <int> [, (<list>)] | (<list>)`
  ⇒ see [unur_ars_set_cpoints], page 98

  `max_intervals [= <int>]`
  ⇒ see [unur_ars_set_max_intervals], page 98

  `pedantic [= <int>]`
  ⇒ see [unur_ars_set_pedantic], page 99

`reinit_ncpoints [= <int>]`
⇒ see [unur_ars_set_reinit_ncpoints], page 99

`reinit_percentiles = <int> [, (<list>)] | (<list>)`
⇒ see [unur_ars_set_reinit_percentiles], page 99

`verify [= <int>]`
⇒ see [unur_ars_set_verify], page 99

- `method = auto` ⇒ `unur_auto_new` (see Section 5.2 [AUTO], page 90)

  `logss [= <int>]`
  ⇒ see [unur_auto_set_logss], page 90

- `method = cstd` ⇒ `unur_cstd_new` (see Section 5.3.4 [CSTD], page 105)

  `variant = <unsigned>`
  ⇒ see [unur_cstd_set_variant], page 105

- `method = dari` ⇒ `unur_dari_new` (see Section 5.8.1 [DARI], page 169)

  `cpfactor = <double>`
  ⇒ see [unur_dari_set_cpfactor], page 170

  `squeeze [= <int>]`
  ⇒ see [unur_dari_set_squeeze], page 169

  `tablesize [= <int>]`
  ⇒ see [unur_dari_set_tablesize], page 169

  `verify [= <int>]`
  ⇒ see [unur_dari_set_verify], page 170

- `method = dau` ⇒ `unur_dau_new` (see Section 5.8.2 [DAU], page 171)

  `urnfactor = <double>`
  ⇒ see [unur_dau_set_urnfactor], page 171

- `method = dgt` ⇒ `unur_dgt_new` (see Section 5.8.4 [DGT], page 176)

  `guidefactor = <double>`
  ⇒ see [unur_dgt_set_guidefactor], page 176

  `variant = <unsigned>`
  ⇒ see [unur_dgt_set_variant], page 177

- `method = dsrou` ⇒ `unur_dsrou_new` (see Section 5.8.5 [DSROU], page 178)

  `cdfatmode = <double>`
  ⇒ see [unur_dsrou_set_cdfatmode], page 178

  `verify [= <int>]`
  ⇒ see [unur_dsrou_set_verify], page 178

- `method = dstd` ⇒ `unur_dstd_new` (see Section 5.8.7 [DSTD], page 181)

  `variant = <unsigned>`
  ⇒ see [unur_dstd_set_variant], page 181

- `method = empk` ⇒ `unur_empk_new` (see Section 5.4.1 [EMPK], page 142)

  `beta = <double>`
  ⇒ see [unur_empk_set_beta], page 144

  `kernel = <unsigned>`
  ⇒ see [unur_empk_set_kernel], page 143

  `positive [= <int>]`
  ⇒ see [unur_empk_set_positive], page 144

  `smoothing = <double>`
  ⇒ see [unur_empk_set_smoothing], page 144

  `varcor [= <int>]`
  ⇒ see [unur_empk_set_varcor], page 144

- `method = gibbs` ⇒ `unur_gibbs_new` (see Section 5.6.1 [GIBBS], page 155)

  `burnin [= <int>]`
  ⇒ see [unur_gibbs_set_burnin], page 157

  `c = <double>`
  ⇒ see [unur_gibbs_set_c], page 156

  `thinning [= <int>]`
  ⇒ see [unur_gibbs_set_thinning], page 156

  `variant_coordinate`
  ⇒ see [unur_gibbs_set_variant_coordinate], page 156

  `variant_random_direction`
  ⇒ see [unur_gibbs_set_variant_random_direction], page 156

- `method = hinv` ⇒ `unur_hinv_new` (see Section 5.3.5 [HINV], page 107)

  `boundary = <double>, <double> | (<list>)`
  ⇒ see [unur_hinv_set_boundary], page 109

  `cpoints = (<list>), <int>`
  ⇒ see [unur_hinv_set_cpoints], page 108

  `guidefactor = <double>`
  ⇒ see [unur_hinv_set_guidefactor], page 109

  `max_intervals [= <int>]`
  ⇒ see [unur_hinv_set_max_intervals], page 109

  `order [= <int>]`
  ⇒ see [unur_hinv_set_order], page 108

  `u_resolution = <double>`
  ⇒ see [unur_hinv_set_u_resolution], page 108

- `method = hitro` ⇒ `unur_hitro_new` (see Section 5.6.2 [HITRO], page 158)

  `adaptive_multiplier = <double>`
  ⇒ see [unur_hitro_set_adaptive_multiplier], page 161

  `burnin [= <int>]`
  ⇒ see [unur_hitro_set_burnin], page 162

`r = <double>`
⇒ see [unur_hitro_set_r], page 161

`thinning [= <int>]`
⇒ see [unur_hitro_set_thinning], page 162

`use_adaptiveline [= <int>]`
⇒ see [unur_hitro_set_use_adaptiveline], page 160

`use_adaptiverectangle [= <int>]`
⇒ see [unur_hitro_set_use_adaptiverectangle], page 161

`use_boundingrectangle [= <int>]`
⇒ see [unur_hitro_set_use_boundingrectangle], page 160

`v = <double>`
⇒ see [unur_hitro_set_v], page 161

`variant_coordinate`
⇒ see [unur_hitro_set_variant_coordinate], page 160

`variant_random_direction`
⇒ see [unur_hitro_set_variant_random_direction], page 160

- `method = hrb` ⇒ `unur_hrb_new` (see Section 5.3.6 [HRB], page 111)

  `upperbound = <double>`
  ⇒ see [unur_hrb_set_upperbound], page 111

  `verify [= <int>]`
  ⇒ see [unur_hrb_set_verify], page 111

- `method = hrd` ⇒ `unur_hrd_new` (see Section 5.3.7 [HRD], page 112)

  `verify [= <int>]`
  ⇒ see [unur_hrd_set_verify], page 112

- `method = hri` ⇒ `unur_hri_new` (see Section 5.3.8 [HRI], page 113)

  `p0 = <double>`
  ⇒ see [unur_hri_set_p0], page 113

  `verify [= <int>]`
  ⇒ see [unur_hri_set_verify], page 113

- `method = itdr` ⇒ `unur_itdr_new` (see Section 5.3.9 [ITDR], page 115)

  `cp = <double>`
  ⇒ see [unur_itdr_set_cp], page 116

  `ct = <double>`
  ⇒ see [unur_itdr_set_ct], page 116

  `verify [= <int>]`
  ⇒ see [unur_itdr_set_verify], page 116

  `xi = <double>`
  ⇒ see [unur_itdr_set_xi], page 116

- `method = mvtdr` ⇒ `unur_mvtdr_new` (see Section 5.5.1 [MVTDR], page 148)

`boundsplitting = <double>`
⇒ see [unur_mvtdr_set_boundsplitting], page 149

`maxcones [= <int>]`
⇒ see [unur_mvtdr_set_maxcones], page 149

`stepsmin [= <int>]`
⇒ see [unur_mvtdr_set_stepsmin], page 149

`verify [= <int>]`
⇒ see [unur_mvtdr_set_verify], page 149

- `method = ninv` ⇒ `unur_ninv_new` (see Section 5.3.10 [NINV], page 117)

`max_iter [= <int>]`
⇒ see [unur_ninv_set_max_iter], page 118

`start = <double>, <double> | (<list>)`
⇒ see [unur_ninv_set_start], page 118

`table [= <int>]`
⇒ see [unur_ninv_set_table], page 118

`usenewton`
⇒ see [unur_ninv_set_usenewton], page 118

`useregula`
⇒ see [unur_ninv_set_useregula], page 118

`x_resolution = <double>`
⇒ see [unur_ninv_set_x_resolution], page 118

- `method = nrou` ⇒ `unur_nrou_new` (see Section 5.3.11 [NROU], page 120)

`center = <double>`
⇒ see [unur_nrou_set_center], page 121

`r = <double>`
⇒ see [unur_nrou_set_r], page 121

`u = <double>, <double> | (<list>)`
⇒ see [unur_nrou_set_u], page 121

`v = <double>`
⇒ see [unur_nrou_set_v], page 121

`verify [= <int>]`
⇒ see [unur_nrou_set_verify], page 121

- `method = srou` ⇒ `unur_srou_new` (see Section 5.3.12 [SROU], page 122)

`cdfatmode = <double>`
⇒ see [unur_srou_set_cdfatmode], page 123

`pdfatmode = <double>`
⇒ see [unur_srou_set_pdfatmode], page 123

`r = <double>`
⇒ see [unur_srou_set_r], page 123

`usemirror [= <int>]`
⇒ see [unur_srou_set_usemirror], page 123

`usesqueeze [= <int>]`
⇒ see [unur_srou_set_usesqueeze], page 123

`verify [= <int>]`
⇒ see [unur_srou_set_verify], page 124

- `method = ssr` ⇒ `unur_ssr_new` (see Section 5.3.13 [SSR], page 125)

  `cdfatmode = <double>`
  ⇒ see [unur_ssr_set_cdfatmode], page 126

  `pdfatmode = <double>`
  ⇒ see [unur_ssr_set_pdfatmode], page 126

  `usesqueeze [= <int>]`
  ⇒ see [unur_ssr_set_usesqueeze], page 126

  `verify [= <int>]`
  ⇒ see [unur_ssr_set_verify], page 126

- `method = tabl` ⇒ `unur_tabl_new` (see Section 5.3.14 [TABL], page 127)

  `areafraction = <double>`
  ⇒ see [unur_tabl_set_areafraction], page 129

  `boundary = <double>, <double> | (<list>)`
  ⇒ see [unur_tabl_set_boundary], page 130

  `cpoints = <int> [, (<list>)] | (<list>)`
  ⇒ see [unur_tabl_set_cpoints], page 128

  `darsfactor = <double>`
  ⇒ see [unur_tabl_set_darsfactor], page 129

  `guidefactor = <double>`
  ⇒ see [unur_tabl_set_guidefactor], page 130

  `max_intervals [= <int>]`
  ⇒ see [unur_tabl_set_max_intervals], page 130

  `max_sqhratio = <double>`
  ⇒ see [unur_tabl_set_max_sqhratio], page 129

  `nstp [= <int>]`
  ⇒ see [unur_tabl_set_nstp], page 128

  `pedantic [= <int>]`
  ⇒ see [unur_tabl_set_pedantic], page 131

  `slopes = (<list>), <int>`
  ⇒ see [unur_tabl_set_slopes], page 130

  `usedars [= <int>]`
  ⇒ see [unur_tabl_set_usedars], page 129

  `useear [= <int>]`
  ⇒ see [unur_tabl_set_useear], page 128

  `variant_ia [= <int>]`
  ⇒ see [unur_tabl_set_variant_ia], page 128

  `variant_splitmode = <unsigned>`
  ⇒ see [unur_tabl_set_variant_splitmode], page 129

`verify [= <int>]`
⇒ see [unur_tabl_set_verify], page 131

- `method = tdr` ⇒ `unur_tdr_new` (see Section 5.3.15 [TDR], page 132)

`c = <double>`
⇒ see [unur_tdr_set_c], page 133

`cpoints = <int> [, (<list>)] | (<list>)`
⇒ see [unur_tdr_set_cpoints], page 134

`darsfactor = <double>`
⇒ see [unur_tdr_set_darsfactor], page 134

`guidefactor = <double>`
⇒ see [unur_tdr_set_guidefactor], page 135

`max_intervals [= <int>]`
⇒ see [unur_tdr_set_max_intervals], page 135

`max_sqhratio = <double>`
⇒ see [unur_tdr_set_max_sqhratio], page 135

`pedantic [= <int>]`
⇒ see [unur_tdr_set_pedantic], page 136

`reinit_ncpoints [= <int>]`
⇒ see [unur_tdr_set_reinit_ncpoints], page 134

`reinit_percentiles = <int> [, (<list>)] | (<list>)`
⇒ see [unur_tdr_set_reinit_percentiles], page 134

`usecenter [= <int>]`
⇒ see [unur_tdr_set_usecenter], page 135

`usedars [= <int>]`
⇒ see [unur_tdr_set_usedars], page 133

`usemode [= <int>]`
⇒ see [unur_tdr_set_usemode], page 135

`variant_gw`
⇒ see [unur_tdr_set_variant_gw], page 133

`variant_ia`
⇒ see [unur_tdr_set_variant_ia], page 133

`variant_ps`
⇒ see [unur_tdr_set_variant_ps], page 133

`verify [= <int>]`
⇒ see [unur_tdr_set_verify], page 136

- `method = utdr` ⇒ `unur_utdr_new` (see Section 5.3.16 [UTDR], page 137)

`cpfactor = <double>`
⇒ see [unur_utdr_set_cpfactor], page 137

`deltafactor = <double>`
⇒ see [unur_utdr_set_deltafactor], page 137

`pdfatmode = <double>`
⇒ see [unur_utdr_set_pdfatmode], page 137

`verify [= <int>]`
⇒ see [unur_utdr_set_verify], page 138

- `method = vempk` ⇒ `unur_vempk_new` (see Section 5.7.1 [VEMPK], page 165)

  `smoothing = <double>`
  ⇒ see [unur_vempk_set_smoothing], page 165

  `varcor [= <int>]`
  ⇒ see [unur_vempk_set_varcor], page 165

- `method = vnrou` ⇒ `unur_vnrou_new` (see Section 5.5.3 [VNROU], page 151)

  `r = <double>`
  ⇒ see [unur_vnrou_set_r], page 152

  `v = <double>`
  ⇒ see [unur_vnrou_set_v], page 152

  `verify [= <int>]`
  ⇒ see [unur_vnrou_set_verify], page 152

3.5 Uniform RNG String

The value of the `urng` key is passed to the PRNG interface (see PRNG manual for details). However it only works when using the PRNG library is enabled, see Section 1.2 [Installation], page 3 for details. There are no other keys.

IMPORTANT: UNU.RAN creates a new uniform random number generator for the generator object. The pointer to this uniform generator has to be read and saved via a `unur_get_urng` call in order to clear the memory *before* the UNU.RAN generator object is destroyed.

If this block is omitted the UNU.RAN default generator is used (which *must not* be destroyed).

4 Handling distribution objects

Objects of type `UNUR_DISTR` are used for handling distributions. All data about a distribution are stored in this object. UNU.RAN provides functions that return instances of such objects for standard distributions (see Chapter 7 [Standard distributions], page 201). It is then possible to change these distribution objects by various set calls. Moreover, it is possible to build a distribution object entirely from scratch. For this purpose there exists `unur_distr_<type>_new` calls that return an empty object of this type for each object type (eg. univariate contiuous) which can be filled with the appropriate set calls.

UNU.RAN distinguishes between several types of distributions, each of which has its own sets of possible parameters (for details see the corresponding sections):

- continuous univariate distributions
- continuous univariate order statistics
- continuous empirical univariate distributions
- continuous multivariate distributions
- continuous empirical multivariate distributions
- matrix distributions
- discrete univariate distributions

Notice that there are essential data about a distribution, eg. the PDF, a list of (shape, scale, location) parameters for the distribution, and the domain of (the possibly truncated) distribution. And there exist parameters that are/can be derived from these, eg. the mode of the distribution or the area below the given PDF (which need not be normalized for many methods). UNU.RAN keeps track of parameters which are known. Thus if one of the essential parameters is changed all derived parameters are marked as unknown and must be set again if these are required for the chosen generation method. Additionally to set calls there are calls for updating derived parameters for objects provided by the UNU.RAN library of standard distributions (one for each parameter to avoid computational overhead since not all parameters are required for all generator methods).

All parameters of distribution objects can be read by corresponding get calls.

Every generator object has its own copy of a distribution object which is accessible by a `unur_get_distr` call. Thus the parameter for this distribution can be read. However, **never** extract the distribution object out of a generator object and run one of the set calls on it to modify the distribution. (How should the poor generator object know what has happend?) Instead there exist calls for each of the generator methods that change particular parameters of the internal copy of the distribution object.

How To Use

UNU.RAN collects all data required for a particular generation method in a *distribution object*. There are two ways to get an instance of a distributions object:

1. Build a distribtion from scratch, by means of the corresponding `unur_distr_<type>_new` call, where `<type>` is the type of the distribution as listed in the below subsections.
2. Use the corresponding `unur_distr_<name>_new` call to get prebuild distribution from the UNU.RAN library of standard distributions. Here `<name>` is the name of the standard distribution in Chapter 7 [Standard distributions], page 201.

In either cases the corresponding `unur_distr_<type>_set_<param>` calls to set the necessary parameters `<param>` (case 1), or change the values of the standard distribution in case 2 (if this makes sense for you). In the latter case `<type>` is the type to which the standard distribution

belongs to. These `set` calls return `UNUR_SUCCESS` when the correspondig parameter has been set successfully. Otherwise an error code is returned.

The parameters of a distribution are divided into *essential* and *derived* parameters.

Notice, that there are some restrictions in setting parameters to avoid possible confusions. Changing essential parameters marks derived parameters as `unknown`. Some of the parameters cannot be changed any more when already set; some parameters block each others. In such a case a new instance of a distribution object has to be build.

Additionally `unur_distr_<type>_upd_<param>` calls can be used for updating derived parameters for objects provided by the UNU.RAN library of standard distributions.

All parameters of a distribution object get be read by means of `unur_distr_<type>_get_<param>` calls.

Every distribution object be identified by its `name` which is a string of arbitrary characters provided by the user. For standard distribution it is automatically set to `<name>` in the corresponding `new` call. It can be changed to any other string.

4.1 Functions for all kinds of distribution objects

The calls in this section can be applied to all distribution objects.

- Destroy `free` an instance of a generator object.
- Ask for the `type` of a generator object.
- Ask for the `dimension` of a generator object.
- Deal with the `name` (identifier string) of a generator object.

Function reference

`void unur_distr_free` (*UNUR_DISTR* distribution*)
Destroy the *distribution* object.

`int unur_distr_set_name` (*UNUR_DISTR* distribution*, *const* `char*` *name*)
`const char* unur_distr_get_name` (*const UNUR_DISTR* distribution*)
Set and get *name* of *distribution*. The *name* can be an arbitrary character string. It can be used to identify generator objects for the user. It is used by UNU.RAN when printing information of the distribution object into a log files.

`int unur_distr_get_dim` (*const UNUR_DISTR* distribution*)
Get number of components of a random vector (its dimension) the *distribution*.

For univariate distributions it returns dimension `1`.

For matrix distributions it returns the number of components (i.e., number of rows times number of columns). When the respective numbers of rows and columns are needed use `unur_distr_matr_get_dim` instead.

`unsigned int unur_distr_get_type` (*const UNUR_DISTR* distribution*)
Get type of *distribution*. Possible types are

`UNUR_DISTR_CONT`
: univariate continuous distribution

`UNUR_DISTR_CEMP`
: empirical continuous univariate distribution (i.e. a sample)

`UNUR_DISTR_CVEC`
: continuous mulitvariate distribution

`UNUR_DISTR_CVEMP`
: empirical continuous multivariate distribution (i.e. a vector sample)

`UNUR_DISTR_DISCR`
: discrete univariate distribution

`UNUR_DISTR_MATR`
: matrix distribution

Alternatively the `unur_distr_is_<TYPE>` calls can be used.

`int unur_distr_is_cont` (*const UNUR_DISTR* distribution*)
`TRUE` if *distribution* is a continuous univariate distribution.

`int unur_distr_is_cvec` (*const UNUR_DISTR* distribution*)
`TRUE` if *distribution* is a continuous multivariate distribution.

`int unur_distr_is_cemp` (*const* `UNUR_DISTR* distribution`)
`TRUE` if *distribution* is an empirical continuous univariate distribution, i.e. a sample.

`int unur_distr_is_cvemp` (*const* `UNUR_DISTR* distribution`)
`TRUE` if *distribution* is an empirical continuous multivariate distribution.

`int unur_distr_is_discr` (*const* `UNUR_DISTR* distribution`)
`TRUE` if *distribution* is a discrete univariate distribution.

`int unur_distr_is_matr` (*const* `UNUR_DISTR* distribution`)
`TRUE` if *distribution* is a matrix distribution.

`int unur_distr_set_extobj` (*UNUR_DISTR** `distribution`, *const* `void* extobj`)
Store a pointer to an external object. This might be usefull if the PDF, PMF, CDF or other functions used to implement a particular distribution a parameter set that cannot be stored as doubles (e.g. pointers to some structure that holds information of the distribution).

Important: When UNU.RAN copies this distribution object into the generator object, then the address *extobj* that this pointer contains is simply copied. Thus the generator holds an address of a non-private object! Once the generator object has been created any change in the external object might effect the generator object.

Warning: External objects must be used with care. Once the generator object has been created or the distribution object has been copied you *must not* destroy this external object.

`const void* unur_distr_get_extobj` (*const* `UNUR_DISTR* distribution`)
Get the pointer to the external object.

Important: Changing this object must be done with with extreme care.

4.2 Continuous univariate distributions

The calls in this section can be applied to continuous univariate distributions.

- Create a `new` instance of a continuous univariate distribution.
- Handle and evaluate distribution function (CDF, `cdf`), probability density function (PDF, `pdf`) and the derivative of the density function (`dpdf`). The following is important:
  - `pdf` need not be normalized, i.e., any integrable nonnegative function can be used.
  - `dpdf` must the derivate of the function provided as `pdf`.
  - `cdf` must be a distribution function, i.e. it must be monotonically increasing with range [0,1].
  - If `cdf` and `pdf` are used together for a pariticular generation method, then `pdf` must be the derivate of the `cdf`, i.e., it must be normalized.
- Handle and evaluate the logarithm of the probability density function (logPDF, `logpdf`) and the derivative of the logarithm of the density function (`dlogpdf`).

  Some methods use the logarithm of the density if available.
- Set (and change) parameters (`pdfparams`) and the area below the graph (`pdfarea`) of the given density.
- Set the `mode` (or pole) of the distribution.
- Set the `center` of the distribution. It is used by some generation methods to adjust the parameters of the generation algorithms to gain better performance. It can be seens as the location of the "central part" of the distribution.
- Some generation methods require the hazard rate (`hr`) of the distribution instead of its `pdf`.
- Alternatively, `cdf`, `pdf`, `dpdf`, and `hr` can be provided as `str`ings instead of function pointers.
- Set the `domain` of the distribution. Notice that the library also can handle truncated distributions, i.e., distributions that are derived from (standard) distributions by simply restricting its domain to a subset. However, there is a subtle difference between changing the domain of a distribution object by a `unur_distr_cont_set_domain` call and changing the (truncated) domain for an existing generator object. The domain of the distribution object is used to create the generator object with hats, squeezes, tables, etc. Whereas truncating the domain of an existing generator object need not necessarily require a recomputation of these data. Thus by a `unur_<method>_chg_truncated` call (if available) the sampling region is restricted to the subset of the domain of the given distribution object. However, generation methods that require a recreation of the generator object when the domain is changed have a `unur_<method>_chg_domain` call instead. For these calls there are of course no restrictions on the given domain (i.e., it is possible to increase the domain of the distribution) (see Chapter 5 [Methods], page 87, for details).

Function reference

`UNUR_DISTR* unur_distr_cont_new` (*void*)
Create a new (empty) object for univariate continuous distribution.

Essential parameters

`int unur_distr_cont_set_pdf` (*UNUR_DISTR* distribution, UNUR_FUNCT_CONT* pdf*)
`int unur_distr_cont_set_dpdf` (*UNUR_DISTR* distribution, UNUR_FUNCT_CONT* dpdf*)

`int unur_distr_cont_set_cdf (`*`UNUR_DISTR* distribution`*`, UNUR_FUNCT_CONT* cdf`*`)`

Set respective pointer to the probability density function (PDF), the derivative of the probability density function (dPDF) and the cumulative distribution function (CDF) of the *distribution*. Each of these function pointers must be of type `double funct(double x, const UNUR_DISTR *distr)`.

Due to the fact that some of the methods do not require a normalized PDF the following is important:

- The given CDF must be the cumulative distribution function of the (non-truncated) distribution. If a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) is truncated, there is no need to change the CDF.
- If both the CDF and the PDF are used (for a method or for order statistics), the PDF must be the derivative of the CDF. If a truncated distribution for one of the standard distributions from the UNU.RAN library of standard distributions is used, there is no need to change the PDF.
- If the area below the PDF is required for a given distribution it must be given by the `unur_distr_cont_set_pdfarea` call. For a truncated distribution this must be of course the integral of the PDF in the given truncated domain. For distributions from the UNU.RAN library of standard distributions this is done automatically by the `unur_distr_cont_upd_pdfarea` call.

It is important to note that all these functions must return a result for all values of x. Eg., if the domain of a given PDF is the interval [-1,1], then the given function must return `0.0` for all points outside this interval. In case of an overflow the PDF should return `UNUR_INFINITY`.

It is not possible to change such a function. Once the PDF or CDF is set it cannot be overwritten. This also holds when the logPDF is given or when the PDF is given by the `unur_distr_cont_set_pdfstr` or `unur_distr_cont_set_logpdfstr` call. A new distribution object has to be used instead.

`UNUR_FUNCT_CONT* unur_distr_cont_get_pdf (`*const UNUR_DISTR* distribution*`)`
`UNUR_FUNCT_CONT* unur_distr_cont_get_dpdf (`*const UNUR_DISTR* distribution*`)`
`UNUR_FUNCT_CONT* unur_distr_cont_get_cdf (`*const UNUR_DISTR* distribution*`)`

Get the respective pointer to the PDF, the derivative of the PDF and the CDF of the *distribution*. The pointer is of type `double funct(double x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the distribution, the `NULL` pointer is returned.

`double unur_distr_cont_eval_pdf (`*double x, const UNUR_DISTR* distribution*`)`
`double unur_distr_cont_eval_dpdf (`*double x, const UNUR_DISTR* distribution*`)`
`double unur_distr_cont_eval_cdf (`*double x, const UNUR_DISTR* distribution*`)`

Evaluate the PDF, derivative of the PDF and the CDF, respectively, at x. Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for the distribution, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

IMPORTANT: In the case of a truncated standard distribution these calls always return the respective values of the *untruncated* distribution!

`int unur_distr_cont_set_logpdf (`*UNUR_DISTR**` distribution`, *UNUR_FUNCT_CONT**` logpdf`)
`int unur_distr_cont_set_dlogpdf (`*UNUR_DISTR**` distribution`, *UNUR_FUNCT_CONT**` dlogpdf`)
`int unur_distr_cont_set_logcdf (`*UNUR_DISTR**` distribution`, *UNUR_FUNCT_CONT**` logcdf`)
`UNUR_FUNCT_CONT* unur_distr_cont_get_logpdf (`*const* `UNUR_DISTR*` distribution`)
`UNUR_FUNCT_CONT* unur_distr_cont_get_dlogpdf (`*const* `UNUR_DISTR*` distribution`)
`UNUR_FUNCT_CONT* unur_distr_cont_get_logcdf (`*const* `UNUR_DISTR*` distribution`)
`double unur_distr_cont_eval_logpdf (`*double* `x`, *const* `UNUR_DISTR*` distribution`)
`double unur_distr_cont_eval_dlogpdf (`*double* `x`, *const* `UNUR_DISTR*` distribution`)
`double unur_distr_cont_eval_logcdf (`*double* `x`, *const* `UNUR_DISTR*` distribution`)

Analogous calls for the logarithm of the density distribution functions.

`int unur_distr_cont_set_pdfstr (`*UNUR_DISTR** `distribution`, *const* `char*` pdfstr`)

This function provides an alternative way to set a PDF and its derivative of the *distribution*. *pdfstr* is a character string that contains the formula for the PDF, see Section 3.3 [Function String], page 43, for details. The derivative of the given PDF is computed automatically. See also the remarks for the `unur_distr_cont_set_pdf` call.

It is not possible to call this funtion twice or to call this function after a `unur_distr_cont_set_pdf` call.

`int unur_distr_cont_set_cdfstr (`*UNUR_DISTR** `distribution`, *const* `char*` cdfstr`)

This function provides an alternative way to set a CDF; analogously to the `unur_distr_cont_set_pdfstr` call. The PDF and its derivative of the given CDF are computed automatically.

`char* unur_distr_cont_get_pdfstr (`*const* `UNUR_DISTR* distribution`)
`char* unur_distr_cont_get_dpdfstr (`*const* `UNUR_DISTR* distribution`)
`char* unur_distr_cont_get_cdfstr (`*const* `UNUR_DISTR* distribution`)

Get pointer to respective string for PDF, derivate of PDF, and CDF of *distribution* that is given as string (instead of a function pointer). This call allocates memory to produce this string. It should be freed when it is not used any more.

`int unur_distr_cont_set_pdfparams (`*UNUR_DISTR** `distribution`, *const* `double* params`, *int* `n_params`)

Sets array of parameters for *distribution*. There is an upper limit for the number of parameters `n_params`. It is given by the macro `UNUR_DISTR_MAXPARAMS` in '`unuran_config.h`'. (It is set to 5 by default but can be changed to any appropriate nonnegative number.) If *n_params* is negative or exceeds this limit no parameters are copied into the distribution object and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.

For standard distributions from the UNU.RAN library the parameters are checked. Moreover, the domain is updated automatically unless it has been changed before by a `unur_distr_cont_set_domain` call. If the given parameters are invalid for the standard distribution, then

no parameters are set and an error code is returned. Notice, that the given parameter list for such a distribution is handled in the same way as in the corresponding `new` calls, i.e. optional parameters for the PDF that are not present in the given list are (re-)set to their default values.

Important: If the parameters of a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) are changed, then neither its mode nor the normalization constant are updated. Please use the respective calls `unur_distr_cont_upd_mode` and `unur_distr_cont_upd_pdfarea`. Moreover, if the domain has been changed by a `unur_distr_cont_set_domain` it is not automatically updated, either. Updating the normalization constant is in particular very important, when the CDF of the distribution is used.

`int unur_distr_cont_get_pdfparams (`*const* `UNUR_DISTR* distribution`, *const* `double** params`)`

Get number of parameters of the PDF and set pointer *params* to array of parameters. If no parameters are stored in the object, an error code is returned and `params` is set to `NULL`.

Important: Do **not** change the entries in *params*!

`int unur_distr_cont_set_pdfparams_vec (`*UNUR_DISTR** `distribution`, *int* `par`, *const* `double* param_vec`, *int* `n_param_vec`)`

This function provides an interface for additional vector parameters for a continuous *distribution*.

It sets the parameter with number *par*. *par* indicates directly which of the parameters is set and must be a number between `0` and `UNUR_DISTR_MAXPARAMS`-1 (the upper limit of possible parameters defined in '`unuran_config.h`'; it is set to 5 but can be changed to any appropriate nonnegative number.)

The entries of a this parameter are given by the array *param_vec* of size *n_param_vec*.

If *param_vec* is `NULL` then the corresponding entry is cleared.

If an error occurs no parameters are copied into the parameter object `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cont_get_pdfparams_vec (`*const* `UNUR_DISTR* distribution`, *int* `par`, *const* `double** param_vecs`)`

Get parameter of the PDF with number *par*. The pointer to the parameter array is stored in *param_vecs*, its size is returned by the function. If the requested parameter is not set, then an error code is returned and `params` is set to `NULL`.

Important: Do **not** change the entries in *param_vecs*!

`int unur_distr_cont_set_logpdfstr (`*UNUR_DISTR** `distribution`, *const* `char* logpdfstr`)`
`char* unur_distr_cont_get_logpdfstr (`*const* `UNUR_DISTR* distribution`)`
`char* unur_distr_cont_get_dlogpdfstr (`*const* `UNUR_DISTR* distribution`)`
`int unur_distr_cont_set_logcdfstr (`*UNUR_DISTR** `distribution`, *const* `char* logcdfstr`)`
`char* unur_distr_cont_get_logcdfstr (`*const* `UNUR_DISTR* distribution`)`

Analogous calls for the logarithm of the density and distribution functions.

`int unur_distr_cont_set_domain (`*UNUR_DISTR** `distribution`, *double* `left`, *double* `right`)`

Set the left and right borders of the domain of the distribution. This can also be used to truncate an existing distribution. For setting the boundary to $\pm\infty$ use `+/- UNUR_INFINITY`.

If *right* is not strictly greater than *left* no domain is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

Important: For some technical reasons it is assumed that the density is unimodal and thus monotone on either side of the mode! This is used in the case when the given mode is outside of the original domain. Then the mode is set to the corresponding boundary of the new domain. If this result is not the desired it must be changed by using a `unur_distr_cont_set_mode` call (or a `unur_distr_cont_upd_mode` call).

`int unur_distr_cont_get_domain` (*const* `UNUR_DISTR* distribution`, *double** `left`, *double** `right`)

Get the left and right borders of the domain of the distribution. If the domain is not set `+/-UNUR_INFINITY` is assumed and returned. No error is reported in this case.

`int unur_distr_cont_get_truncated` (*const* `UNUR_DISTR* distribution`, *double** `left`, *double** `right`)

Get the left and right borders of the (truncated) domain of the distribution. For non-truncated distribution this call is equivalent to the `unur_distr_cont_get_domain` call.

This call is only useful in connection with a `unur_get_distr` call to get the boundaries of the sampling region of a generator object.

`int unur_distr_cont_set_hr` (*UNUR_DISTR** `distribution`, *UNUR_FUNCT_CONT** `hazard`)

Set pointer to the hazard rate (HR) of the *distribution*.

The *hazard rate* (or failure rate) is a mathematical way of describing aging. If the lifetime X is a random variable with density *f(x)* and CDF *F(x)* the hazard rate *h(x)* is defined as *h(x) = f(x) / (1-F(x))*. In other words, *h(x)* represents the (conditional) rate of failure of a unit that has survived up to time x with probability *1-F(x)*. The key distribution is the exponential distribution as it has constant hazard rate of value 1. Hazard rates tending to infinity describe distributions with sub-exponential tails whereas distributions with hazard rates tending to zero have heavier tails than the exponential distribution.

It is important to note that all these functions must return a result for all floats x. In case of an overflow the PDF should return `UNUR_INFINITY`.

Important: Do not simply use *f(x) / (1-F(x))*, since this is numerically very unstable and results in numerical noise if *F(x)* is (very) close to 1. Moreover, if the density *f(x)* is known a generation method that uses the density is more appropriate.

It is not possible to change such a function. Once the HR is set it cannot be overwritten. This also holds when the HR is given by the `unur_distr_cont_set_hrstr` call. A new distribution object has to be used instead.

`UNUR_FUNCT_CONT* unur_distr_cont_get_hr` (*const* `UNUR_DISTR* distribution`)

Get the pointer to the hazard rate of the *distribution*. The pointer is of type `double funct(double x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the distribution, the `NULL` pointer is returned.

`double unur_distr_cont_eval_hr` (*double* `x`, *const* `UNUR_DISTR* distribution`)

Evaluate the hazard rate at x. Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for the distribution, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cont_set_hrstr (UNUR_DISTR* distribution, const char* hrstr)`
This function provides an alternative way to set a hazard rate and its derivative of the *distribution*. *hrstr* is a character string that contains the formula for the HR, see Section 3.3 [Function String], page 43, for details. See also the remarks for the `unur_distr_cont_set_hr` call.

It is not possible to call this funtion twice or to call this function after a `unur_distr_cont_set_hr` call.

`char* unur_distr_cont_get_hrstr (const UNUR_DISTR* distribution)`
Get pointer to string for HR of *distribution* that is given via the string interface. This call allocates memory to produce this string. It should be freed when it is not used any more.

Derived parameters

The following paramters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

`int unur_distr_cont_set_mode (UNUR_DISTR* distribution, double mode)`
Set mode of *distribution*. The *mode* must be contained in the domain of *distribution*. Otherwise the mode is not set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`. For distributions with unbounded density, this call is used to set the pole of the PDF. Notice that the PDF should then return UNUR_INFINITY at the pole. Notice that the mode is adjusted when the domain is set, see the remark for the `unur_distr_cont_set_domain` call.

`int unur_distr_cont_upd_mode (UNUR_DISTR* distribution)`
Recompute the mode of the *distribution*. This call works properly for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. Otherwise a (slow) numerical mode finder based on Brent's algorithm is used. If it failes `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`double unur_distr_cont_get_mode (UNUR_DISTR* distribution)`
Get mode of *distribution*. If the mode is not marked as known, `unur_distr_cont_upd_mode` is called to compute the mode. If this is not successful `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the distribution at all.)

`int unur_distr_cont_set_center (UNUR_DISTR* distribution, double center)`
Set center of the *distribution*. The center is used by some methods to shift the distribution in order to decrease numerical round-off error. If not given explicitly a default is used.

Important: This call does not check whether the center is contained in the given domain. Similarly `unur_distr_cont_set_domain` does not adjust the center properly.

Default: The mode, if set by a `unur_distr_cont_set_mode` or `unur_distr_cont_upd_mode` call; otherwise `0`.

`double unur_distr_cont_get_center (const UNUR_DISTR* distribution)`
Get center of the *distribution*. It always returns some point as there always exists a default for the center, see `unur_distr_cont_set_center`.

`int unur_distr_cont_set_pdfarea` (*UNUR_DISTR** ***distribution***, *double* **area**)
Set the area below the PDF. If `area` is non-positive, no area is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

For a distribution object created by the UNU.RAN library of standard distributions you always should use the `unur_distr_cont_upd_pdfarea`. Otherwise there might be ambiguous side-effects.

`int unur_distr_cont_upd_pdfarea` (*UNUR_DISTR** ***distribution***)
Recompute the area below the PDF of the distribution. It only works for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. Otherwise `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

This call also sets the normalization constant such that the given PDF is the derivative of a given CDF, i.e. the area is 1. However, for truncated distributions the area is smaller than 1.

The call does not work for distributions from the UNU.RAN library of standard distributions with truncated domain when the CDF is not available.

`double unur_distr_cont_get_pdfarea` (*UNUR_DISTR** ***distribution***)
Get the area below the PDF of the distribution. If this area is not known, `unur_distr_cont_upd_pdfarea` is called to compute it. If this is not successful `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

4.3 Continuous univariate order statistics

These are special cases of a continuous univariate distributions and thus they have most of these parameters (with the exception that functions cannot be changed). Additionally,

- there is a call to extract the underlying distribution,
- and a call to handle the `rank` of the order statistics.

Function reference

`UNUR_DISTR* unur_distr_corder_new` (*const* `UNUR_DISTR* distribution`, *int* `n`, *int* `k`)

Create an object for order statistics of sample size n and rank k. *distribution* must be a pointer to a univariate continuous distribution. The resulting generator object is of the same type as of a `unur_distr_cont_new` call. (However, it cannot be used to make an order statistics out of an order statistics.)

To have a PDF for the order statistics, the given distribution object must contain a CDF and a PDF. Moreover, it is assumed that the given PDF is the derivative of the given CDF. Otherwise the area below the PDF of the order statistics is not computed correctly.

Important: There is no warning when the computed area below the PDF of the order statistics is wrong.

`const UNUR_DISTR* unur_distr_corder_get_distribution` (*const* `UNUR_DISTR* distribution`)

Get pointer to distribution object for underlying distribution.

Essential parameters

`int unur_distr_corder_set_rank` (*UNUR_DISTR** `distribution`, *int* `n`, *int* `k`)

Change sample size n and rank k of order statistics. In case of invalid data, no parameters are changed. The area below the PDF can be set to that of the underlying distribution by a `unur_distr_corder_upd_pdfarea` call.

`int unur_distr_corder_get_rank` (*const* `UNUR_DISTR* distribution`, *int** `n`, *int** `k`)

Get sample size n and rank k of order statistics. In case of error an error code is returned.

Additionally most of the set and get calls for continuous univariate distributions work. The most important exceptions are that the PDF and CDF cannot be changed and `unur_distr_cont_upd_mode` uses in any way a (slow) numerical method that might fail.

`UNUR_FUNCT_CONT* unur_distr_corder_get_pdf` (*UNUR_DISTR** `distribution`)
`UNUR_FUNCT_CONT* unur_distr_corder_get_dpdf` (*UNUR_DISTR** `distribution`)
`UNUR_FUNCT_CONT* unur_distr_corder_get_cdf` (*UNUR_DISTR** `distribution`)

Get the respective pointer to the PDF, the derivative of the PDF and the CDF of the distribution, respectively. The pointer is of type `double funct(double x, UNUR_DISTR *distr)`. If the corresponding function is not available for the distribution, the `NULL` pointer is returned. See also `unur_distr_cont_get_pdf`. (Macro)

`double unur_distr_corder_eval_pdf` (*double* `x`, *UNUR_DISTR** `distribution`)
`double unur_distr_corder_eval_dpdf` (*double* `x`, *UNUR_DISTR** `distribution`)
`double unur_distr_corder_eval_cdf` (*double* `x`, *UNUR_DISTR** `distribution`)

Evaluate the PDF, derivative of the PDF. and the CDF, respectively, at x. Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for

the distribution, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`. See also `unur_distr_cont_eval_pdf`. (Macro)

IMPORTANT: In the case of a truncated standard distribution these calls always return the respective values of the *untruncated* distribution!

`int unur_distr_corder_set_pdfparams (`*UNUR_DISTR* distribution*, *double* params*, *int n_params*`)`
Set array of parameters for underlying distribution. See `unur_distr_cont_set_pdfparams` for details. (Macro)

`int unur_distr_corder_get_pdfparams (`*UNUR_DISTR* distribution*, *double** params*`)`
Get number of parameters of the PDF of the underlying distribution and set pointer *params* to array of parameters. See `unur_distr_cont_get_pdfparams` for details. (Macro)

`int unur_distr_corder_set_domain (`*UNUR_DISTR* distribution*, *double left*, *double right*`)`
Set the left and right borders of the domain of the distribution. See `unur_distr_cont_set_domain` for details. (Macro)

`int unur_distr_corder_get_domain (`*UNUR_DISTR* distribution*, *double* left*, *double* right*`)`
Get the left and right borders of the domain of the distribution. See `unur_distr_cont_get_domain` for details. (Macro)

`int unur_distr_corder_get_truncated (`*UNUR_DISTR* distribution*, *double* left*, *double* right*`)`
Get the left and right borders of the (truncated) domain of the distribution. See `unur_distr_cont_get_truncated` for details. (Macro)

Derived parameters

The following paramters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

`int unur_distr_corder_set_mode (`*UNUR_DISTR* distribution*, *double mode*`)`
Set mode of distribution. See also `unur_distr_corder_set_mode`. (Macro)

`double unur_distr_corder_upd_mode (`*UNUR_DISTR* distribution*`)`
Recompute the mode of the distribution numerically. Notice that this routine is slow and might not work properly in every case. See also `unur_distr_cont_upd_mode` for further details. (Macro)

`double unur_distr_corder_get_mode (`*UNUR_DISTR* distribution*`)`
Get mode of distribution. See `unur_distr_cont_get_mode` for details. (Macro)

`int unur_distr_corder_set_pdfarea (`*UNUR_DISTR* distribution*, *double area*`)`
Set the area below the PDF. See `unur_distr_cont_set_pdfarea` for details. (Macro)

`double unur_distr_corder_upd_pdfarea (`*UNUR_DISTR* distribution*`)`
Recompute the area below the PDF of the distribution. It only works for order statistics for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. `unur_distr_cont_upd_pdfarea` assumes that the PDF of the underlying distribution is normalized, i.e. it is the derivative of its CDF. Otherwise the computed area is wrong and there is **no** warning about this failure. See `unur_distr_cont_upd_pdfarea` for further details. (Macro)

`double unur_distr_corder_get_pdfarea (`*UNUR_DISTR* distribution*`)`
Get the area below the PDF of the distribution. See `unur_distr_cont_get_pdfarea` for details. (Macro)

4.4 Continuous empirical univariate distributions

Empirical univariate distributions are derived from observed data. There are two ways to create such a generator object:

1. By a list of *raw data* by means of a `unur_distr_cemp_set_data` call.
2. By a *histogram* (i.e. preprocessed data) by means of a `unur_distr_cemp_set_hist` call.

How these data are used to sample from the empirical distribution depends from the chosen generation method.

Function reference

`UNUR_DISTR* unur_distr_cemp_new` (*void*)
Create a new (empty) object for empirical univariate continuous distribution.

Essential parameters

`int unur_distr_cemp_set_data` (*UNUR_DISTR** ***distribution***, *const* ***double**** ***sample***, *int* ***n_sample***)
Set observed sample for empirical distribution.

`int unur_distr_cemp_read_data` (*UNUR_DISTR** ***distribution***, *const* ***char**** ***filename***)
Read data from file '`filename`'. It reads the first number from each line. Numbers are parsed by means of the C standard routine `strtod`. Lines that do not start with `+`, `-`, `.`, or a digit are ignored. (Beware of lines starting with a blank!)

In case of an error (file cannot be opened, invalid string for double in line) no data are copied into the distribution object and an error code is returned.

`int unur_distr_cemp_get_data` (*const* ***UNUR_DISTR* distribution***, *const* ***double** sample***)
Get number of samples and set pointer *sample* to array of observations. If no sample has been given, an error code is returned and `sample` is set to `NULL`.

Important: Do **not** change the entries in *sample*!

`int unur_distr_cemp_set_hist` (*UNUR_DISTR** ***distribution***, *const* ***double**** ***prob***, *int* ***n_prob***, *double* ***xmin***, *double* ***xmax***)
Set a histogram with bins of equal width. *prob* is an array of length *n_prob* that contains the probabilities for the bins (in ascending order). *xmin* and *xmax* give the lower and upper bound of the histogram, respectively. The bins are assumed to have equal width.

Remark: This is shortcut for calling `unur_distr_cemp_set_hist_prob` and `unur_distr_cemp_set_hist_domain`. *Notice:* All sampling methods either use raw data or histogram. It is possible to set both types of data; however, it is not checked whether the given histogran corresponds to possibly given raw data.

`int unur_distr_cemp_set_hist_prob` (*UNUR_DISTR** ***distribution***, *const* ***double**** ***prob***, *int* ***n_prob***)
Set probabilities of a histogram with *n_prob* bins. Hence *prob* must be an array of length *n_prob* that contains the probabilities for the bins in ascending order. It is important also to set the location of the bins either with a `unur_distr_cemp_set_hist_domain` for bins of equal width or `unur_distr_cemp_set_hist_bins` when the bins have different width.

Notice: All sampling methods either use raw data or histogram. It is possible to set both types of data; however, it is not checked whether the given histogram corresponds to possibly given raw data.

`int unur_distr_cemp_set_hist_domain` (*UNUR_DISTR** `distribution`, *double* `xmin`, *double* `xmax`)

Set a domain of a histogram with bins of equal width. *xmin* and *xmax* give the lower and upper bound of the histogram, respectively.

`int unur_distr_cemp_set_hist_bins` (*UNUR_DISTR** `distribution`, *const* `double* bins`, *int* `n_bins`)

Set location of bins of a histogram with *n_bins* bins. Hence *bins* must be an array of length *n_bins*. The domain of the *distribution* is automatically set by this call and overrides any calls to `unur_distr_cemp_set_hist_domain`. *Important:* The probabilities of the bins of the *distribution* must be already be set by a `unur_distr_cemp_set_hist_prob` (or a `unur_distr_cemp_set_hist` call) and the value of *n_bins* must equal *n_prob*`+1` from the corresponding value of the respective call.

4.5 Continuous multivariate distributions

The following calls handle multivariate distributions. However, the requirements of particular generation methods is not as unique as for univariate distributions. Moreover, random vector generation methods are still under development. The below functions are a first attempt to handle this situation.

Notice that some of the parameters – when given carelessly – might contradict to others. For example: Some methods require the marginal distribution and some methods need a standardized form of the marginal distributions, where the actual mean and variance is stored in the mean vector and the covariance matrix, respectively.

We also have to mention that some methods might abuse some of the parameters. Please read the discription of the chosen sampling method carfully.

The following kind of calls exists:

- Create a `new` instance of a continuous multivariate distribution;
- Handle and evaluate probability density function (PDF, `pdf`) and the gradient of the density function (`dpdf`). The following is important:
  - `pdf` need not be normalized, i.e., any integrable nonnegative function can be used.
  - `dpdf` must the derivate of the function provided as `pdf`.
- Handle and evaluate the logarithm of the probability density function (logPDF, `logpdf`) and the gradient of the logarithm of the density function (`dlogpdf`).

  Some methods use the logarithm of the density if available.
- Set (and change) parameters (`pdfparams`) and the volume below the graph (`pdfvol`) of the given density.
- Set `mode` and `mean` of the distribution.
- Set the `center` of the distribution. It is used by some generation methods to adjust the parameters of the generation algorithms to gain better performance. It can be seens as the location of the "central part" of the distribution.
- Handle the `cova`riance matrix of the distribution and its `cholesky` and `inv`verse matrices.
- Set the `rankcorr`elation matrix of the distribution.
- Deal with `marginal` distributions.
- Set domain of the distribution.

Function reference

`UNUR_DISTR* unur_distr_cvec_new` (*int* ***dim***)

Create a new (empty) object for multivariate continuous distribution. *dim* is the number of components of the random vector (i.e. its dimension). It is also possible to use dimension 1. Notice, however, that this is treated as a distribution of random vectors with only one component and not as a distribution of real numbers. For the latter `unur_distr_cont_new` should be used to create an object for a univariate distribution.

Essential parameters

`int unur_distr_cvec_set_pdf` (*UNUR_DISTR** ***distribution***,
*UNUR_FUNCT_CVEC** ***pdf***)

Set respective pointer to the PDF of the *distribution*. This function must be of type `double funct(const double *x, UNUR_DISTR *distr)`, where *x* must be a pointer to a double array of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call).

It is not necessary that the given PDF is normalized, i.e. the integral need not be 1. Nevertheless the volume below the PDF can be provided by a `unur_distr_cvec_set_pdfvol` call.

It is not possible to change the PDF. Once the PDF is set it cannot be overwritten. This also holds when the logPDF is given. A new distribution object has to be used instead.

`int unur_distr_cvec_set_dpdf` (*UNUR_DISTR* distribution*, *UNUR_VFUNCT_CVEC* dpdf*)

Set pointer to the gradient of the PDF. The type of this function must be `int funct(double *result, const double *x, UNUR_DISTR *distr)`, where *result* and *x* must be pointers to double arrays of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call). The gradient of the PDF is stored in the array *result*. The function should return an error code in case of an error and must return `UNUR_SUCCESS` otherwise.

The given function must be the gradient of the function given by a `unur_distr_cvec_set_pdf` call.

It is not possible to change the gradient of the PDF. Once the dPDF is set it cannot be overwritten. This also holds when the gradient of the logPDF is given. A new distribution object has to be used instead.

`int unur_distr_cvec_set_pdpdf` (*UNUR_DISTR* distribution*, *UNUR_FUNCTD_CVEC* pdpdf*)

Set pointer to partial derivatives of the PDF. The type of this function must be `double funct(const double *x, int coord, UNUR_DISTR *distr)`, where *x* must be a pointer to a double array of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call). *coord* is the coordinate for which the partial dervative should be computed.

Notice that *coord* must be an integer from {0,. . . ,dim-1}.

It is not possible to change the partial derivative of the PDF. Once the pdPDF is set it cannot be overwritten. This also holds when the partial derivative of the logPDF is given. A new distribution object has to be used instead.

`UNUR_FUNCT_CVEC* unur_distr_cvec_get_pdf` (*const UNUR_DISTR* distribution*)

Get the pointer to the PDF of the *distribution*. The pointer is of type `double funct(const double *x, UNUR_DISTR *distr)`. If the corresponding function is not available for the *distribution*, the `NULL` pointer is returned.

`UNUR_VFUNCT_CVEC* unur_distr_cvec_get_dpdf` (*const UNUR_DISTR* distribution*)

Get the pointer to the gradient of the PDF of the *distribution*. The pointer is of type `int double funct(double *result, const double *x, UNUR_DISTR *distr)`. If the corresponding function is not available for the *distribution*, the `NULL` pointer is returned.

`double unur_distr_cvec_eval_pdf` (*const double* x, UNUR_DISTR* distribution*)

Evaluate the PDF of the *distribution* at *x*. *x* must be a pointer to a double array of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call) that contains the vector for which the function has to be evaluated.

Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for the *distribution*, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cvec_eval_dpdf` (*double** **`result`**, *const* **`double* x`**, *UNUR_DISTR** **`distribution`**)

Evaluate the gradient of the PDF of the *distribution* at *x*. The result is stored in the double array *result*. Both *result* and *x* must be pointer to double arrays of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call).

Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for the *distribution*, an error code is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA` (*result* is left unmodified).

`double unur_distr_cvec_eval_pdpdf` (*const* **`double* x`**, *int* **`coord`**, *UNUR_DISTR** **`distribution`**)

Evaluate the partial derivative of the PDF of the *distribution* at *x* for the coordinate *coord*. *x* must be a pointer to a double array of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call) that contains the vector for which the function has to be evaluated.

Notice that *coord* must be an integer from {0,. . .,dim-1}.

Notice that *distribution* must not be the `NULL` pointer. If the corresponding function is not available for the *distribution*, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cvec_set_logpdf` (*UNUR_DISTR** **`distribution`**, *UNUR_FUNCT_CVEC** **`logpdf`**)
`int unur_distr_cvec_set_dlogpdf` (*UNUR_DISTR** **`distribution`**, *UNUR_VFUNCT_CVEC** **`dlogpdf`**)
`int unur_distr_cvec_set_pdlogpdf` (*UNUR_DISTR** **`distribution`**, *UNUR_FUNCTD_CVEC** **`pdlogpdf`**)
`UNUR_FUNCT_CVEC* unur_distr_cvec_get_logpdf` (*const* **`UNUR_DISTR* distribution`**)
`UNUR_VFUNCT_CVEC* unur_distr_cvec_get_dlogpdf` (*const* **`UNUR_DISTR* distribution`**)
`double unur_distr_cvec_eval_logpdf` (*const* **`double* x`**, *UNUR_DISTR** **`distribution`**)
`int unur_distr_cvec_eval_dlogpdf` (*double** **`result`**, *const* **`double* x`**, *UNUR_DISTR** **`distribution`**)
`double unur_distr_cvec_eval_pdlogpdf` (*const* **`double* x`**, *int* **`coord`**, *UNUR_DISTR** **`distribution`**)

Analogous calls for the logarithm of the density function.

`int unur_distr_cvec_set_mean` (*UNUR_DISTR** **`distribution`**, *const* **`double* mean`**)

Set mean vector for multivariate *distribution*. *mean* must be a pointer to an array of size `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. A `NULL` pointer for *mean* is interpreted as the zero vector (0,. . .,0).

Important: If the parameters of a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) are changed, then neither its mode nor the normalization constant are updated. Please use the respective calls `unur_distr_cvec_upd_mode` and `unur_distr_cvec_upd_pdfvol`.

`const double* unur_distr_cvec_get_mean` (*const* **`UNUR_DISTR* distribution`**)

Get the mean vector of the *distribution*. The function returns a pointer to an array of size `dim`. If the mean vector is not marked as known the `NULL` pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

Important: Do **not** modify the array that holds the mean vector!

`int unur_distr_cvec_set_covar (`*UNUR_DISTR* distribution, const double* covar*`)`

Set covariance matrix for multivariate *distribution*. *covar* must be a pointer to an array of size `dim` x `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. The rows of the matrix have to be stored consecutively in this array.

covar must be a variance-covariance matrix of the *distribution*, i.e. it must be symmetric and positive definit and its diagonal entries (i.e. the variance of the components of the random vector) must be strictly positive. The Cholesky factor is computed (and stored) to verify the positive definiteness condition. Notice that the inverse of the given covariance matrix is automatically computed when it is requested by some routine. Notice that the computation of this inverse matrix is unstable in case of high correlations and/or high dimensions. Thus it might fail and methods that require this inverse cannot be used. As an alternative the inverse of the covariance matrix can be directly set by a `unur_distr_cvec_set_covar_inv` call.

A `NULL` pointer for *covar* is interpreted as the identity matrix.

Important: This entry is abused in some methods which do not require the covariance matrix. It is then used to perform some transformation to obtain better performance.

Important: In case of an error (e.g. because *covar* is not a valid covariance matrix) an error code is returned. Moreover, the covariance matrix is not set and is marked as unknown. A previously set covariance matrix is then no longer available.

Important: If the parameters of a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) are changed, then neither its mode nor the normalization constant are updated. Please use the respective calls `unur_distr_cvec_upd_mode` and `unur_distr_cvec_upd_pdfvol`. *Remark:* UNU.RAN does not check whether the an eventually set covariance matrix and a rank-correlation matrix do not contradict each other.

`int unur_distr_cvec_set_covar_inv (`*UNUR_DISTR* distribution, const double* covar_inv*`)`

Set inverse of the covariance matrix for multivariate *distribution*. *covar_inv* must be a pointer to an array of size `dim` x `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. The rows of the matrix have to be stored consecutively in this array.

covar_inv must be symmetric and positive definit. Only the symmetry of the matrix is checked.

A `NULL` pointer for *covar_inv* is interpreted as the identity matrix.

Important: In case of an error (because *covar_inv* is not symetric) an error code is returned. Moreover, the inverse of the covariance matrix is not set and is marked as unknown. A previously set inverse matrix is then no longer available.

Remark: UNU.RAN does not check whether the given matrix is positive definit.

Remark: UNU.RAN does not check whether the matrix *covar_inv* is the inverse of the eventually set covariance matrix.

`const double* unur_distr_cvec_get_covar (`*const UNUR_DISTR* distribution*`)`
`const double* unur_distr_cvec_get_cholesky (`*const UNUR_DISTR* distribution*`)`

`const double* unur_distr_cvec_get_covar_inv` (*UNUR_DISTR* `distribution`*)

Get covariance matrix of *distribution*, its Cholesky factor, and its inverse, respectively. The function returns a pointer to an array of size `dim` x `dim`. The rows of the matrix are stored consecutively in this array. If the requested matrix is not marked as known the `NULL` pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

Important: Do **not** modify the array that holds the covariance matrix!

Remark: The inverse of the covariance matrix is computed if it is not already stored.

`int unur_distr_cvec_set_rankcorr` (*UNUR_DISTR* `distribution`, *const* `double* rankcorr`*)

Set rank-correlation matrix (Spearman's correlation) for multivariate *distribution*. *rankcorr* must be a pointer to an array of size `dim` x `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. The rows of the matrix have to be stored consecutively in this array.

rankcorr must be a rank-correlation matrix of the *distribution*, i.e. it must be symmetric and positive definite and its diagonal entries must be equal to `1`.

The Cholesky factor is computed (and stored) to verify the positive definiteness condition.

A `NULL` pointer for *rankcorr* is interpreted as the identity matrix.

Important: In case of an error (e.g. because *rankcorr* is not a valid rank-correlation matrix) an error code is returned. Moreover, the rank-correlation matrix is not set and is marked as unknown. A previously set rank-correlation matrix is then no longer available.

Remark: UNU.RAN does not check whether the an eventually set covariance matrix and a rank-correlation matrix do not contradict each other.

`const double* unur_distr_cvec_get_rankcorr` (*const UNUR_DISTR** `distribution`*)*

`const double* unur_distr_cvec_get_rk_cholesky` (*const UNUR_DISTR** `distribution`*)*

Get rank-correlation matrix and its cholesky factor, respectively, of *distribution*. The function returns a pointer to an array of size `dim` x `dim`. The rows of the matrix are stored consecutively in this array. If the requested matrix is not marked as known the `NULL` pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

Important: Do **not** modify the array that holds the rank-correlation matrix!

`int unur_distr_cvec_set_marginals` (*UNUR_DISTR* `distribution`, *UNUR_DISTR* `marginal`*)

Sets marginal distributions of the given *distribution* to the same *marginal* distribution object. The *marginal* distribution must be an instance of a continuous univariate distribution object. Notice that the marginal distribution is copied into the *distribution* object.

`int unur_distr_cvec_set_marginal_array` (*UNUR_DISTR* `distribution`, *UNUR_DISTR*** `marginals`*)

Analogously to the above `unur_distr_cvec_set_marginals` call. However, now an array *marginals* of the pointers to each of the marginal distributions must be given. It **must** be an array of size `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. *Notice*: Local copies for each of the entries are stored in the *distribution* object. If some of these entries are identical (i.e. contain the same pointer), then for each of these a new copy is made.

`int unur_distr_cvec_set_marginal_list (UNUR_DISTR* distribution, ...)`
Similar to the above `unur_distr_cvec_set_marginal_array` call. However, now the pointers to the particular marginal distributions can be given as parameter and does not require an array of pointers. Additionally the given distribution objects are immediately destroyed. Thus calls like `unur_distr_normal` can be used as arguments. (With `unur_distr_cvec_set_marginal_array` the result of such call has to be stored in a pointer since it has to be freed afterwarts to avoid memory leaks!)

The number of pointers to in the list of function arguments **must** be equal to the dimension of the *distribution*, i.e. the dimension returned by `unur_distr_get_dim`. If one of the given pointer to marginal distributions is the `NULL` pointer then the marginal distributions of *distribution* are not set (or previous settings are not changed) and an error code is returned.

Important: All distribution objects given in the argument list are destroyed!

`const UNUR_DISTR* unur_distr_cvec_get_marginal (const UNUR_DISTR* distribution, int n)`
Get pointer to the *n*-th marginal distribution object from the given multivariate *distribution*. If this does not exist, `NULL` is returned. The marginal distributions are enumerated from `1` to `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`.

`int unur_distr_cvec_set_pdfparams (UNUR_DISTR* distribution, const double* params, int n_params)`
Sets array of parameters for *distribution*. There is an upper limit for the number of parameters `n_params`. It is given by the macro `UNUR_DISTR_MAXPARAMS` in '`unuran_config.h`'. (It is set to 5 by default but can be changed to any appropriate nonnegative number.) If *n_params* is negative or exceeds this limit no parameters are copied into the distribution object and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.

For standard distributions from the UNU.RAN library the parameters are checked. Moreover, the domain is updated automatically. If the given parameters are invalid for the standard distribution, then no parameters are set and an error code is returned. Notice that the given parameter list for such a distribution is handled in the same way as in the corresponding `new` calls, i.e. optional parameters for the PDF that are not present in the given list are (re-)set to their default values.

Important: If the parameters of a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) are changed, then neither its mode nor the normalization constant are updated. Please use the respective calls `unur_distr_cvec_upd_mode` and `unur_distr_cvec_upd_pdfvol`.

`int unur_distr_cvec_get_pdfparams (const UNUR_DISTR* distribution, const double** params)`
Get number of parameters of the PDF and set pointer *params* to array of parameters. If no parameters are stored in the object, an error code is returned and `params` is set to `NULL`.

Important: Do **not** change the entries in *params*!

`int unur_distr_cvec_set_pdfparams_vec (UNUR_DISTR* distribution, int par, const double* param_vec, int n_params)`
This function provides an interface for additional vector parameters for a multivariate *distribution* besides mean vector and covariance matrix which have their own calls.

It sets the parameter with number *par*. *par* indicates directly which of the parameters is set and must be a number between `0` and `UNUR_DISTR_MAXPARAMS`-1 (the upper limit of possible

parameters defined in '`unuran_config.h`'; it is set to 5 but can be changed to any appropriate nonnegative number.)

The entries of a this parameter are given by the array *param_vec* of size *n_params*. Notice that using this interface an An (n x m)-matrix has to be stored in an array of length *n_params* = n times m; where the rows of the matrix are stored consecutively in this array.

Due to great variety of possible parameters for a multivariate *distribution* there is no simpler interface.

If *param_vec* is `NULL` then the corresponding entry is cleared.

Important: If the parameters of a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) are changed, then neither its mode nor the normalization constant are updated. Please use the respective calls `unur_distr_cvec_upd_mode` and `unur_distr_cvec_upd_pdfvol`. If an error occurs no parameters are copied into the parameter object `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cvec_get_pdfparams_vec` (*const* `UNUR_DISTR* distribution`, *int* `par`, *const* `double** param_vecs`)

Get parameter of the PDF with number *par*. The pointer to the parameter array is stored in *param_vecs*, its size is returned by the function. If the requested parameter is not set, then an error code is returned and `params` is set to `NULL`.

Important: Do **not** change the entries in *param_vecs*!

`int unur_distr_cvec_set_domain_rect` (*UNUR_DISTR** `distribution`, *const* `double* lowerleft`, *const* `double* upperright`)

Set rectangular domain for *distribution* with *lowerleft* and *upperright* vertices. Both must be pointer to an array of the size returned by `unur_distr_get_dim`. A `NULL` pointer is interpreted as the zero vector (0,. . .,0). For setting a coordinate of the boundary to $\pm\infty$ use `+/- UNUR_INFINITY`. The *lowerleft* vertex must be strictly smaller than *upperright* in each component. Otherwise no domain is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

By default the domain of a distribution is unbounded. Thus one can use this call to truncate an existing distribution.

Important: Changing the domain of *distribution* marks derived parameters like the mode or the center as unknown and must be set *after* changing the domain. This is important for the already set (or default) value for the center does not fall into the given domain. Notice that calls of the PDF and derived functions return `0.` when the parameter is not contained in the domain.

`int unur_distr_cvec_is_indomain` (*const* `double* x`, *const* `UNUR_DISTR* distribution`)

Check whether x falls into the domain of *distribution*.

Derived parameters

The following paramters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

`int unur_distr_cvec_set_mode` (*UNUR_DISTR** `distribution`, *const* `double* mode`)

Set mode of the *distribution*. *mode* must be a pointer to an array of the size returned by `unur_distr_get_dim`. A `NULL` pointer for *mode* is interpreted as the zero vector (0,. . .,0).

`int unur_distr_cvec_upd_mode (`*UNUR_DISTR* distribution*`)`
Recompute the mode of the *distribution*. This call works properly for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. If it failes `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`const double* unur_distr_cvec_get_mode (`*UNUR_DISTR* distribution*`)`
Get mode of the *distribution*. The function returns a pointer to an array of the size returned by `unur_distr_get_dim`. If the mode is not marked as known the `NULL` pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the *distribution* at all.)

Important: Do **not** modify the array that holds the mode!

`int unur_distr_cvec_set_center (`*UNUR_DISTR* distribution*, *const double* center*`)`
Set center of the *distribution*. *center* must be a pointer to an array of the size returned by `unur_distr_get_dim`. A `NULL` pointer for *center* is interpreted as the zero vector (0,. . . ,0).

The center is used by some methods to shift the distribution in order to decrease numerical round-off error. If not given explicitly a default is used. Moreover, it is used as starting point for several numerical search algorithm (e.g. for the mode). Then *center* must be a pointer where the call to the PDF returns a non-zero value. In particular *center* must contained in the domain of the distribution.

Default: The mode, if given by a `unur_distr_cvec_set_mode` call; else the mean, if given by a `unur_distr_cvec_set_mean` call; otherwise the null vector (0,. . . ,0).

`const double* unur_distr_cvec_get_center (`*UNUR_DISTR* distribution*`)`
Get center of the *distribution*. The function returns a pointer to an array of the size returned by `unur_distr_get_dim`. It always returns some point as there always exists a default for the center, see `unur_distr_cvec_set_center`. *Important:* Do **not** modify the array that holds the center!

`int unur_distr_cvec_set_pdfvol (`*UNUR_DISTR* distribution*, *double volume*`)`
Set the volume below the PDF. If *vol* is non-positive, no volume is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

`int unur_distr_cvec_upd_pdfvol (`*UNUR_DISTR* distribution*`)`
Recompute the volume below the PDF of the distribution. It only works for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. Otherwise `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

This call also sets the normalization constant such that the given PDF is the derivative of a given CDF, i.e. the volume is 1.

`double unur_distr_cvec_get_pdfvol (`*UNUR_DISTR* distribution*`)`
Get the volume below the PDF of the *distribution*. If this volume is not known, `unur_distr_cont_upd_pdfarea` is called to compute it. If this is not successful `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

4.6 Continuous univariate full conditional distribution

Full conditional distribution for a given continuous multivariate distributiion. The condition is a position vector and either a variable that is variated or a vector that indicates the direction on which the random vector can variate.

There is a subtle difference between using direction vector and using the k-th variable. When a direction vector is given the PDF of the conditional distribution is defined by $f(t) = PDF(pos + t \cdot dir)$. When a variable is selected the full conditional distribution with all other variables fixed is used.

This is a special case of a continuous univariate distribution and thus they have most of these parameters (with the exception that functions cannot be changed). Additionally,

- there is a call to extract the underlying multivariate distribution,
- and a call to handle the variables that are fixed and the direction for changing the random vector.

This distibution type is primarily used for evaluation the conditional distribution and its derivative (as required for, e.g., the Gibbs sampler). The density is not normalized (i.e. does not integrate to one). Mode and area are not available and it does not make sense to use any call to set or change parameters except the ones given below.

Function reference

`UNUR_DISTR* unur_distr_condi_new` (*const* `UNUR_DISTR* distribution`, *const* `double* pos`, *const* `double* dir`, *int* `k`)

Create an object for full conditional distribution for the given *distribution*. The condition is given by a position vector *pos* and either the k-th variable that is variated or the vector *dir* that contains the direction on which the random vector can variate.

distribution must be a pointer to a multivariate continuous distribution. *pos* must be a pointer to an array of size `dim`, where `dim` is the dimension of the underlying distribution object. *dir* must be a pointer to an array if size `dim` or `NULL`. k must be in the range `0, ..., dim-1`. If the k-th variable is used, *dir* must be set to `NULL`.

Notice: There is a subtle difference between using direction vector *dir* and using the k-th variable. When *dir* is given, the current position *pos* is mapped into 0 of the conditional distribution and the derivative is taken from the function PDF(*pos*+t**dir*) w.r.t. *t*. On the other hand, when the coordinate k is used (i.e., when *dir* is set to `NULL`), the full conditional distribution of the distribution is considered (as used for the Gibbs sampler). In particular, the current point is just projected into the one-dimensional subspace without mapping it into the point 0.

Notice: If a coordinate k is used, then the k-th partial derivative is used if it as available. Otherwise the gradient is computed and the k-th component is returned.

The resulting generator object is of the same type as of a `unur_distr_cont_new` call.

`int unur_distr_condi_set_condition` (*struct* `unur_distr* distribution`, *const* `double* pos`, *const* `double* dir`, *int* `k`)

Set/change condition for conditional *distribution*. Change values of fixed variables to *pos* and use direction *dir* or k-th variable of conditional *distribution*.

pos must be a pointer to an array of size `dim`, where `dim` is the dimension of the underlying distribution object. *dir* must be a pointer to an array if size `dim` or `NULL`. k must be in the range `0, ..., dim-1`. If the k-th variable is used, *dir* must be set to `NULL`.

Notice: There is a subtle difference between using direction vector *dir* and using the k-th variable. When *dir* is given, the current position *pos* is mapped into 0 of the conditional

distribution and the derivative is taken from the function PDF(*pos*+t**dir*) w.r.t. *t*. On the other hand, when the coordinate *k* is used (i.e., when *dir* is set to `NULL`), the full conditional distribution of the distribution is considered (as used for the Gibbs sampler). In particular, the current point is just projected into the one-dimensional subspace without mapping it into the point 0.

`int unur_distr_condi_get_condition` (*struct* `unur_distr* distribution`, *const* `double** pos`, *const* `double** dir`, *int** `k`)

Get condition for conditional *distribution*. The values for the fixed variables are stored in *pos*, which must be a pointer to an array of size `dim`. The condition is stored in *dir* and *k*, respectively.

Important: Do **not** change the entries in *pos* and *dir*!

`const UNUR_DISTR* unur_distr_condi_get_distribution` (*const* `UNUR_DISTR* distribution`)

Get pointer to distribution object for underlying distribution.

4.7 Continuous empirical multivariate distributions

Empirical multivariate distributions are just lists of vectors (with the same dimension). Thus there are only calls to insert these data. How these data are used to sample from the empirical distribution depends from the chosen generation method.

Function reference

`UNUR_DISTR* unur_distr_cvemp_new` (*int* ***dim***)

Create a new (empty) object for an empirical multivariate continuous distribution. *dim* is the number of components of the random vector (i.e. its dimension). It must be at least 2; otherwise `unur_distr_cemp_new` should be used to create an object for an empirical univariate distribution.

Essential parameters

`int unur_distr_cvemp_set_data` (*UNUR_DISTR** ***distribution***, *const* ***double**** ***sample***, *int* ***n_sample***)

Set observed sample for empirical *distribution*. *sample* is an array of doubles of size `dim` x *n_sample*, where `dim` is the dimension of the *distribution* returned by `unur_distr_get_dim`. The data points must be stored consecutively in *sample*, i.e., data points (x1, y1), (x2, y2), ... are given as an array {x1, y1, x2, y2, ...}.

`int unur_distr_cvemp_read_data` (*UNUR_DISTR** ***distribution***, *const* ***char**** ***filename***)

Read data from file '`filename`'. It reads the first `dim` numbers from each line, where `dim` is the dimension of the *distribution* returned by `unur_distr_get_dim`. Numbers are parsed by means of the C standard routine `strtod`. Lines that do not start with `+`, `-`, `.`, or a digit are ignored. (Beware of lines starting with a blank!)

In case of an error (file cannot be opened, too few entries in a line, invalid string for double in line) no data are copied into the distribution object and an error code is returned.

`int unur_distr_cvemp_get_data` (*const* ***UNUR_DISTR* distribution***, *const* ***double** sample***)

Get number of samples and set pointer *sample* to array of observations. If no sample has been given, an error code is returned and *sample* is set to `NULL`. If successful *sample* points to an array of length `dim` x `n_sample`, where `dim` is the dimension of the distribution returned by `unur_distr_get_dim` and `n_sample` the return value of the function.

Important: Do **not** modify the array *sample*.

4.8 MATRix distributions

Distributions for random matrices. Notice that UNU.RAN uses arrays of `doubles` to handle matrices. The rows of the matrix are stored consecutively.

Function reference

`UNUR_DISTR* unur_distr_matr_new` (*int* `n_rows`, *int* `n_cols`)

Create a new (empty) object for a matrix distribution. *n_rows* and *n_cols* are the respective numbers of rows and columns of the random matrix (i.e. its dimensions). It is also possible to have only one number or rows and/or columns. Notice, however, that this is treated as a distribution of random matrices with only one row or column or component and not as a distribution of vectors or real numbers. For the latter `unur_distr_cont_new` or `unur_distr_cvec_new` should be used to create an object for a univariate distribution and a multivariate (vector) distribution, respectively.

Essential parameters

`int unur_distr_matr_get_dim` (*const* `UNUR_DISTR* distribution`, *int** `n_rows`, *int** `n_cols`)

Get number of rows and columns of random matrix (its dimension). It returns the total number of components. If successfull `UNUR_SUCCESS` is returned.

4.9 Discrete univariate distributions

The calls in this section can be applied to discrete univariate distributions.

- Create a `new` instance of a discrete univariate distribution.
- Handle and evaluate distribution function (CDF, `cdf`) and probability mass function (PMF, `pmf`). The following is important:
  - `pmf` need not be normalized, i.e., any summable nonnegative function on the set of intergers can be used.
  - `cdf` must be a distribution function, i.e. it must be monotonically increasing with range [0,1].
  - If `cdf` and `pdf` are used together for a pariticular generation method, then `pmf` must be normalized, i.e. it must sum to 1.
- Alternatively, `cdf` and `pdf` can be provided as `str`ings instead of function pointers.
- Some generation methods require a (finite) probability vector (PV, `pv`), i.e. an array of `double`s. It can be automatically computed if the `pmf` is given but `pv` is not.
- Set (and change) parameters (`pmfparams`) and the total sum (`pmfsum`) of the given PMF or PV.
- Set the `mode` of the distribution.
- Set the `domain` of the distribution.

Function reference

`UNUR_DISTR* unur_distr_discr_new` (*void*)
Create a new (empty) object for a univariate discrete distribution.

Essential parameters

There are two interfaces for discrete univariate distributions: Either provide a (finite) probability vector (PV). Or provide a probability mass function (PMF). For the latter case there are also a couple of derived parameters that are not required when a PV is given.

It is not possible to set both a PMF and a PV directly. However, the PV can be computed from the PMF (or the CDF if no PMF is available) by means of a `unur_distr_discr_make_pv` call. If both the PV and the PMF are given in the distribution object it depends on the generation method which of these is used.

`int unur_distr_discr_set_pv` (*UNUR_DISTR* distribution, const double* pv, int n_pv*)
Set finite probability vector (PV) for the *distribution*. It is not necessary that the entries in the given PV sum to 1. *n_pv* must be positive. However, there is no testing whether all entries in *pv* are non-negative.

If no domain has been set, then the left boundary is set to `0`, by default. If *n_pv* is too large, e.g. because left boundary `+` *n_pv* exceeds the range of integers, then the call fails.

Notice that it is not possible to set both a PV and a PMF or CDF. If the PMF or CDF is set first one cannot set the PV. If the PMF or CDF is set first after a PV is set, the latter is removed (and recomputed using `unur_distr_discr_make_pv` when required).

`int unur_distr_discr_make_pv` (*UNUR_DISTR* distribution*)
Compute a PV when a PMF or CDF is given. However, when the domain is not given or is too large and the sum over the PMF is given then the (right) tail of the *distribution* is chopped off such that the probability for the tail region is less than 1.e-8. If the sum over the PMF is not given a PV of maximal length is computed.

The maximal size of the created PV is bounded by the macro `UNUR_MAX_AUTO_PV` that is defined in '`unuran_config.h`'.

If successful, the length of the generated PV is returned. If the sum over the PMF on the chopped tail is not neglible small (i.e. greater than 1.e-8 or unknown) than the negative of the length of the PV is returned and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

Notice that the left boundary of the PV is set to `0` by default when a discrete distribution object is created from scratch.

If computing a PV fails for some reasons, an error code is returned and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

`int unur_distr_discr_get_pv (`*const UNUR_DISTR* distribution*, *const double** pv*`)`

Get length of PV of the *distribution* and set pointer *pv* to array of probabilities. If no PV is given, an error code is returned and *pv* is set to `NULL`.
(It does not call `unur_distr_discr_make_pv` !)

`int unur_distr_discr_set_pmf (`*UNUR_DISTR* distribution*, *UNUR_FUNCT_DISCR* pmf*`)`
`int unur_distr_discr_set_cdf (`*UNUR_DISTR* distribution*, *UNUR_FUNCT_DISCR* cdf*`)`

Set respective pointer to the PMF and the CDF of the *distribution*. These functions must be of type `double funct(int k, const UNUR_DISTR *distr)`.

It is important to note that all these functions must return a result for all integers k. E.g., if the domain of a given PMF is the interval {1,2,3,. . .,100}, than the given function must return `0.0` for all points outside this interval.

The default domain for the PMF or CDF is [`0`, `INT_MAX`]. The domain can be changed using a `unur_distr_discr_set_domain` call.

It is not possible to change such a function. Once the PMF or CDF is set it cannot be overwritten. A new distribution object has to be used instead.

Notice that it is not possible to set both a PV and a PMF or CDF. If the PMF or CDF is set first one cannot set the PV. If the PMF or CDF is set first after a PV is set, the latter is removed (and recomputed using `unur_distr_discr_make_pv` when required).

`double unur_distr_discr_eval_pv (`*int k, const UNUR_DISTR* distribution*`)`
`double unur_distr_discr_eval_pmf (`*int k, const UNUR_DISTR* distribution*`)`
`double unur_distr_discr_eval_cdf (`*int k, const UNUR_DISTR* distribution*`)`

Evaluate the PV, PMF, and the CDF, respectively, at k. Notice that *distribution* must not be the `NULL` pointer. If no PV is set for the *distribution*, then `unur_distr_discr_eval_pv` behaves like `unur_distr_discr_eval_pmf`. If the corresponding function is not available for the *distribution*, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

IMPORTANT: In the case of a truncated standard distribution these calls always return the respective values of the *untruncated* distribution!

`int unur_distr_discr_set_pmfstr (`*UNUR_DISTR* distribution*, *const char* pmfstr*`)`

This function provides an alternative way to set a PMF of the *distribution*. *pmfstr* is a character string that contains the formula for the PMF, see Section 3.3 [Function String], page 43, for details. See also the remarks for the `unur_distr_discr_set_pmf` call.

It is not possible to call this funtion twice or to call this function after a `unur_distr_discr_set_pmf` call.

`int unur_distr_discr_set_cdfstr` (*UNUR_DISTR** **distribution**, *const* **char* cdfstr**)

This function provides an alternative way to set a CDF; analogously to the `unur_distr_discr_set_pmfstr` call.

`char* unur_distr_discr_get_pmfstr` (*const* **UNUR_DISTR* distribution**)
`char* unur_distr_discr_get_cdfstr` (*const* **UNUR_DISTR* distribution**)

Get pointer to respective string for PMF and CDF of *distribution* that is given via the string interface. This call allocates memory to produce this string. It should be freed when it is not used any more.

`int unur_distr_discr_set_pmfparams` (*UNUR_DISTR** **distribution**, *const* **double* params**, *int* **n_params**)

Set array of parameters for *distribution*. There is an upper limit for the number of parameters *n_params*. It is given by the macro `UNUR_DISTR_MAXPARAMS` in '`unuran_config.h`'. (It is set to 5 but can be changed to any appropriate nonnegative number.) If *n_params* is negative or exceeds this limit no parameters are copied into the *distribution* object and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.

For standard distributions from the UNU.RAN library the parameters are checked. Moreover, the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. If the given parameters are invalid for the standard distribution, then no parameters are set and an error code is returned. Notice that the given parameter list for such a distribution is handled in the same way as in the corresponding `new` calls, i.e. optional parameters for the PDF that are not present in the given list are (re-)set to their default values.

Important: Integer parameter must be given as `double`s.

`int unur_distr_discr_get_pmfparams` (*const* **UNUR_DISTR* distribution**, *const* **double** params**)

Get number of parameters of the PMF and set pointer *params* to array of parameters. If no parameters are stored in the object, an error code is returned and `params` is set to `NULL`.

`int unur_distr_discr_set_domain` (*UNUR_DISTR** **distribution**, *int* **left**, *int* **right**)

Set the left and right borders of the domain of the *distribution*. This can also be used to truncate an existing distribution. For setting the boundary to $\pm\infty$ use `INT_MIN` and `INT_MAX`, respectively. If *right* is not strictly greater than *left* no domain is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`. It is allowed to use this call to increase the domain. If the PV of the discrete distribution is used, than the right boudary is ignored (and internally set to *left* + size of PV -1). Notice that `INT_MIN` and `INT_MAX` are interpreted as (minus/plus) infinity.

Default: [`0`, `INT_MAX`].

`int unur_distr_discr_get_domain` (*const* **UNUR_DISTR* distribution**, *int** **left**, *int** **right**)

Get the left and right borders of the domain of the *distribution*. If the domain is not set explicitly the interval [`INT_MIN`, `INT_MAX`] is assumed and returned. When a PV is given then the domain is set automatically to [`0`,size of PV -1].

Derived parameters

The following paramters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

`int unur_distr_discr_set_mode` (*UNUR_DISTR** `distribution`, *int* `mode`)
Set mode of *distribution*.

`int unur_distr_discr_upd_mode` (*UNUR_DISTR** `distribution`)
Recompute the mode of the *distribution*. This call works properly for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. Otherwise a (slow) numerical mode finder is used. It only works properly for unimodal probability mass functions. If it failes `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_discr_get_mode` (*UNUR_DISTR** `distribution`)
Get mode of *distribution*. If the mode is not marked as known, `unur_distr_discr_upd_mode` is called to compute the mode. If this is not successful `INT_MAX` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the distribution at all.)

`int unur_distr_discr_set_pmfsum` (*UNUR_DISTR** `distribution`, *double* `sum`)
Set the sum over the PMF. If `sum` is non-positive, no sum is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.

For a distribution object created by the UNU.RAN library of standard distributions you always should use the `unur_distr_discr_upd_pmfsum`. Otherwise there might be ambiguous side-effects.

`int unur_distr_discr_upd_pmfsum` (*UNUR_DISTR** `distribution`)
Recompute the sum over the PMF of the *distribution*. In most cases the normalization constant is recomputed and thus the sum is 1. This call works for distribution objects from the UNU.RAN library of standard distributions when the corresponding function is available. When a PV, a PMF with finite domain, or a CDF is given, a simple generic function which uses a naive summation loop is used. If this computation is not possible, an error code is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

The call does not work for distributions from the UNU.RAN library of standard distributions with truncated domain when the CDF is not available.

`double unur_distr_discr_get_pmfsum` (*UNUR_DISTR** `distribution`)
Get the sum over the PMF of the *distribution*. If this sum is not known, `unur_distr_discr_upd_pmfsum` is called to compute it. If this is not successful `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`.

5 Methods for generating non-uniform random variates

Sampling from a particular distribution with UNU.RAN requires the following steps:

1. Create a distribution object (see Chapter 4 [Handling distribution objects], page 55).
2. Select a method and create a parameter object.
3. Initizialize the generator object using `unur_init`.

   Important: Initialization of the generator object might fail. `unur_init` returns a `NULL` pointer then, which **must** not be used for sampling.
4. Draw a sample from the generator object using the corresponding sampling function (depending on the type of distribution: univariate continuous, univariate discrete, multivariate continuous, and random matrix).
5. It is possible for a generator object to change the parameters and the domain of the underlying distribution. This must be done by extracting this object by means of a `unur_get_distr` call and changing the distribution using the correspondig set calls, see Chapter 4 [Handling distribution objects], page 55. The generator object **must** then be reinitialized by means of the `unur_reinit` call.

   Important: Currently not all methods allow reinitialization, see the description of the particular method (keyword *Reinit*).

   Important: Reinitialization of the generator object might fail. Thus one **must** check the return code of the `unur_reinit` call.

   Important: When reinitialization fails then sampling routines always return INFINITY (for continuous distributions) or `0` (for discrete distributions), respectively. However, it is still possible to change the underlying distribution and try to reinitialize again.

5.1 Routines for all generator objects

Routines for all generator objects.

Function reference

`UNUR_GEN* unur_init (`*`UNUR_PAR* parameters`*`)`

Initialize a generator object. All necessary information must be stored in the parameter object.

Important: If an error has occurred a `NULL` pointer is return. This must not be used for the sampling routines (this causes a segmentation fault).

Always check whether the call was successful or not!

Important: This call destroys the *parameter* object automatically. Thus it is not necessary/allowed to free it.

`int unur_reinit (`*`UNUR_GEN* generator`*`)`

Update an existing generator object after the underlying distribution has been modified (using `unur_get_distr` together with corresponding set calls. It **must** be executed before sampling using this generator object is continued as otherwise it produces an invalid sample or might even cause a segmentation fault.

Important: Currently not all methods allow reinitialization, see the description of the particular method (keyword *Reinit*).

Important: Reinitialization of the generator object might fail. Thus one **must** check the return code:

`UNUR_SUCCESS (0x0u)`
: success (no error)

`UNUR_ERR_NO_REINIT`
: reinit routine not implemented.

other values
: some error has occured while trying to reinitialize the generator object.

Important: When reinitialization fails then sampling routines always return INFINITY (for continuous distributions) or `0` (for discrete distributions), respectively. However, it is still possible to change the underlying distribution and try to reinitialize again.

Important: When one tries to run `unur_reinit`, but reinitialization is not implemented, then the generator object cannot be used any more and must be destroyed and a new one has to be built from scratch.

`int unur_sample_discr (`*UNUR_GEN* generator*`)`
`double unur_sample_cont (`*UNUR_GEN* generator*`)`
`int unur_sample_vec (`*UNUR_GEN* generator, double* vector*`)`
`int unur_sample_matr (`*UNUR_GEN* generator, double* matrix*`)`

Sample from generator object. The three routines depend on the type of the generator object (discrete or continuous univariate distribution, multivariate distribution, or random matrix).

Notice: UNU.RAN uses arrays of `double`s to handle matrices. There the rows of the matrix are stored consecutively.

Notice: The routines `unur_sample_vec` and `unur_sample_matr` return `UNUR_SUCCESS` if generation was successful and some error code otherwise.

Important: These routines do **not** check whether *generator* is an invalid `NULL` pointer.

`void unur_free (`*UNUR_GEN* generator*`)`

Destroy (free) the given generator object.

`const char* unur_gen_info (`*UNUR_GEN* generator, int help*`)`

Get a string with informations about the given *generator*. These informations allow some fine tuning of the generation method. If *help* is `TRUE`, some hints on setting parameters are given.

This function is intented for using in interactive environments (like `R`).

If an error occurs, then `NULL` is returned.

`int unur_get_dimension (`*const UNUR_GEN* generator*`)`

Get the number of dimension of a (multivariate) distribution. For a univariate distribution `1` is return.

`const char* unur_get_genid (`*const UNUR_GEN* generator*`)`

Get identifier string for generator.

`UNUR_DISTR* unur_get_distr (`*const UNUR_GEN* generator*`)`

Get pointer to distribution object from generator object. This function can be used to change the parameters of the distribution and reinitialize the generator object. Notice that currently **not all** generating methods have a reinitialize routine. This function should be used with extreme care. Changing the distribution is changed and using the generator object without reinitializing might cause wrong samples or segmentation faults. Moreover, if the corresponding generator object is freed, the pointer must not be used.

Important: The returned distribution object must not be freed. If the distribution object is changed then one **must** run `unur_reinit` !

`int unur_set_use_distr_privatecopy (`*UNUR_PAR* parameters*, *int use_privatecopy*`)`

Set flag whether the generator object should make a private copy of the given distribution object or just stores the pointer to this distribution object. Values for *use_privatecopy*:

`TRUE` make a private copy (default)

`FALSE` do not make a private copy and store pointer to given (external) distribution object.

By default, generator objects keep their own private copy of the given distribution object. Thus the generator object can be handled independently from other UNU.RAN objects (with uniform random number generators as the only exception). When the generator object is initialized the given distribution object is cloned and stored.

However, in some rare situations it can be useful when only the pointer to the given distribution object is stored without making a private copy. A possible example is when only one random variate has to be drawn from the distribution. This behavior can be achieved when *use_localcopy* is set to `FALSE`.

Warning! Using a pointer to the external distribution object instead of a private copy must be done with **extreme care**! When the distrubtion object is changed or freed then the generator object does not work any more, might case a segmentation fault, or (even worse) produces garbage. On the other hand, when the generator object is initialized or used to draw a random sampling the distribution object may be changed.

Notice: The prototypes of all `unur_<method>_new` calls use a `const` qualifier for the distribution argument. However, if *use_privatecopy* is set to `FALSE` this qualifier is discarded and the distribution might be changed.

Important! If *use_localcopy* is set to `FALSE` and the corresponding distribution object is changed then one must run `unur_reinit` on the generator object. (Notice that currently not all generation methods support reinitialization.)

Default: *use_privatecopy* is `TRUE`.

5.2 AUTO – Select method automatically

AUTO selects a an appropriate method for the given distribution object automatically. There are no parameters for this method, yet. But it is planned to give some parameter to describe the task for which the random variate generator is used for and thus make the choice of the generating method more appropriate. Notice that the required sampling routine for the generator object depends on the type of the given distribution object.

The chosen method also depends on the sample size for which the generator object will be used. If only a few random variates the order of magnitude of the sample size should be set via a `unur_auto_set_logss` call.

IMPORTANT: This is an experimental version and the method chosen may change in future releases of UNU.RAN.

For an example see Section 2.1 [Example: As short as possible], page 14.

How To Use

Create a generator object for the given distribution object.

Function reference

`UNUR_PAR* unur_auto_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_auto_set_logss` (*UNUR_PAR** `parameters`, *int* `logss`)
Set the order of magnitude for the size of the sample that will be generated by the generator, i.e., the the common logarithm of the sample size.

Default is 10.

Notice: This feature will be used in future releases of UNU.RAN only.

5.3 Methods for continuous univariate distributions

Overview of methods

Methods for **continuous univariate distributions**
sample with `unur_sample_cont`

method	PDF	dPDF	CDF	mode	area	other
AROU	x	x		[x]		T-concave
HINV	[x]	[x]	x			
HRB						bounded hazard rate
HRD						decreasing hazard rate
HRI						increasing hazard rate
ITDR	x	x		x		monotone with pole
NINV	[x]		x			
NROU	x			[x]		
SROU	x			x	x	T-concave
SSR	x			x	x	T-concave
TABL	x			x	[~]	all local extrema
TDR	x	x				T-concave
TDRGW	x	x				T-concave
UTDR	x			x	~	T-concave
CSTD						build-in standard distribution
CEXT						wrapper for external generator

Example

```
/* ---------------------------------------------------------- */
/* File: example_cont.c                                       */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

/* Example how to sample from a continuous univariate         */
/* distribution.                                              */
/*                                                            */
/* We build a distribution object from scratch and sample.    */

/* ---------------------------------------------------------- */

/* Define the PDF and dPDF of our distribution.               */
/*                                                            */
/* Our distribution has the PDF                               */
/*                                                            */
/*          /  1 - x*x  if |x| <= 1                           */
/*  f(x) = <                                                  */
/*          \  0        otherwise                             */
/*                                                            */

/* The PDF of our distribution:                               */
double mypdf( double x, const UNUR_DISTR *distr )
     /* The second argument (‘distr’) can be used for parameters */
     /* for the PDF. (We do not use parameters in our example.)  */
{
  if (fabs(x) >= 1.)
```

```
    return 0.;
  else
    return (1.-x*x);
} /* end of mypdf() */

/* The derivative of the PDF of our distribution:                */
double mydpdf( double x, const UNUR_DISTR *distr )
{
  if (fabs(x) >= 1.)
    return 0.;
  else
    return (-2.*x);
} /* end of mydpdf() */

/* ------------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                                 */
  double x;     /* will hold the random number                   */

  /* Declare the three UNURAN objects.                           */
  UNUR_DISTR *distr;    /* distribution object                   */
  UNUR_PAR   *par;      /* parameter object                      */
  UNUR_GEN   *gen;      /* generator object                      */

  /* Create a new distribution object from scratch.              */

  /* Get empty distribution object for a continuous distribution */
  distr = unur_distr_cont_new();

  /* Fill the distribution object -- the provided information    */
  /* must fulfill the requirements of the method choosen below.  */
  unur_distr_cont_set_pdf(distr,  mypdf);     /* PDF             */
  unur_distr_cont_set_dpdf(distr, mydpdf);    /* its derivative  */
  unur_distr_cont_set_mode(distr, 0.);        /* mode            */
  unur_distr_cont_set_domain(distr, -1., 1.); /* domain          */

  /* Choose a method: TDR.                                       */
  par = unur_tdr_new(distr);

  /* Set some parameters of the method TDR.                      */
  unur_tdr_set_variant_gw(par);
  unur_tdr_set_max_sqhratio(par, 0.90);
  unur_tdr_set_c(par, -0.5);
  unur_tdr_set_max_intervals(par, 100);
  unur_tdr_set_cpoints(par, 10, NULL);

  /* Create the generator object.                                */
  gen = unur_init(par);

  /* Notice that this call has also destroyed the parameter      */
  /* object 'par' as a side effect.                              */

  /* It is important to check if the creation of the generator   */
  /* object was successful. Otherwise 'gen' is the NULL pointer  */
  /* and would cause a segmentation fault if used for sampling.  */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create   */
  /* another generator object. If you do not need it any more,   */
  /* it should be destroyed to free memory.                      */
```

```
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------------- */
```

Example (String API)

```
/* ---------------------------------------------------------------- */
/* File: example_cont_str.c                                         */
/* ---------------------------------------------------------------- */
/* String API.                                                      */
/* ---------------------------------------------------------------- */

/* Include UNURAN header file.                                      */
#include <unuran.h>

/* ---------------------------------------------------------------- */

/* Example how to sample from a continuous univariate               */
/* distribution.                                                    */

/* We use a generic distribution object and sample.                 */
/*                                                                  */
/* The PDF of our distribution is given by                          */
/*                                                                  */
/*          /  1 - x*x  if |x| <= 1                                 */
/*  f(x) = <                                                        */
/*          \  0        otherwise                                   */
/*                                                                  */

/* ---------------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                                   */
  double x;     /* will hold the random number                     */

  /* Declare UNURAN generator object.                               */
  UNUR_GEN   *gen;      /* generator object                        */

  /* Create the generator object.                                   */
  /* Use a generic continuous distribution.                         */
  /* Choose a method: TDR.                                          */
  gen = unur_str2gen(
           "distr = cont; pdf=\"1-x*x\"; domain=(-1,1); mode=0. & \
            method=tdr; variant_gw; max_sqhratio=0.90; c=-0.5; \
            max_intervals=100; cpoints=10");

  /* It is important to check if the creation of the generator      */
  /* object was successful. Otherwise 'gen' is the NULL pointer     */
  /* and would cause a segmentation fault if used for sampling.     */
```

```
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

5.3.1 AROU – Automatic Ratio-Of-Uniforms method

Required: T-concave PDF, dPDF

Optional: mode

Speed: Set-up: slow, Sampling: fast

Reinit: not implemented

Reference: [LJa00]

AROU is a variant of the ratio-of-uniforms method that uses the fact that the transformed region is convex for many distributions. It works for all T-concave distributions with T(x) = -1/sqrt(x).

It is possible to use this method for correlation induction by setting an auxiliary uniform random number generator via the `unur_set_urng_aux` call. (Notice that this must be done after a possible `unur_set_urng` call.) When an auxiliary generator is used then the number of used uniform random numbers that is used up for one generated random variate is constant and equal to 1.

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on by calling `unur_arou_set_verify` and `unur_arou_chg_verify`, respectively. Notice however that sampling is (much) slower then.

For densities with modes not close to 0 it is suggested to set either the mode or the center of the distribution by the `unur_distr_cont_set_mode` or `unur_distr_cont_set_center` call. The latter is the approximate location of the mode or the mean of the distribution. This location provides some information about the main part of the PDF and is used to avoid numerical problems.

Function reference

`UNUR_PAR* unur_arou_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_arou_set_usedars` (*UNUR_PAR** `parameters`, *int* `usedars`)
If *usedars* is set to `TRUE`, "derandomized adaptive rejection sampling" (DARS) is used in setup. Segments where the area between hat and squeeze is too large compared to the average area between hat and squeeze over all intervals are split. This procedure is repeated until the ratio between area below squeeze and area below hat exceeds the bound given by `unur_arou_set_max_sqhratio` call or the maximum number of segments is reached. Moreover, it also aborts when no more segments can be found for splitting.

Segments are split such that the angle of the segments are halved (corresponds to arc-mean rule of method TDR (see Section 5.3.15 [TDR], page 132)).

Default is `TRUE`.

`int unur_arou_set_darsfactor` (*UNUR_PAR** `parameters`, *double* `factor`)
Set factor for "derandomized adaptive rejection sampling". This factor is used to determine the segments that are "too large", that is, all segments where the area between squeeze and hat is larger than *factor* times the average area over all intervals between squeeze and hat. Notice that all segments are split when *factor* is set to `0.`, and that there is no splitting at all when *factor* is set to `UNUR_INFINITY`.

Default is `0.99`. There is no need to change this parameter.

`int unur_arou_set_max_sqhratio (`*UNUR_PAR* parameters*, *double* `max_ratio`)
Set upper bound for the ratio (area inside squeeze) / (area inside envelope). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via adaptive rejection sampling. Use `0` if no construction points should be added after the setup. Use `1` if adding new construction points should not be stopped until the maximum number of construction points is reached.

Default is `0.99`.

`double unur_arou_get_sqhratio (`*const UNUR_GEN* generator*`)`
Get the current ratio (area inside squeeze) / (area inside envelope) for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_arou_get_hatarea (`*const UNUR_GEN* generator*`)`
Get the area below the hat for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_arou_get_squeezearea (`*const UNUR_GEN* generator*`)`
Get the area below the squeeze for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`int unur_arou_set_max_segments (`*UNUR_PAR* parameters*, *int* `max_segs`)
Set maximum number of segements. No construction points are added *after* the setup when the number of segments succeeds *max_segs*.

Default is `100`.

`int unur_arou_set_cpoints (`*UNUR_PAR* parameters*, *int* `n_stp`, *const* `double\* stp`)
Set construction points for enveloping polygon. If *stp* is `NULL`, then a heuristical rule of thumb is used to get *n_stp* construction points. This is the default behavior when this routine is not called. The (default) number of construction points is `30`, then.

`int unur_arou_set_usecenter (`*UNUR_PAR* parameters*, *int* `usecenter`)
Use the center as construction point. Default is `TRUE`.

`int unur_arou_set_guidefactor (`*UNUR_PAR* parameters*, *double* `factor`)
Set factor for relative size of the guide table for indexed search (see also method DGT Section 5.8.4 [DGT], page 176). It must be greater than or equal to `0`. When set to `0`, then sequential search is used.

Default is `2`.

`int unur_arou_set_verify (`*UNUR_PAR* parameters*, *int* `verify`)
`int unur_arou_chg_verify (`*UNUR_GEN* generator*, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_arou_set_pedantic (`*UNUR_PAR* parameters*, *int* `pedantic`)
Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not T-concave.

With *pedantic* being `TRUE`, the sampling routine is then exchanged by a routine that simply returns `UNUR_INFINITY`. Otherwise the new point is not added to the list of construction points. At least the hat function remains T-concave.

Setting *pedantic* to `FALSE` allows sampling from a distribution which is "almost" T-concave and small errors are tolerated. However it might happen that the hat function cannot be improved significantly. When the hat function that has been constructed by the `unur_init` call is extremely large then it might happen that the generation times are extremely high (even hours are possible in extremely rare cases).

Default is `FALSE`.

5.3.2 ARS – Adaptive Rejection Sampling

Required: concave logPDF, derivative of logPDF

Optional: mode

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [GWa92] [HLD04: Cha.4]

ARS is an acceptance/rejection method that uses the concavity of the log-density function to construct hat function and squeezes automatically. It is very similar to method TDR (see Section 5.3.15 [TDR], page 132) with variant GW, parameter `c = 0`, and DARS switched off. Moreover, method ARS requires the logPDF and its derivative dlogPDF to run. On the other hand, it is designed to draw only a (very) small samples and it is much more robust against densities with very large or small areas below the PDF as it occurs, for example, in conditional distributions of (high dimensional) multivariate distributions. Additionally, it can be re-initialized when the underlying distribution has been modified. Thus it is well suited for Gibbs sampling.

Notice, that method ARS is a restricted version of TDR. If the full functionally of Transformed Density Rejection is needed use method Section 5.3.15 [TDR], page 132.

How To Use

Method ARS is designed for distributions with log-concave densities. To use this method you need a distribution object with the logarithm of the PDF and its derivative given.

The number of construction points as well as a set of such points can be provided using `unur_ars_set_cpoints`. Notice that addition construction points are added by means of adaptive rejection sampling until the maximal number of intervals given by `unur_ars_set_max_intervals` is reached.

A generated distribution object can be reinitialized using the `unur_reinit` call. When `unur_reinit` is called construction points for the new generator are necessary. There are two options: Either the same construction points as for the initial generator (given by a `unur_ars_set_cpoints` call) are used (this is the default), or percentiles of the old hat function can be used. This can be set or changed using `unur_ars_set_reinit_percentiles` and `unur_ars_chg_reinit_percentiles`. This feature is usefull when the underlying distribution object is only moderately changed. (An example is Gibbs sampling with small correlations.)

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_ars_set_verify` and `unur_ars_chg_verify`, respectively. Notice however that sampling is (much) slower then.

Function reference

`UNUR_PAR* unur_ars_new` (*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

`int unur_ars_set_max_intervals` (*UNUR_PAR** `parameters`, *int* `max_ivs`)

Set maximum number of intervals. No construction points are added after the setup when the number of intervals suceeds *max_ivs*. It is increased automatically to twice the number of construction points if this is larger.

Default is `200`.

`int unur_ars_set_cpoints (`*UNUR_PAR* parameters*, *int n_cpoints*, *const double* cpoints*`)`

Set construction points for the hat function. If *cpoints* is `NULL` then a heuristic rule of thumb is used to get *n_cpoints* construction points. This is the default behavior. *n_cpoints* should be at least `2`, otherwise defaults are used.

The default number of construction points is 2.

`int unur_ars_set_reinit_percentiles (`*UNUR_PAR* parameters*, *int n_percentiles*, *const double* percentiles*`)`
`int unur_ars_chg_reinit_percentiles (`*UNUR_GEN* generator*, *int n_percentiles*, *const double* percentiles*`)`

By default, when the *generator* object is reinitialized, it used the same construction points as for the initialization procedure. Often the underlying distribution object has been changed only moderately. For example, the full conditional distribution of a multivariate distribution. In this case it might be more appropriate to use percentilesm of the hat function for the last (unchanged) distribution. *percentiles* must then be a pointer to an ordered array of numbers between `0.01` and `0.99`. If *percentiles* is `NULL`, then a heuristic rule of thumb is used to get *n_percentiles* values for these percentiles. Notice that *n_percentiles* must be at least `2`, otherwise defaults are used. (Then the first and third quartiles are used by default.)

`int unur_ars_set_reinit_ncpoints (`*UNUR_PAR* parameters*, *int ncpoints*`)`
`int unur_ars_chg_reinit_ncpoints (`*UNUR_GEN* generator*, *int ncpoints*`)`

When reinit fails with the given construction points or the percentiles of the old hat function, another trial is undertaken with *ncpoints* construction points. *ncpoints* must be at least `10`.

Default: `30`

`int unur_ars_set_verify (`*UNUR_PAR* parameters*, *int verify*`)`
`int unur_ars_chg_verify (`*UNUR_GEN* generator*, *int verify*`)`

Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_ars_set_pedantic (`*UNUR_PAR* parameters*, *int pedantic*`)`

Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not log-concave.

With *pedantic* being `TRUE`, the sampling routine is exchanged by a routine that simply returns `UNUR_INFINITY`. Otherwise the new point is not added to the list of construction points. At least the hat function remains log-concave.

Setting *pedantic* to `FALSE` allows sampling from a distribution which is "almost" log-concave and small errors are tolerated. However it might happen that the hat function cannot be improved significantly. When the hat functions that has been constructed by the `unur_init` call is extremely large then it might happen that the generation times are extremely high (even hours are possible in extremely rare cases).

Default is `FALSE`.

`double unur_ars_get_loghatarea (`*const UNUR_GEN* generator*`)`

Get the logarithm of area below the hat for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_ars_eval_invcdfhat` (*const* `UNUR_GEN* generator`, *double* `u`)

Evaluate the inverse of the CDF of the hat distribution at u.

If u is out of the domain [0,1] then `unur_errno` is set to `UNUR_ERR_DOMAIN` and the respective bound of the domain of the distribution are returned (which is `-UNUR_INFINITY` or `UNUR_INFINITY` in the case of unbounded domains).

5.3.3 CEXT – wrapper for Continuous EXTernal generators

Required: routine for sampling continuous random variates

Speed: depends on external generator

Reinit: supported

Method CEXT is a wrapper for external generators for continuous univariate distributions. It allows the usage of external random variate generators within the UNU.RAN framework.

How To Use

The following steps are required to use some external generator within the UNU.RAN framework (some of these are optional):

1. Make an empty generator object using a `unur_cext_new` call. The argument *distribution* is optional and can be replaced by `NULL`. However, it is required if you want to pass parameters of the generated distribution to the external generator or for running some validation tests provided by UNU.RAN.
2. Create an initialization routine of type `int (*init)(UNUR_GEN *gen)` and plug it into the generator object using the `unur_cext_set_init` call. Notice that the *init* routine must return `UNUR_SUCCESS` when it has been executed successfully and `UNUR_FAILURE` otherwise. It is possible to get the size of and the pointer to the array of parameters of the underlying distribution object by the respective calls `unur_cext_get_ndistrparams` and `unur_cext_get_distrparams`. Parameters for the external generator that are computed in the *init* routine can be stored in a single array or structure which is available by the `unur_cext_get_params` call.

   Using an *init* routine is optional and can be omitted.
3. Create a sampling routine of type `double (*sample)(UNUR_GEN *gen)` and plug it into the generator object using the `unur_cext_set_sample` call.

   Uniform random numbers are provided by the `unur_sample_urng` call. Do not use your own implementation of a uniform random number generator directly. If you want to use your own random number generator we recommend to use the UNU.RAN interface (see see Chapter 6 [Using uniform random number generators], page 189).

   The array or structure that contains parameters for the external generator that are computed in the *init* routine are available using the `unur_cext_get_params` call.

   Using a *sample* routine is of course obligatory.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. The *init* routine is then called again.

Here is a short example that demonstrates the application of this method by means of the exponential distribution:

```
/* ------------------------------------------------------------ */
/* File: example_cext.c                                         */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

/* This example shows how an external generator for the         */
/* exponential distribution with one scale parameter can be     */
/* used within the UNURAN framework.                            */
/*                                                              */
```

```
/* Notice, that this example does not provide the simplest       */
/* solution.                                                     */

/* ------------------------------------------------------------- */
/* Initialization routine.                                       */
/*                                                               */
/*   Here we simply read the scale parameter of the exponential  */
/*   distribution and store it in an array for parameters of     */
/*   the external generator.                                     */
/*   [ Of course we could do this in the sampling routine as     */
/*   and avoid the necessity of this initialization routine. ]   */

int exponential_init (UNUR_GEN *gen)
{
  /* Get pointer to parameters of exponential distribution       */
  double *params = unur_cext_get_distrparams(gen);

  /* The scale parameter is the first entry (see manual)         */
  double lambda = (params) ? params[0] : 1.;

  /* Get array to store this parameter for external generator    */
  double *genpar = unur_cext_get_params(gen, sizeof(double));
  genpar[0] = lambda;

  /* Executed successfully                                       */
  return UNUR_SUCCESS;
}

/* ------------------------------------------------------------- */
/* Sampling routine.                                             */
/*                                                               */
/*   Contains the code for the external generator.               */

double exponential_sample (UNUR_GEN *gen)
{
  /* Get scale parameter                                         */
  double *genpar = unur_cext_get_params(gen,0);
  double lambda = genpar[0];

  /* Sample a uniformly distributed random number                */
  double U = unur_sample_urng(gen);

  /* Transform into exponentially distributed random variate     */
  return ( -log(1. - U) * lambda );
}

/* ------------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                                 */
  double x;     /* will hold the random number                   */

  /* Declare the three UNURAN objects.                           */
  UNUR_DISTR *distr;    /* distribution object                   */
  UNUR_PAR   *par;      /* parameter object                      */
  UNUR_GEN   *gen;      /* generator object                      */

  /* Use predefined exponential distribution with scale param. 2 */
  double fpar[1] = { 2. };
  distr = unur_distr_exponential(fpar, 1);

  /* Use method CEXT                                             */
  par = unur_cext_new(distr);
```

```
    /* Set initialization and sampling routines.                  */
    unur_cext_set_init(par, exponential_init);
    unur_cext_set_sample(par, exponential_sample);

    /* Create the generator object.                               */
    gen = unur_init(par);

    /* It is important to check if the creation of the generator  */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
       fprintf(stderr, "ERROR: cannot create generator object\n");
       exit (EXIT_FAILURE);
    }

    /* It is possible to reuse the distribution object to create  */
    /* another generator object. If you do not need it any more,  */
    /* it should be destroyed to free memory.                     */
    unur_distr_free(distr);

    /* Now you can use the generator object 'gen' to sample from  */
    /* the standard Gaussian distribution.                        */
    /* Eg.:                                                       */
    for (i=0; i<10; i++) {
      x = unur_sample_cont(gen);
      printf("%f\n",x);
    }

    /* When you do not need the generator object any more, you    */
    /* can destroy it.                                            */
    unur_free(gen);

    exit (EXIT_SUCCESS);

  } /* end of main() */

  /* ---------------------------------------------------------- */
```

Function reference

`UNUR_PAR* unur_cext_new` (*const* ***`UNUR_DISTR* distribution`***)

Get default parameters for new generator.

`int unur_cext_set_init` (*UNUR_PAR** ***`parameters`***, *int* (* ***`init`***)(*UNUR_GEN* *gen*))

Set initialization routine for external generator. Inside the

Important: The routine *init* must return `UNUR_SUCCESS` when the generator was initialized successfully and `UNUR_FAILURE` otherwise.

Parameters that are computed in the *init* routine can be stored in an array or structure that is avaiable by means of the `unur_cext_get_params` call. Parameters of the underlying distribution object can be obtained by the `unur_cext_get_distrparams` call.

`int unur_cext_set_sample` (*UNUR_PAR** ***`parameters`***, *double* (* ***`sample`***)(*UNUR_GEN* *gen*))

Set sampling routine for external generator.

Important: Use `unur_sample_urng(gen)` to get a uniform random number. The pointer to the array or structure that contains the parameters that are precomputed in the *init* routine are available by `unur_cext_get_params(gen,0)`. Additionally one can use the `unur_cext_get_distrparams` call.

`void* unur_cext_get_params` (*UNUR_GEN* **`generator`**, *size_t* **`size`**)

Get pointer to memory block for storing parameters of external generator. A memory block of size *size* is automatically (re-) allocated if necessary and the pointer to this block is stored in the *generator* object. If one only needs the pointer to this memory block set *size* to `0`.

Notice, that *size* is the size of the memory block and not the length of an array.

Important: This rountine should only be used in the initialization and sampling routine of the external generator.

`double* unur_cext_get_distrparams` (*UNUR_GEN* **`generator`**)

`int unur_cext_get_ndistrparams` (*UNUR_GEN* **`generator`**)

Get size of and pointer to array of parameters of underlying distribution in *generator* object.

Important: These rountines should only be used in the initialization and sampling routine of the external generator.

5.3.4 CSTD – Continuous STandarD distributions

Required: standard distribution from UNU.RAN library (see Chapter 7 [Standard distributions], page 201).

Speed: Set-up: fast, Sampling: depends on distribution and generator

Reinit: supported

CSTD is a wrapper for special generators for continuous univariate standard distributions. It only works for distributions in the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201). If a distribution object is provided that is build from scratch, or if no special generator for the given standard distribution is provided, the `NULL` pointer is returned.

For some distributions more than one special generator is possible.

How To Use

Create a distribution object for a standard distribution from the UNU.RAN library (see Chapter 7 [Standard distributions], page 201). For some distributions more than one special generator (*variants*) is possible. These can be choosen by a `unur_cstd_set_variant` call. For possible variants see Chapter 7 [Standard distributions], page 201. However the following are common to all distributions:

`UNUR_STDGEN_DEFAULT`
: the default generator.

`UNUR_STDGEN_FAST`
: the fastest available special generator.

`UNUR_STDGEN_INVERSION`
: the inversion method (if available).

Notice that the variant `UNUR_STDGEN_FAST` for a special generator may be slower than one of the universal algorithms! Additional variants may exist for particular distributions.

Sampling from truncated distributions (which can be constructed by changing the default domain of a distribution by means of `unur_distr_cont_set_domain` or `unur_cstd_chg_truncated` calls) is possible but requires the inversion method.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_cstd_new` (*const* `UNUR_DISTR* distribution`)
: Get default parameters for new generator. It requires a distribution object for a continuous univariant distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201).

  Using a truncated distribution is allowed only if the inversion method is available and selected by the `unur_cstd_set_variant` call immediately after creating the parameter object. Use a `unur_distr_cont_set_domain` call to get a truncated distribution. To change the domain of a (truncated) distribution of a generator use the `unur_cstd_chg_truncated` call.

`int unur_cstd_set_variant` (*UNUR_PAR** `parameters`, *unsigned* `variant`)
: Set variant (special generator) for sampling from a given distribution. For possible variants see Chapter 7 [Standard distributions], page 201.

Common variants are `UNUR_STDGEN_DEFAULT` for the default generator, `UNUR_STDGEN_FAST` for (one of the) fastest implemented special generators, and `UNUR_STDGEN_INVERSION` for the inversion method (if available). If the selected variant number is not implemented, then an error code is returned and the variant is not changed.

`int unur_cstd_chg_truncated (`*UNUR_GEN* `generator`, double `left`, double `right`*`)`

Change left and right border of the domain of the (truncated) distribution. This is only possible if the inversion method is used. Otherwise this call has no effect and an error code is returned.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call.

It is not required to run `unur_reinit` after this call has been used.

Important: If the CDF is (almost) the same for *left* and *right* and (almost) equal to `0` or `1`, then the truncated domain is not chanced and the call returns an error code.

Notice: If the parameters of the distribution has been changed it is recommended to set the truncated domain again, since the former call might change the domain of the distribution but not update the values for the boundaries of the truncated distribution.

5.3.5 HINV – Hermite interpolation based INVersion of CDF

Required: CDF

Optional: PDF, dPDF

Speed: Set-up: (very) slow, Sampling: (very) fast

Reinit: supported

Reference: [HLa03] [HLD04: Sect.7.2; Alg.7.1]

HINV is a variant of numerical inversion, where the inverse CDF is approximated using Hermite interpolation, i.e., the interval [0,1] is split into several intervals and in each interval the inverse CDF is approximated by polynomials constructed by means of values of the CDF and PDF at interval boundaries. This makes it possible to improve the accuracy by splitting a particular interval without recomputations in unaffected intervals. Three types of splines are implemented: linear, cubic, and quintic interpolation. For linear interpolation only the CDF is required. Cubic interpolation also requires PDF and quintic interpolation PDF and its derivative.

These splines have to be computed in a setup step. However, it only works for distributions with bounded domain; for distributions with unbounded domain the tails are chopped off such that the probability for the tail regions is small compared to the given u-resolution.

The method is not exact, as it only produces random variates of the approximated distribution. Nevertheless, the maximal numerical error in "u-direction" (i.e. |U-CDF(X)|, for X = "approximate inverse CDF"(U) |U-CDF(X)|) can be set to the required resolution (within machine precision). Notice that very small values of the u-resolution are possible but may increase the cost for the setup step.

As the possible maximal error is only estimated in the setup it may be necessary to set some special design points for computing the Hermite interpolation to guarantee that the maximal u-error can not be bigger than desired. Such points are points where the density is not differentiable or has a local extremum. Notice that there is no necessity to do so. However, if you do not provide these points to the algorithm there might be a small chance that the approximation error is larger than the given u-resolution, or that the required number of intervals is larger than necessary.

How To Use

HINV works for continuous univariate distribution objects with given CDF and (optional) PDF. It uses Hermite interpolation of order 1, 3 [default] or 5. The order can be set by means of `unur_hinv_set_order`. For distributions with unbounded domains the tails are chopped off such that the probability for the tail regions is small compared to the given u-resulution. For finding these cut points the algorithm starts with the region `[-1.e20,1.e20]`. For the exceptional case where this might be too small (or one knows this region and wants to avoid this search heuristics) it can be directly set via a `unur_hinv_set_boundary` call.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an `unur_hinv_chg_truncated` call.

This method is not exact, as it only produces random variates of the approximated distribution. Nevertheless, the numerical error in "u-direction" (i.e. |U-CDF(X)|, for X = "approximate inverse CDF"(U) |U-CDF(X)|) can be controlled by means of `unur_hinv_set_u_resolution`. The possible maximal error is only estimated in the setup. Thus it might be necessary to set some special design points for computing the Hermite interpolation to guarantee that the maximal u-error can not be bigger than desired. Such points (e.g. extremal points of the PDF, points with infinite derivative) can be set using using the `unur_hinv_set_cpoints`

call. If the mode for a unimodal distribution is set in the distribution object this mode is automatically used as design-point if the `unur_hinv_set_cpoints` call is not used.

As already mentioned the maximal error of this approximation is only estimated. If this error is crucial for an application we recommend to compute this error using `unur_hinv_estimate_error` which runs a small Monte Carlo simulation.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. The values given by the last `unur_hinv_chg_truncated` call will be then changed to the values of the domain of the underlying distribution object. Moreover, starting construction points (nodes) that are given by a `unur_hinv_set_cpoints` call are ignored when `unur_reinit` is called. It is important to note that for a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) the normalization constant has to be updated using the `unur_distr_cont_upd_pdfarea` call whenever its parameters have been changed by means of a `unur_distr_cont_set_pdfparams` call.

Function reference

`UNUR_PAR* unur_hinv_new` (*const UNUR_DISTR* distribution*)
Get default parameters for generator.

`int unur_hinv_set_order` (*UNUR_PAR* parameters*, *int order*)
Set order of Hermite interpolation. Valid orders are `1`, `3`, and `5`. Notice that *order* greater than `1` requires the density of the distribution, and *order* greater than `3` even requires the derivative of the density. Using *order* `1` results for most distributions in a huge number of intervals and is therefore not recommended. If the maximal error in u-direction is very small (say smaller than `1.e-10`), *order* `5` is recommended as it leads to considerably fewer design points, as long there are no poles or heavy tails.

Remark: When the target distribution has poles or (very) heavy tails *order* `5` (i.e., quintic interpolation) is numerically less stable and more sensitive to round-off errors than *order* `3` (i.e., cubic interpolation).

Default is `3` if the density is given and `1` otherwise.

`int unur_hinv_set_u_resolution` (*UNUR_PAR* parameters*, *double u_resolution*)
Set maximal error in u-direction. However, the given u-error must not be smaller than machine epsilon (`DBL_EPSILON`) and should not be too close to this value. As the resoultion of most uniform random number sources is 2^(-32) = `2.3e-10`, a value of `1.e-10` leads to an inversion algorithm that could be called exact. For most simulations slighly bigger values for the maximal error are enough as well.

Default is `1.e-10`.

`int unur_hinv_set_cpoints` (*UNUR_PAR* parameters*, *const double* stp*, *int n_stp*)
Set starting construction points (nodes) for Hermite interpolation.

As the possible maximal error is only estimated in the setup it may be necessary to set some special design points for computing the Hermite interpolation to guarantee that the maximal u-error can not be bigger than desired. We suggest to include as special design points all local extrema of the density, all points where the density is not differentiable, and isolated points inside of the domain with density 0. If there is an interval with density constant equal to 0 inside of the given domain of the density, both endpoints of this interval should be included as special design points. Notice that there is no necessity to do so. However, if these points

are not provided to the algorithm the approximation error might be larger than the given u-resolution, or the required number of intervals could be larger than necessary.

Important: Notice that the given points must be in increasing order and they must be disjoint.

Important: The boundary point of the computational region must not be given in this list! Points outside the boundary of the computational region are ignored.

Default is for unimodal densities - if known - the mode of the density, if it is not equal to the border of the domain.

`int unur_hinv_set_boundary (`*UNUR_PAR* parameters*, *double* `left`, *double* `right`)

Set the left and right boundary of the computational interval. Of course `+/- UNUR_INFINITY` is not allowed. If the CDF at *left* and *right* is not close to the respective values `0.` and `1.` then this interval is increased by a (rather slow) search algorithm.

Important: This call does not change the domain of the given distribution itself. But it restricts the domain for the resulting random variates.

Default is `1.e20`.

`int unur_hinv_set_guidefactor (`*UNUR_PAR* parameters*, *double* `factor`)

Set factor for relative size of the guide table for indexed search (see also method DGT Section 5.8.4 [DGT], page 176). It must be greater than or equal to `0`. When set to `0`, then sequential search is used.

Default is `1`.

`int unur_hinv_set_max_intervals (`*UNUR_PAR* parameters*, *int* `max_ivs`)

Set maximum number of intervals. No generator object is created if the necessary number of intervals for the Hermite interpolation exceeds *max_ivs*. It is used to prevent the algorithm to eat up all memory for very badly shaped CDFs.

Default is `1000000` (1.e6).

`int unur_hinv_get_n_intervals (`*const UNUR_GEN* generator*`)`

Get number of nodes (design points) used for Hermite interpolation in the generator object. The number of intervals is the number of nodes minus 1. It returns an error code in case of an error.

`double unur_hinv_eval_approxinvcdf (`*const UNUR_GEN* generator*, *double* `u`)

Evaluate Hermite interpolation of inverse CDF at *u*. If *u* is out of the domain [0,1] then `unur_errno` is set to `UNUR_ERR_DOMAIN` and the respective bound of the domain of the distribution are returned (which is `-UNUR_INFINITY` or `UNUR_INFINITY` in the case of unbounded domains).

Notice: This function always evaluates the inverse CDF of the given distribution. A call to `unur_hinv_chg_truncated` call has no effect.

`int unur_hinv_chg_truncated (`*UNUR_GEN* generator*, *double* `left`, *double* `right`)

Changes the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. The tables of splines are not recomputed. Thus it might happen that the relative error for the generated variates from the truncated distribution is

greater than the bound for the non-truncated distribution. This call also fails when the CDF values of the boundary points are too close, i.e. when only a few different floating point numbers would be computed due to round-off errors with floating point arithmetic.

When failed an error code is returned.

Important: Always check the return code since the domain is not changed in case of an error.

`int unur_hinv_estimate_error` (*const* `UNUR_GEN* generator`, *int* `samplesize`, *double** `max_error`, *double** `MAE`)

Estimate maximal u-error and mean absolute error (MAE) for *generator* by means of Monte-Carlo simulation with sample size *samplesize*. The results are stored in *max_error* and *MAE*, respectively.

It returns `UNUR_SUCCESS` if successful.

5.3.6 HRB – Hazard Rate Bounded

Required: bounded hazard rate

Optional: upper bound for hazard rate

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [HLD04: Sect.9.1.4; Alg.9.4]

Generates random variate with given hazard rate which must be bounded from above. It uses the thinning method with a constant dominating hazard function.

How To Use

HRB requires a hazard function for a continuous distribution together with an upper bound. The latter has to be set using the `unur_hrb_set_upperbound` call. If no such upper bound is given it is assumed that the upper bound can be achieved by evaluating the hazard rate at the left hand boundary of the domain of the distribution. Notice, however, that for decreasing hazard rate the method HRD (see Section 5.3.7 [Hazard Rate Decreasing], page 112) is much faster and thus the prefered method.

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, the left border must not be negative. Otherwise it is set to `0`. This is also the default if no domain is given at all. For computational reasons the right border is always set to `UNUR_INFINITY` independently of the given domain. Thus for domains bounded from right the function for computing the hazard rate should return `UNUR_INFINITY` right of this domain.

For distributions with increasing hazard rate method HRI (see Section 5.3.8 [Hazard Rate Increasing], page 113) is required.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that the upper bound given by the `unur_hrb_set_upperbound` call cannot be changed and must be valid for the changed distribution.

Function reference

`UNUR_PAR* unur_hrb_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_hrb_set_upperbound` (*UNUR_PAR** `parameters`, *double* `upperbound`)
Set upper bound for hazard rate. If this call is not used it is assumed that the the maximum of the hazard rate is achieved at the left hand boundary of the domain of the distribution.

`int unur_hrb_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_hrb_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by the given bound, then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`.

Default is `FALSE`.

5.3.7 HRD – Hazard Rate Decreasing

Required: decreasing (non-increasing) hazard rate

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [HLD04: Sect.9.1.5; Alg.9.5]

Generates random variate with given non-increasing hazard rate. It is necessary that the distribution object contains this hazard rate. Decreasing hazard rate implies that the corresponding PDF of the distribution has heavier tails than the exponential distribution (which has constant hazard rate).

How To Use

HRD requires a hazard function for a continuous distribution with non-increasing hazard rate. There are no parameters for this method.

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, only the left hand boundary is used. For computational reasons the right hand boundary is always reset to `UNUR_INFINITY`. If no domain is given by the user then the left hand boundary is set to `0`.

For distributions which do not have decreasing hazard rates but are bounded from above use method HRB (see Section 5.3.6 [Hazard Rate Bounded], page 111). For distributions with increasing hazard rate method HRI (see Section 5.3.8 [Hazard Rate Increasing], page 113) is required.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_hrd_new` (*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

`int unur_hrd_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_hrd_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)

Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by the given bound, then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`.

Default is `FALSE`.

5.3.8 HRI – Hazard Rate Increasing

Required: increasing (non-decreasing) hazard rate

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [HLD04: Sect.9.1.6; Alg.9.6]

Generates random variate with given non-increasing hazard rate. It is necessary that the distribution object contains this hazard rate. Increasing hazard rate implies that the corresponding PDF of the distribution has heavier tails than the exponential distribution (which has constant hazard rate).

The method uses a decomposition of the hazard rate into a main part which is constant for all x beyond some point *p0* and a remaining part. From both of these parts points are sampled using the thinning method and the minimum of both is returned. Sampling from the first part is easier as we have a constant dominating hazard rate. Thus *p0* should be large. On the other hand, if *p0* is large than the thinning algorithm needs many iteration. Thus the performance of the the algorithm deponds on the choice of *p0*. We found that values close to the expectation of the generated distribution result in good performance.

How To Use

HRI requires a hazard function for a continuous distribution with non-decreasing hazard rate. The parameter *p0* should be set to a value close to the expectation of the required distribution using `unur_hri_set_p0`. If performance is crucial one may try other values as well.

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, only the left hand boundary is used. For computational reasons the right hand boundary is always reset to `UNUR_INFINITY`. If no domain is given by the user then the left hand boundary is set to `0`.

For distributions with decreasing hazard rate method HRD (see Section 5.3.8 [Hazard Rate Decreasing], page 113) is required. For distributions which do not have increasing or decreasing hazard rates but are bounded from above use method HRB (see Section 5.3.6 [Hazard Rate Bounded], page 111).

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Notice, that the upper bound given by the `unur_hrb_set_upperbound` call cannot be changed and must be valid for the changed distribution. Notice that the parameter *p0* which has been set by a `unur_hri_set_p0` call cannot be changed and must be valid for the changed distribution.

Function reference

`UNUR_PAR* unur_hri_new (const UNUR_DISTR* distribution)`
Get default parameters for generator.

`int unur_hri_set_p0 (UNUR_PAR* parameters, double p0)`
Set design point for algorithm. It is used to split the domain of the distribution. Values for *p0* close to the expectation of the distribution results in a relatively good performance of the algorithm. It is important that the hazard rate at this point must be greater than `0` and less than `UNUR_INFINITY`.

Default: left boundary of domain `+ 1.`

`int unur_hri_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_hri_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)

Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by the given bound, then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`.

Default is `FALSE`.

5.3.9 ITDR – Inverse Transformed Density Rejection

Required: monotone PDF, dPDF, pole

Optional: splitting point between pole and tail region, c-values

Speed: Set-up: moderate, Sampling: moderate

Reinit: supported

Reference: [HLDa07]

ITDR is an acceptance/rejection method that works for monotone densities. It is especially designed for PDFs with a single pole. It uses different hat functions for the pole region and for the tail region. For the tail region *Transformed Density Rejection* with a single construction point is used. For the pole region a variant called *Inverse Transformed Density Rejection* is used. The optimal splitting point between the two regions and the respective maximum local concavity and inverse local concavity (see Appendix B [Glossary], page 235) that guarantee valid hat functions for each regions are estimated. This splitting point is set to the intersection point of local concavity and inverse local concavity. However, it is assumed that both, the local concavity and the inverse local concavity do not have a local minimum in the interior of the domain (which is the case for all standard distributions with a single pole). In other cases (or when the built-in search routines do not compute non-optimal values) one can provide the splitting point, and the c-values.

How To Use

Method ITDR requires a distribution object with given PDF and its derivative and the location of the pole (or mode). The PDF must be monotone and may contain a pole. It must be set via the `unur_distr_cont_set_pdf` and `unur_distr_cont_set_dpdf` calls. The PDF should return UNUR_INFINITY for the pole. Alternatively, one can also set the logarithm of the PDF and its derivative via the `unur_distr_cont_set_logpdf` and `unur_distr_cont_set_dlogpdf` calls. This is in especially useful since then the setup and search routines are numerically more stable. Moreover, for many distributions computing the logarithm of the PDF is less expensive then computing the PDF directly.

The pole of the distribution is given by a `unur_distr_cont_set_mode` call. Notice that distributions with "heavy" poles may have numerical problems caused by the resultion of the floating point numbers used by computers. While the minimal distance between two different floating point numbers is about `1.e-320` near `0.` it increases to `1.e-16` near `1.` Thus any random variate generator implemented on a digital computer in fact draws samples from a discrete distribution that approximates the desired continuous distribution. For distributions with "heavy" poles not at 0 this approximation may be too crude and thus every goodness-of-fit test will fail. Besides this theoretic problem that cannot be resolved we have to take into consideration that round-off errors occur more frequently when we have PDFs with poles far away from `0.` Method ITDR tries to handles this situation as good as possible by moving the pole into `0.` Thus do not use a wrapper for your PDF that hides this shift since the information about the resolution of the floating point numbers near the pole gets lost.

Method ITDR uses different hats for the pole region and for the tail region. The splitting point between these two regions, the optimal c-value and design points for constructing the hats using Transformed Density Rejection are computed automatically. (The results of these computations can be read using the respective calls `unur_itdr_get_xi`, `unur_itdr_get_cp` , and `unur_itdr_get_ct` for the intersection point between local concavity and inverse local concavity, the c-value for the pole and the tail region.) However, one can also analyze the local concavity and inverse local concavity set the corresponding values using `unur_itdr_set_xi`, `unur_itdr_set_cp` , and `unur_itdr_set_ct` calls. Notice, that c-values greater than -1/2 can

be set to `-0.5`. Although this results in smaller acceptance probabities sampling from the hat distribution is much faster than for other values of c. Depending on the expenses of evaluating the PDF the resulting algorithm is usually faster.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. However, the values given by `unur_itdr_set_xi`, `unur_itdr_set_cp` , or `unur_itdr_set_ct` calls are then ignored when `unur_reinit` is called.

Function reference

`UNUR_PAR* unur_itdr_new` (*const UNUR_DISTR* distribution*)
Get default parameters for generator.

`int unur_itdr_set_xi` (*UNUR_PAR* parameters*, *double xi*)
Sets points where local concavity and inverse local concavity are (almost) equal. It is used to estimate the respective c-values for pole region and hat regions and to determine the splitting point bx between pole and tail region. If no such point is provided it will be computed automatically.

Default: not set.

`int unur_itdr_set_cp` (*UNUR_PAR* parameters*, *double cp*)
Sets parameter cp for transformation T for inverse density in pole region. It must be at most 0 and greater than -1. A value of `-0.5` is treated separately and usually results in faster marginal generation time (at the expense of smaller acceptance probabilities. If no cp-value is given it is estimated automatically.

Default: not set.

`int unur_itdr_set_ct` (*UNUR_PAR* parameters*, *double ct*)
Sets parameter ct for transformation T for density in tail region. It must be at most 0. For densities with unbounded domain it must be greater than -1. A value of `-0.5` is treated separately and usually results in faster marginal generation time (at the expense of smaller acceptance probabilities. If no ct-value is given it is estimated automatically.

Default: not set.

`double unur_itdr_get_xi` (*UNUR_GEN* generator*)
`double unur_itdr_get_cp` (*UNUR_GEN* generator*)
`double unur_itdr_get_ct` (*UNUR_GEN* generator*)
Get intersection point xi, and c-values cp and ct, respectively. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_itdr_get_area` (*UNUR_GEN* generator*)
Get area below hat. (In case of an error `UNUR_INFINITY` is returned.)

`int unur_itdr_set_verify` (*UNUR_PAR* parameters*, *int verify*)
Turn verifying of algorithm while sampling on/off.

If the condition $PDF(x) \leq hat(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However, notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_itdr_chg_verify` (*UNUR_GEN* generator*, *int verify*)
Change the verifying of algorithm while sampling on/off.

5.3.10 NINV – Numerical INVersion

Required: CDF

Optional: PDF

Speed: Set-up: optional, Sampling: (very) slow

Reinit: supported

NINV is the implementation of numerical inversion. For finding the root it is possible to choose between Newton's method and the regula falsi (combined with interval bisectioning). The regula falsi requires only the CDF while Newton's method also requires the PDF. To speed up the marginal generation time a table with suitable starting points can be computed in the setup. The performance of the algorithm can adjusted by desired accuracy of the method. It is possible to use this method for generating from truncated distributions. The truncated domain can be changed for an existing generator object.

How To Use

The method works generates random variates by numerical inversion and requires a continuous univariate distribution objects with given CDF. Two methods are available:

- Regula falsi [default]
- Newton's method

Newton's method additionally requires the PDF of the distribution and cannot be used otherwise (NINV automatically switches to regula falsi then. Default algorithm is regula falsi. It is slightly slower but numerically much more stable than Newton's algorithm.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an `unur_ninv_chg_truncated` call.

To speed up the marginal generation time a table with suitable starting points can be computed in the setup. Using such a table can be switched on by means of a `unur_ninv_set_table` call where the table size is given as a parameter. The table is still useful when the (truncated) domain is changed often, since it is computed for the domain of the given distribution. (It is not possible to enlarge this domain.) If it is necessary to recalculate the table during sampling, the command `unur_ninv_chg_table` can be used. As a rule of thumb using such a table is appropriate when the number of generated points exceeds the table size by a factor of 100.

The default number of iterations of NINV should be enough for all reasonable cases. Nevertheless, it is possible to adjust the maximal number of iterations with the commands `unur_ninv_set_max_iter` and `unur_ninv_chg_max_iter`. In particular this might be necessary when the PDF has a pole (where it is not bounded from below).

It is also possible to set/change the accuracy of the method (which also heavily influencies the generation time). For this it is possible to change the maximum error allowed in x with `unur_ninv_set_x_resolution` and `unur_ninv_chg_x_resolution`, respectively.

NINV tries to use proper starting values for both the regala falsi and Newton's method. Of course the user might have more knowledge about the properties of the underlying distribution and is able to share his wisdom with NINV using the respective commands `unur_ninv_set_start` and `unur_ninv_chg_start`

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. The values given by the last `unur_ninv_chg_truncated` call will be then changed to the values of the domain of the underlying distribution object. It is important to note that for a distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201) the normalization constant has

to be updated using the `unur_distr_cont_upd_pdfarea` call whenever its parameters have been changed by means of a `unur_distr_cont_set_pdfparams` call.

It might happen that NINV aborts `unur_sample_cont` without computing the correct value (because the maximal number iterations has been exceeded). Then the last approximate value for x is returned (with might be fairly false) and `unur_error` is set to `UNUR_ERR_GEN_SAMPLING`.

Function reference

`UNUR_PAR* unur_ninv_new` (*const UNUR_DISTR* distribution*)
Get default parameters for generator.

`int unur_ninv_set_useregula` (*UNUR_PAR* parameters*)
Switch to regula falsi combined with interval bisectioning. (This the default.)

`int unur_ninv_set_usenewton` (*UNUR_PAR* parameters*)
Switch to Newton's method. Notice that it is numerically less stable than regula falsi. It it is not possible to invert the CDF for a particular uniform random number U when calling `unur_sample_cont`, `unur_error` is set to `UNUR_ERR_GEN_SAMPLING`. Thus it is recommended to check `unur_error` before using the result of the sampling routine.

`int unur_ninv_set_max_iter` (*UNUR_PAR* parameters*, *int max_iter*)
`int unur_ninv_chg_max_iter` (*UNUR_GEN* generator*, *int max_iter*)
Set and change number of maximal iterations. Default is `40`.

`int unur_ninv_set_x_resolution` (*UNUR_PAR* parameters*, *double x_resolution*)
`int unur_ninv_chg_x_resolution` (*UNUR_GEN* generator*, *double x_resolution*)
Set and change the maximal relative error in x. Default is `1.e-8`.

`int unur_ninv_set_start` (*UNUR_PAR* parameters*, *double left*, *double right*)
Set starting points. If not set, suitable values are chosen automatically.

Newton:	*left*:	starting point
Regula falsi:	*left*, *right*:	boundary of starting interval

If the starting points are not set then the follwing points are used by default:

Newton:	*left*:	CDF(*left*) = 0.5
Regula falsi:	*left*:	CDF(*left*) = 0.1
	right:	CDF(*right*) = 0.9

If *left* == *right*, then UNU.RAN always uses the default starting points!

`int unur_ninv_chg_start` (*UNUR_GEN* gen*, *double left*, *double right*)
Change the starting points for numerical inversion. If left==right, then UNU.RAN uses the default starting points (see `unur_ninv_set_start`).

`int unur_ninv_set_table` (*UNUR_PAR* parameters*, *int no_of_points*)
Generates a table with *no_of_points* points containing suitable starting values for the iteration. The value of *no_of_points* must be at least 10 (otherwise it will be set to 10 automatically).

The table points are chosen such that the CDF at these points form an equidistance sequence in the interval (0,1).

If a table is used, then the starting points given by `unur_ninv_set_start` are ignored.

No table is used by default.

`int unur_ninv_chg_table` (*UNUR_GEN* gen*, *int no_of_points*)
Recomputes a table as described in `unur_ninv_set_table`.

`int unur_ninv_chg_truncated` (*UNUR_GEN* gen*, *double left*, *double right*)
Changes the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. Moreover the starting point(s) will not be changed.

Important: If the CDF is (almost) the same for *left* and *right* and (almost) equal to `0` or `1`, then the truncated domain is *not* chanced and the call returns an error code.

Notice: If the parameters of the distribution has been changed by a `unur_distr_cont_set_pdfparams` call it is recommended to set the truncated domain again, since the former call might change the domain of the distribution but not update the values for the boundaries of the truncated distribution.

`double unur_ninv_eval_approxinvcdf` (*UNUR_GEN* generator*, *double u*)
Get approximate approximate value of inverse CDF at *u*. If *u* is out of the domain [0,1] then `unur_errno` is set to `UNUR_ERR_DOMAIN` and the respective bound of the domain of the distribution are returned (which is `-UNUR_INFINITY` or `UNUR_INFINITY` in the case of unbounded domains).

Notice: This function always evaluates the inverse CDF of the given distribution. A call to `unur_ninv_chg_truncated` call has no effect.

5.3.11 NROU – Naive Ratio-Of-Uniforms method

Required: PDF

Optional: mode, center, bounding rectangle for acceptance region

Speed: Set-up: slow or fast, Sampling: moderate

Reinit: supported

Reference: [HLD04: Sect.2.4 and Sect.6.4]

NROU is an implementation of the (generalized) ratio-of-uniforms method which uses (minimal) bounding rectangles, see Section A.4 [Ratio-of-Uniforms], page 232. It uses a positive control parameter r for adjusting the algorithm to the given distribution to improve performance and/or to make this method applicable. Larger values of r increase the class of distributions for which the method works at the expense of a higher rejection constant. For computational reasons *r=1* should be used if possible (this is the default). Moreover, this implementation uses the center μ of the distribution (see `unur_distr_cont_get_center` for details of its default values).

For the special case with $r = 1$ the coordinates of the minimal bounding rectangles are given by

$$v^+ = \sup_x \sqrt{PDF(x)},$$
$$u^- = \inf_x (x-\mu)\sqrt{PDF(x)},$$
$$u^+ = \sup_x (x-\mu)\sqrt{PDF(x)},$$

where μ is the center of the distribution. For other values of r we have

$$v^+ = \sup_x (PDF(x))^{1/(r+1)},$$
$$u^- = \inf_x (x-\mu)(PDF(x))^{r/(r+1)},$$
$$u^+ = \sup_x (x-\mu)(PDF(x))^{r/(r+1)}.$$

These bounds can be given directly. Otherwise they are computed automatically by means of a (slow) numerical routine. Of course this routine can fail, especially when this rectangle is not bounded.

It is important to note that the algorithm works with $PDF(x-\mu)$ instead of $PDF(x)$. This is important as otherwise the acceptance region can become a very long and skinny ellipsoid along a diagonal of the (huge) bounding rectangle.

How To Use

For using the NROU method UNU.RAN needs the PDF of the distribution. Additionally, the parameter r can be set via a `unur_vnrou_set_r` call. Notice that the acceptance probability decreases when r is increased. On the other hand is is more unlikely that the bounding rectangle does not exist if r is small.

A bounding rectangle can be given by the `unur_vnrou_set_u` and `unur_vnrou_set_v` calls.

Important: The bounding rectangle has to be provided for the function $PDF(x - center)$! Notice that `center` is the center of the given distribution, see `unur_distr_cont_set_center`. If in doubt or if this value is not optimal, it can be changed (overridden) by a `unur_nrou_set_center` call.

If the coordinates of the bounding rectangle are not provided by the user then the minimal bounding rectangle is computed automatically.

By means of `unur_vnrou_set_verify` and `unur_vnrou_chg_verify` one can run the sampling algorithm in a checking mode, i.e., in every cycle of the rejection loop it is checked whether

the used rectangle indeed enclosed the acceptance region of the distribution. When in doubt (e.g., when it is not clear whether the numerical routine has worked correctly) this can be used to run a small Monte Carlo study.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that derived parameters like the mode must also be (re-) set if the parameters or the domain has be changed. Notice, however, that then the values that has been set by `unur_vnrou_set_u` and `unur_vnrou_set_v` calls are removed and the coordinates of the bounding box are computed numerically.

Function reference

`UNUR_PAR* unur_nrou_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_nrou_set_u` (*UNUR_PAR** `parameters`, *double* `umin`, *double* `umax`)
Sets left and right boundary of bounding rectangle. If no values are given, the boundary of the minimal bounding rectangle is computed numerically.

Notice: Computing the minimal bounding rectangle may fail under some circumstances. Moreover, for multimodal distributions the bounds might be too small as only local extrema are computed. Nevertheless, for T_c -concave distributions with $c = -1/2$ it should work.

Important: The bounding rectangle that has to be provided is for the function $PDF(x - center)$!

Default: not set.

`int unur_nrou_set_v` (*UNUR_PAR** `parameters`, *double* `vmax`)
Set upper boundary for bounding rectangle. If this value is not given then $\sqrt{PDF(mode)}$ is used instead.

Notice: When the mode is not given for the distribution object, then it will be computed numerically.

Default: not set.

`int unur_nrou_set_r` (*UNUR_PAR** `parameters`, *double* `r`)
Sets the parameter r of the generalized ratio-of-uniforms method.

Notice: This parameter must satisfy $r>0$.

Default: `1`.

`int unur_nrou_set_center` (*UNUR_PAR** `parameters`, *double* `center`)
Set the center (μ) of the PDF. If not set the center of the given distribution object is used.

Default: see `unur_distr_cont_set_center`.

`int unur_nrou_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
Turn verifying of algorithm while sampling on/off.

If the condition $PDF(x) \le hat(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However, notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_nrou_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Change the verifying of algorithm while sampling on/off.

5.3.12 SROU – Simple Ratio-Of-Uniforms method

Required: T-concave PDF, mode, area

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [LJa01] [LJa02] [HLD04: Sect.6.3.1; Sect.6.3.2; Sect.6.4.1; Alg.6.4; Alg.6.5; Alg.6.7]

SROU is based on the ratio-of-uniforms method (see Section A.4 [Ratio-of-Uniforms], page 232) that uses universal inequalities for constructing a (universal) bounding rectangle. It works for all T-concave distributions, including log-concave and T-concave distributions with $T(x) = -1/\sqrt{x}$.

Moreover an (optional) parameter `r` can be given, to adjust the generator to the given distribution. This parameter is strongly related to the parameter `c` for transformed density rejection (see Section 5.3.15 [TDR], page 132) via the formula *c = -r/(r+1)*. The rejection constant increases with higher values for `r`. On the other hand, the given density must be T_c -concave for the corresponding *c*. The default setting for `r` is 1 which results in a very simple code. (For other settings, sampling uniformly from the acceptance region is more complicated.)

Optionally the CDF at the mode can be given to increase the performance of the algorithm. Then the rejection constant is reduced by 1/2 and (if `r=1`) even a universal squeeze can (but need not be) used. A way to increase the performance of the algorithm when the CDF at the mode is not provided is the usage of the mirror principle (only if `r=1`). However, using squeezes and using the mirror principle is only recommended when the PDF is expensive to compute.

The exact location of the mode and/or the area below the PDF can be replace by appropriate bounds. Then the algorithm still works but has larger rejection constants.

How To Use

SROU works for any continuous univariate distribution object with given T_c -concave PDF with $c < 1$,) mode and area below PDF. Optional the CDF at the mode can be given to increase the performance of the algorithm by means of the `unur_srou_set_cdfatmode` call. Additionally squeezes can be used and switched on via `unur_srou_set_usesqueeze` (only if `r=1`). A way to increase the performance of the algorithm when the CDF at the mode is not provided is the usage of the mirror principle which can be swithced on by means of a `unur_srou_set_usemirror` call (only if `r=1`) . However using squeezes and using the mirror principle is only recommended when the PDF is expensive to compute.

The parameter `r` can be given, to adjust the generator to the given distribution. This parameter is strongly related parameter `c` for transformed density rejection via the formula *c = -r/(r+1)*. The parameter `r` can be any value larger than or equal to 1. Values less then 1 are automatically set to 1. The rejection constant depends on the chosen parameter `r` but not on the particular distribution. It is 4 for `r` equal to 1 and higher for higher values of `r`. It is important to note that different algorithms for different values of `r`: If `r` equal to 1 this is much faster than the algorithm for `r` greater than 1. The default setting for `r` is 1.

If the (exact) area below the PDF is not known, then an upper bound can be used instead (which of course increases the rejection constant). But then the squeeze flag must not be set and `unur_srou_set_cdfatmode` must not be used.

If the exact location of the mode is not known, then use the approximate location and provide the (exact) value of the PDF at the mode by means of the `unur_srou_set_pdfatmode` call. But then `unur_srou_set_cdfatmode` must not be used. Notice, that a (slow) numerical mode finder will be used if no mode is given at all. It is even possible to give an upper bound for the PDF only. However, then the (upper bound for the) area below the PDF has to be multiplied by the

ratio between the upper bound and the lower bound of the PDF at the mode. Again setting the squeeze flag and using `unur_srou_set_cdfatmode` is not allowed.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that derived parameters like the mode must also be (re-) set if the parameters or the domain has be changed. Moreover, if the PDF at the mode has been provided by a `unur_srou_set_pdfatmode` call, additionally `unur_srou_chg_pdfatmode` must be used (otherwise this call is not necessary since then this figure is computed directly from the PDF).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on by calling `unur_srou_set_verify` and `unur_srou_chg_verify`, respectively. Notice however that sampling is (a little bit) slower then.

Function reference

`UNUR_PAR* unur_srou_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_srou_set_r` (*UNUR_PAR** `parameters`, *double* `r`)
Set parameter r for transformation. Only values greater than or equal to 1 are allowed. The performance of the generator decreases when r is increased. On the other hand r must not be set to small, since the given density must be T_c-concave for $c = -r/(r+1)$.

Notice: If r is set to `1` a simpler and much faster algorithm is used then for r greater than one.

For computational reasons values of r that are greater than `1` but less than `1.01` are always set to `1.01`.

Default is `1`.

`int unur_srou_set_cdfatmode` (*UNUR_PAR** `parameters`, *double* `Fmode`)
Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_srou_chg_cdfatmode` has to be used to update this value.

Default: not set.

`int unur_srou_set_pdfatmode` (*UNUR_PAR** `parameters`, *double* `fmode`)
Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters of the distribution.

IMPORTANT: This call has to be executed after a possible call of `unur_srou_set_r`. Default: not set.

`int unur_srou_set_usesqueeze` (*UNUR_PAR** `parameters`, *int* `usesqueeze`)
Set flag for using universal squeeze (default: off). Using squeezes is only useful when the evaluation of the PDF is (extremely) expensive. Using squeezes is automatically disabled when the CDF at the mode is not given (then no universal squeezes exist).

Squeezes can only be used if `r=1`.

Default is `FALSE`.

`int unur_srou_set_usemirror (`*UNUR_PAR* parameters*, *int usemirror*`)`
Set flag for using mirror principle (default: off). Using the mirror principle is only useful when the CDF at the mode is not known and the evaluation of the PDF is rather cheap compared to the marginal generation time of the underlying uniform random number generator. It is automatically disabled when the CDF at the mode is given. (Then there is no necessity to use the mirror principle. However disabling is only done during the initialization step but not at a re-initialization step.)

The mirror principle can only be used if `r=1`.

Default is `FALSE`.

`int unur_srou_set_verify (`*UNUR_PAR* parameters*, *int verify*`)`
`int unur_srou_chg_verify (`*UNUR_GEN* generator*, *int verify*`)`
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_srou_chg_cdfatmode (`*UNUR_GEN* generator*, *double Fmode*`)`
Change CDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

`int unur_srou_chg_pdfatmode (`*UNUR_GEN* generator*, *double fmode*`)`
Change PDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

5.3.13 SSR – Simple Setup Rejection

Required: T-concave PDF, mode, area

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [LJa01] [HLD04: Sect.6.3.3; Alg.6.6]

SSR is an acceptance/rejection method that uses universal inequalities for constructing (universal) hats and squeezes (see Section A.2 [Rejection], page 230). It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

It requires the PDF, the (exact) location of the mode and the area below the given PDF. The rejection constant is 4 for all T-concave distributions with unbounded domain and is less than 4 when the domain is bounded. Optionally the CDF at the mode can be given to increase the performance of the algorithm. Then the rejection constant is at most 2 and a universal squeeze can (but need not be) used. However, using squeezes is not recommended unless the evaluation of the PDF is expensive.

The exact location of the mode and/or the area below the PDF can be replace by appropriate bounds. Then the algorithm still works but has larger rejection constants.

How To Use

SSR works for any continuous univariate distribution object with given T-concave PDF (with $T(x) = -1/\sqrt{x},$) mode and area below PDF. Optional the CDF at the mode can be given to increase the performance of the algorithm by means of the `unur_ssr_set_cdfatmode` call. Additionally squeezes can be used and switched on via `unur_ssr_set_usesqueeze`. If the (exact) area below the PDF is not known, then an upper bound can be used instead (which of course increases the rejection constant). But then the squeeze flag must not be set and `unur_ssr_set_cdfatmode` must not be used.

If the exact location of the mode is not known, then use the approximate location and provide the (exact) value of the PDF at the mode by means of the `unur_ssr_set_pdfatmode` call. But then `unur_ssr_set_cdfatmode` must not be used. Notice, that a (slow) numerical mode finder will be used if no mode is given at all. It is even possible to give an upper bound for the PDF only. However, then the (upper bound for the) area below the PDF has to be multiplied by the ratio between the upper bound and the lower bound of the PDF at the mode. Again setting the squeeze flag and using `unur_ssr_set_cdfatmode` is not allowed.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that derived parameters like the mode must also be (re-) set if the parameters or the domain has be changed. Moreover, if the PDF at the mode has been provided by a `unur_ssr_set_pdfatmode` call, additionally `unur_ssr_chg_pdfatmode` must be used (otherwise this call is not necessary since then this figure is computed directly from the PDF).

Important: If any of mode, PDF or CDF at the mode, or the area below the mode has been changed, then `unur_reinit` must be executed. (Otherwise the generator produces garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on/off by calling `unur_ssr_set_verify` and `unur_ssr_chg_verify`, respectively. Notice, however, that sampling is (a little bit) slower then.

Function reference

`UNUR_PAR* unur_ssr_new (const UNUR_DISTR* distribution)`

Get default parameters for generator.

`int unur_ssr_set_cdfatmode (`*UNUR_PAR* parameters*, *double Fmode*`)`
Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_ssr_chg_cdfatmode` has to be used to update this value.

Default: not set.

`int unur_ssr_set_pdfatmode (`*UNUR_PAR* parameters*, *double fmode*`)`
Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters for the distribution.

Default: not set.

`int unur_ssr_set_usesqueeze (`*UNUR_PAR* parameters*, *int usesqueeze*`)`
Set flag for using universal squeeze (default: off). Using squeezes is only useful when the evaluation of the PDF is (extremely) expensive. Using squeezes is automatically disabled when the CDF at the mode is not given (then no universal squeezes exist).

Default is `FALSE`.

`int unur_ssr_set_verify (`*UNUR_PAR* parameters*, *int verify*`)`
`int unur_ssr_chg_verify (`*UNUR_GEN* generator*, *int verify*`)`
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_ssr_chg_cdfatmode (`*UNUR_GEN* generator*, *double Fmode*`)`
Change CDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

`int unur_ssr_chg_pdfatmode (`*UNUR_GEN* generator*, *double fmode*`)`
Change PDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

5.3.14 TABL – a TABLe method with piecewise constant hats

Required: PDF, all local extrema, cut-off values for the tails

Optional: approximate area

Speed: Set-up: (very) slow, Sampling: fast

Reinit: not implemented

Reference: [AJa93] [AJa95] [HLD04: Cha.5.1]

TABL (called Ahrens method in [HLD04]) is an acceptance/rejection method (see Section A.2 [Rejection], page 230) that uses a decomposition of the domain of the distribution into many short subintervals. Inside of these subintervals constant hat and squeeze functions are utilized. Thus it is easy to use the idea of immediate acceptance for points below the squeeze. This reduces the expected number of uniform random numbers per generated random variate to less than two. Using a large number of subintervals only little more than one random number is necessary on average. Thus this method becomes very fast.

Due to the constant hat function this method only works for distributions with bounded domains. Thus for unbounded domains the left and right tails have to be cut off. This is no problem when the probability of falling into these tail regions is beyond computational relevance (e.g. smaller than `1.e-12`).

For easy construction of hat and squeeze functions it is necessary to know the regions of monotonicity (called *slopes*) or equivalently all local maxima and minima of the density. The main problem for this method in the setup is the choice of the subintervals. A simple and close to optimal approach is the "equal area rule" [HLD04: Cha.5.1] . There the subintervals are selected such that the area below the hat is the same for each subinterval which can be realized with a simple recursion. If more subintervals are necessary it is possible to split either randomly chosen intervals (adaptive rejection sampling, ARS) or those intervals, where the ratio between squeeze and hat is smallest. This version of the setup is called derandomized ARS (DARS). With the default settings TABL is first calculating approximately 30 subintervals with the equal area rule. Then DARS is used till the desired fit of the hat is reached.

A convenient measure to control the quality of the fit of hat and squeeze is the ratio (area below squeeze)/(area below hat) called `sqhratio` which must be smaller or equal to one. The expected number of iterations in the rejection algorithm is known to be smaller than 1/sqhratio and the expected number of evaluations of the density is bounded by `1/sqhratio - 1`. So values of the sqhratio close to one (e.g. `0.95` or `0.99`) lead to many subintervals. Thus a better fitting hat is constructed and the sampling algorithm becomes fast; on the other hand large tables are needed and the setup is very slow. For moderate values of sqhratio (e.g. `0.9` or `0.8`) the sampling is slower but the required tables are smaller and the setup is not so slow.

It follows from the above explanations that TABL is always requiring a slow setup and that it is not very well suited for heavy-tailed distributions.

How To Use

For using the TABL method UNU.RAN needs a bounded interval to which the generated variates can be restricted and information about all local extrema of the distribution. For unimodal densities it is sufficient to provide the mode of the distribution. For the case of a built-in unimodal distribution with bounded domain all these information is present in the distribution object and thus no extra input is necessary (see example_TABL1 below).

For a built-in unimodal distribution with unbounded domain we should specify the cut-off values for the tails. This can be done with the `unur_tabl_set_boundary` call (see example_TABL2

below). For the case that we do not set these boundaries the default values of `+/- 1.e20` are used. We can see in example_TABL1 that this still works fine for many standard distributions.

For the case of a multimodal distribution we have to set the regions of monotonicity (called slopes) explicitly using the `unur_tabl_set_slopes` command (see example_TABL3 below).

To controll the fit of the hat and the size of the tables and thus the speed of the setup and the sampling it is most convenient to use the `unur_tabl_set_max_sqhratio` call. The default is `0.9` which is a sensible value for most distributions and applications. If very large samples of a distribution are required or the evaluation of a density is very slow it may be useful to increase the sqhratio to eg. `0.95` or even `0.99`. With the `unur_tabl_get_sqhratio` call we can check which sqhratio was really reached. If that value is below the desired value it is necessary to increase the maximal number of subintervals, which defaults to `1000`, using the `unur_tabl_set_max_intervals` call. The `unur_tabl_get_n_intervals` call can be used to find out the number of subintervals the setup calculated.

It is also possible to set the number of intervals and their respective boundaries by means of the `unur_tabl_set_cpoints` call.

It is also possible to use method TABL for correlation induction (variance reduction) by setting of an auxiliary uniform random number generator via the `unur_set_urng_aux` call. (Notice that this must be done after a possible `unur_set_urng` call.) However, this only works when immediate acceptance is switched of by a `unur_tabl_set_variant_ia` call.

Function reference

`UNUR_PAR* unur_tabl_new` (*const UNUR_DISTR* distribution*)
Get default parameters for generator.

`int unur_tabl_set_variant_ia` (*UNUR_PAR* parameters*, *int use_ia*)
Use immediate acceptance when *use_ia* is set to `TRUE`. This technique requires less uniform. If it is set to `FALSE`, "classical" acceptance/rejection from hat distribution is used.

Notice: Auxiliary uniform random number generators for correlation induction (variance reduction) can only be used when "classical" acceptance/rejection is used.

Default: `TRUE`.

`int unur_tabl_set_cpoints` (*UNUR_PAR* parameters*, *int n_cpoints*, *const double* cpoints*)
Set construction points for the hat function. If *stp* is `NULL` than a heuristic rule of thumb is used to get *n_stp* construction points. This is the default behavior.

The default number of construction points is `30`.

`int unur_tabl_set_nstp` (*UNUR_PAR* parameters*, *int n_stp*)
Set number of construction points for the hat function. *n_stp* must be greater than zero. After the setup there are about *n_stp* construction points. However it might be larger when a small fraction is given by the `unur_tabl_set_areafraction` call. It also might be smaller for some variants.

Default is `30`.

`int unur_tabl_set_useear` (*UNUR_PAR* parameters*, *int useear*)
If *useear* is set to `TRUE`, the "equal area rule" is used, the given slopes are partitioned in such a way that the area below the hat function in each subinterval ("stripe") has the same area (except the last the last interval which can be smaller). The area can be set by means of the `unur_tabl_set_areafraction` call.

Default is `TRUE`.

`int unur_tabl_set_areafraction` (*UNUR_PAR** `parameters`, *double* `fraction`)
Set parameter for the equal area rule. During the setup a piecewise constant hat is constructed, such that the area below each of these pieces (strips) is the same and equal to the (given) area below the PDF times *fraction* (which must be greater than zero).

Important: If the area below the PDF is not set in the distribution object, then 1 is assumed.

Default is `0.1`.

`int unur_tabl_set_usedars` (*UNUR_PAR** `parameters`, *int* `usedars`)
If *usedars* is set to `TRUE`, "derandomized adaptive rejection sampling" (DARS) is used in the setup. Intervals, where the area between hat and squeeze is too large compared to the average area between hat and squeeze over all intervals, are split. This procedure is repeated until the ratio between squeeze and hat exceeds the bound given by `unur_tabl_set_max_sqhratio` call or the maximum number of intervals is reached. Moreover, it also aborts when no more intervals can be found for splitting.

For finding splitting points the arc-mean rule (a mixture of arithmetic mean and harmonic mean) is used.

Default is `TRUE`.

`int unur_tabl_set_darsfactor` (*UNUR_PAR** `parameters`, *double* `factor`)
Set factor for "derandomized adaptive rejection sampling". This factor is used to determine the segments that are "too large", that is, all segments where the area between squeeze and hat is larger than *factor* times the average area over all intervals between squeeze and hat. Notice that all segments are split when *factor* is set to `0.`, and that there is no splitting at all when *factor* is set to `UNUR_INFINITY`.

Default is `0.99`. There is no need to change this parameter.

`int unur_tabl_set_variant_splitmode` (*UNUR_PAR** `parameters`, *unsigned* `splitmode`)
There are three variants for adaptive rejection sampling. These differ in the way how an interval is split:

splitmode `1`
: use the generated point to split the interval.

splitmode `2`
: use the mean point of the interval.

splitmode `3`
: use the arcmean point; suggested for distributions with heavy tails.

Default is splitmode `2`.

`int unur_tabl_set_max_sqhratio` (*UNUR_PAR** `parameters`, *double* `max_ratio`)
Set upper bound for the ratio (area below squeeze) / (area below hat). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via DARS in the setup.

For the case of ARS (unur_tabl_set_usedars() must be set to `FALSE`): Use `0` if no construction points should be added after the setup. Use `1` if added new construction points should not be stopped until the maximum number of construction points is reached. If *max_ratio* is close to one, many construction points are used.

Default is `0.9`.

`double unur_tabl_get_sqhratio (const UNUR_GEN* generator)`
Get the current ratio (area below squeeze) / (area below hat) for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_tabl_get_hatarea (const UNUR_GEN* generator)`
Get the area below the hat for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_tabl_get_squeezearea (const UNUR_GEN* generator)`
Get the area below the squeeze for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`int unur_tabl_set_max_intervals (UNUR_PAR* parameters, int max_ivs)`
Set maximum number of intervals. No construction points are added in or after the setup when the number of intervals suceeds *max_ivs*.

Default is `1000`.

`int unur_tabl_get_n_intervals (const UNUR_GEN* generator)`
Get the current number of intervals. (In case of an error 0 is returned.)

`int unur_tabl_set_slopes (UNUR_PAR* parameters, const double* slopes, int n_slopes)`
Set slopes for the PDF. A slope <a,b> is an interval [a,b] or [b,a] where the PDF is monotone and PDF(a) >= PDF(b). The list of slopes is given by an array *slopes* where each consecutive tuple (i.e. `(slopes[0], slopes[1])`, `(slopes[2], slopes[3])`, etc.) defines one slope. Slopes must be sorted (i.e. both `slopes[0]` and `slopes[1]` must not be greater than any entry of the slope `(slopes[2], slopes[3])`, etc.) and must not be overlapping. Otherwise no slopes are set and *unur_errno* is set to `UNUR_ERR_PAR_SET`.

Notice: *n_slopes* is the number of slopes (and not the length of the array *slopes*).

Notice that setting slopes resets the given domain for the distribution. However, in case of a standard distribution the area below the PDF is not updated.

`int unur_tabl_set_guidefactor (UNUR_PAR* parameters, double factor)`
Set factor for relative size of the guide table for indexed search (see also method DGT Section 5.8.4 [DGT], page 176). It must be greater than or equal to `0`. When set to `0`, then sequential search is used.

Default is `1`.

`int unur_tabl_set_boundary (UNUR_PAR* parameters, double left, double right)`
Set the left and right boundary of the computation interval. The piecewise hat is only constructed inside this interval. The probability outside of this region must not be of computational relevance. Of course `+/- UNUR_INFINITY` is not allowed.

Default is `-1.e20,1.e20`.

`int unur_tabl_chg_truncated (UNUR_GEN* gen, double left, double right)`
Change the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. The hat function will not be changed.

Important: The ratio between the area below the hat and the area below the squeeze changes when the sampling region is restricted. In particalur it becomes (very) large when sampling from the (far) tail of the distribution. Then it is better to create a generator object for the tail of distribution only.

Important: This call does not work for variant `IA` (immediate acceptance). In this case UNU.RAN switches *automatically* to variant `RH` (use "classical" acceptance/rejection from hat distribution) and does revert to the variant originally set by the user.

Important: It is not a good idea to use adaptave rejection sampling while sampling from a domain that is a strict subset of the domain that has been used to construct the hat. For that reason adaptive adding of construction points is *automatically disabled* by this call.

Important: If the CDF of the hat is (almost) the same for *left* and *right* and (almost) equal to `0` or `1`, then the truncated domain is not changed and the call returns an error code.

`int unur_tabl_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_tabl_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)

Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_tabl_set_pedantic` (*UNUR_PAR** `parameters`, *int* `pedantic`)

Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not monotone in the given slopes.

With *pedantic* being `TRUE`, the sampling routine is exchanged by a routine that simply returns `UNUR_INFINITY` indicating an error.

Default is `FALSE`.

5.3.15 TDR – Transformed Density Rejection

Required: T-concave PDF, dPDF

Optional: mode

Speed: Set-up: slow, Sampling: fast

Reinit: supported

Reference: [GWa92] [HWa95] [HLD04: Cha.4]

TDR is an acceptance/rejection method that uses the concavity of a transformed density to construct hat function and squeezes automatically. Such PDFs are called T-concave. Currently the following transformations are implemented and can be selected by setting their `c`-values by a `unur_tdr_set_c` call:

`c = 0` T(x) = log(x)

`c = -0.5` T(x) = -1/sqrt(x) (Default)

In future releases the transformations T(x) = -(x)^c will be available for any c with 0 > c > -1. Notice that if a PDF is T-concave for a c then it also T-concave for every c'<c. However the performance decreases when c' is smaller than c. For computational reasons we suggest the usage of c = -0.5 (this is the default). For c <= -1 the hat is not bounded any more if the domain of the PDF is unbounded. But in the case of a bounded domain using method TABL is preferred to a TDR with c < -1 (except in a few special cases).

We offer three variants of the algorithm.

`GW` squeezes between construction points

`PS` squeezes proportional to hat function (Default)

`IA` same as variant PS but uses a compositon method with "immediate acceptance" in the region below the squeeze.

`GW` has a slightly faster setup but higher marginal generation times. `PS` is faster than `GW`. `IA` uses less uniform random numbers and is therefore faster than `PS`.

It is also possible to evaluate the inverse of the CDF of the hat distribution directly using the `unur_tdr_eval_invcdfhat` call.

There are lots of parameters for these methods, see below.

It is possible to use this method for correlation induction by setting an auxiliary uniform random number generator via the `unur_set_urng_aux` call. (Notice that this must be done after a possible `unur_set_urng` call.) When an auxiliary generator is used then the number of uniform random numbers from the first URNG that are used for one generated random variate is constant and given in the following table:

`GW ... 2`

`PS ... 2`

`IA ... 1`

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_tdr_set_verify` and `unur_tdr_chg_verify`, respectively. Notice however that sampling is (much) slower then.

For densities with modes not close to 0 it is suggested to set either the mode or the center of the distribution by the `unur_distr_cont_set_mode` or `unur_distr_cont_set_center` call. The latter is the approximate location of the mode or the mean of the distribution. This location

provides some information about the main part of the PDF and is used to avoid numerical problems.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an `unur_tdr_chg_truncated` call.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Important: The ratio between the area below the hat and the area below the squeeze changes when the sampling region is restricted. Especially it becomes (very) small when sampling from the (far) tail of the distribution. Then it is better to create a new generator object for the tail of the distribution only.

Function reference

`UNUR_PAR* unur_tdr_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_tdr_set_c` (*UNUR_PAR** `parameters`, *double* `c`)
Set parameter c for transformation T. Currently only values between 0 and -0.5 are allowed. If `c` is between 0 and -0.5 it is set to -0.5.

Default is `-0.5`.

`int unur_tdr_set_variant_gw` (*UNUR_PAR** `parameters`)
Use original version with squeezes between construction points as proposed by Gilks & Wild (1992).

`int unur_tdr_set_variant_ps` (*UNUR_PAR** `parameters`)
Use squeezes proportional to the hat function. This is faster than the original version. This is the default.

`int unur_tdr_set_variant_ia` (*UNUR_PAR** `parameters`)
Use squeezes proportional to the hat function together with a composition method that required less uniform random numbers.

`int unur_tdr_set_usedars` (*UNUR_PAR** `parameters`, *int* `usedars`)
If *usedars* is set to `TRUE`, "derandomized adaptive rejection sampling" (DARS) is used in setup. Intervals where the area between hat and squeeze is too large compared to the average area between hat and squeeze over all intervals are split. This procedure is repeated until the ratio between area below squeeze and area below hat exceeds the bound given by `unur_tdr_set_max_sqhratio` call or the maximum number of intervals is reached. Moreover, it also aborts when no more intervals can be found for splitting.

For finding splitting points the following rules are used (in this order, i.e., is if the first rule cannot be applied, the next one is used):

1. Use the expected value of adaptive rejection sampling.
2. Use the arc-mean rule (a mixture of arithmetic mean and harmonic mean).
3. Use the arithmetic mean of the interval boundaries.

Notice, however, that for unbounded intervals neither rule 1 nor rule 3 can be used.

As an additional feature, it is possible to choose amoung these rules. If *usedars* is set to `1` or `TRUE` the expected point (rule 1) is used (it switches to rule 2 for a particular interval if rule 1 cannot be applied). If it is set to `2` the arc-mean rule is used. If it is set to `3` the mean

is used. Notice that rule 3 can only be used if the domain of the distribution is bounded. It is faster than the other two methods but for heavy-tailed distribution and large domain the hat converges extremely slowly.

The default depends on the given construction points. If the user has provided such points via a `unur_tdr_set_cpoints` call, then *usedars* is set to `FALSE` by default, i.e., there is no further splitting. If the user has only given the number of construction points (or only uses the default number), then *usedars* is set to `TRUE` (i.e., use rule 1).

`int unur_tdr_set_darsfactor (`*UNUR_PAR* parameters*, *double* `factor`)

Set factor for "derandomized adaptive rejection sampling". This factor is used to determine the intervals that are "too large", that is, all intervals where the area between squeeze and hat is larger than *factor* times the average area over all intervals between squeeze and hat. Notice that all intervals are split when *factor* is set to `0.`, and that there is no splitting at all when *factor* is set to `UNUR_INFINITY`.

Default is `0.99`. There is no need to change this parameter.

`int unur_tdr_set_cpoints (`*UNUR_PAR* parameters*, *int* `n_stp`, *const double** `stp`)

Set construction points for the hat function. If *stp* is `NULL` than a heuristic rule of thumb is used to get *n_stp* construction points. This is the default behavior.

The default number of construction points is 30.

`int unur_tdr_set_reinit_percentiles (`*UNUR_PAR* parameters*, *int* `n_percentiles`, *const double** `percentiles`)
`int unur_tdr_chg_reinit_percentiles (`*UNUR_GEN* generator*, *int* `n_percentiles`, *const double** `percentiles`)

By default, when the *generator* object is reinitialized, it used the same construction points as for the initialization procedure. Often the underlying distribution object has been changed only moderately. For example, the full conditional distribution of a multivariate distribution. In this case it might be more appropriate to use percentilesm of the hat function for the last (unchanged) distribution. *percentiles* must then be a pointer to an ordered array of numbers between `0.01` and `0.99`. If *percentiles* is `NULL`, then a heuristic rule of thumb is used to get *n_percentiles* values for these percentiles. Notice that *n_percentiles* must be at least `2`, otherwise defaults are used. (Then the first and third quartiles are used by default.)

`int unur_tdr_set_reinit_ncpoints (`*UNUR_PAR* parameters*, *int* `ncpoints`)
`int unur_tdr_chg_reinit_ncpoints (`*UNUR_GEN* generator*, *int* `ncpoints`)

When reinit fails with the given construction points or the percentiles of the old hat function, another trial is undertaken with *ncpoints* construction points. *ncpoints* must be at least `10`.

Default: `50`

`int unur_tdr_chg_truncated (`*UNUR_GEN* gen*, *double* `left`, *double* `right`)

Change the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. The hat function will not be changed and there is no need to run `unur_reinit`. *Important:* The ratio between the area below the hat and the area below the squeeze changes when the sampling region is restricted. In particular it becomes (very) large when sampling from the (far) tail of the distribution. Then it is better to create a generator object for the tail of distribution only.

Important: This call does not work for variant `IA` (immediate acceptance). In this case UNU.RAN switches *automatically* to variant `PS`.

Important: It is not a good idea to use adaptave rejection sampling while sampling from a domain that is a strict subset of the domain that has been used to construct the hat. For that reason adaptive adding of construction points is *automatically disabled* by this call.

Important: If the CDF of the hat is (almost) the same for *left* and *right* and (almost) equal to `0` or `1`, then the truncated domain is not changed and the call returns an error code.

`int unur_tdr_set_max_sqhratio` (*UNUR_PAR** `parameters`, *double* `max_ratio`)
Set upper bound for the ratio (area below squeeze) / (area below hat). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via adaptive rejection sampling. Use 0 if no construction points should be added after the setup. Use 1 if added new construction points should not be stopped until the maximum number of construction points is reached.

Default is `0.99`.

`double unur_tdr_get_sqhratio` (*const UNUR_GEN** `generator`)
Get the current ratio (area below squeeze) / (area below hat) for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_tdr_get_hatarea` (*const UNUR_GEN** `generator`)
Get the area below the hat for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`double unur_tdr_get_squeezearea` (*const UNUR_GEN** `generator`)
Get the area below the squeeze for the generator. (In case of an error `UNUR_INFINITY` is returned.)

`int unur_tdr_set_max_intervals` (*UNUR_PAR** `parameters`, *int* `max_ivs`)
Set maximum number of intervals. No construction points are added after the setup when the number of intervals suceeds *max_ivs*. It is increased automatically to twice the number of construction points if this is larger.

Default is `100`.

`int unur_tdr_set_usecenter` (*UNUR_PAR** `parameters`, *int* `usecenter`)
Use the center as construction point. Default is `TRUE`.

`int unur_tdr_set_usemode` (*UNUR_PAR** `parameters`, *int* `usemode`)
Use the (exact!) mode as construction point. Notice that the behavior of the algorithm is different to simply adding the mode in the list of construction points via a `unur_tdr_set_cpoints` call. In the latter case the mode is treated just like any other point. However, when `usemode` is `TRUE`, the tangent in the mode is always set to 0. Then the hat of the transformed density can never cut the x-axis which must never happen if c < 0, since otherwise the hat would not be bounded.

Default is `TRUE`.

`int unur_tdr_set_guidefactor` (*UNUR_PAR** `parameters`, *double* `factor`)
Set factor for relative size of the guide table for indexed search (see also method DGT Section 5.8.4 [DGT], page 176). It must be greater than or equal to `0`. When set to `0`, then sequential search is used.

Default is 2.

`int unur_tdr_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_tdr_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)

Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_tdr_set_pedantic` (*UNUR_PAR** `parameters`, *int* `pedantic`)

Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not T-concave.

With *pedantic* being `TRUE`, the sampling routine is exchanged by a routine that simply returns `UNUR_INFINITY`. Otherwise the new point is not added to the list of construction points. At least the hat function remains T-concave.

Setting *pedantic* to `FALSE` allows sampling from a distribution which is "almost" T-concave and small errors are tolerated. However it might happen that the hat function cannot be improved significantly. When the hat functions that has been constructed by the `unur_init` call is extremely large then it might happen that the generation times are extremely high (even hours are possible in extremely rare cases).

Default is `FALSE`.

`double unur_tdr_eval_invcdfhat` (*const UNUR_GEN** `generator`, *double* `u`, *double** `hx`, *double** `fx`, *double** `sqx`)

Evaluate the inverse of the CDF of the hat distribution at u. As a side effect the values of the hat, the density, and the squeeze at the computed point x are stored in *hx*, *fx*, and *sqx*, respectively. However, these computations are suppressed if the corresponding variable is set to `NULL`.

If u is out of the domain [0,1] then `unur_errno` is set to `UNUR_ERR_DOMAIN` and the respective bound of the domain of the distribution are returned (which is `-UNUR_INFINITY` or `UNUR_INFINITY` in the case of unbounded domains).

Important: This call does not work for variant `IA` (immediate acceptance). In this case the hat CDF is evaluated as if variant `PS` is used.

Notice: This function always evaluates the inverse CDF of the hat distribution. A call to `unur_tdr_chg_truncated` call has no effect.

5.3.16 UTDR – Universal Transformed Density Rejection

Required: T-concave PDF, mode, approximate area

Speed: Set-up: moderate, Sampling: Moderate

Reinit: supported

Reference: [HWa95] [HLD04: Sect.4.5.4; Alg.4.4]

UTDR is based on the transformed density rejection and uses three almost optimal points for constructing hat and squeezes. It works for all T-concave distributions with $T(x) = -1/\sqrt{(x)}$.

It requires the PDF and the (exact) location of the mode. Notice that if no mode is given at all, a (slow) numerical mode finder will be used. Moreover the approximate area below the given PDF is used. (If no area is given for the distribution the algorithm assumes that it is approximately 1.) The rejection constant is bounded from above by 4 for all T-concave distributions.

How To Use

UTDR works for any continuous univariate distribution object with given T-concave PDF (with $T(x) = -1/\sqrt{x},$) mode and approximate area below PDF.

When the PDF does not change at the mode for varying parameters, then this value can be set with `unur_utdr_set_pdfatmode` to avoid some computations. Since this value will not be updated any more when the parameters of the distribution are changed, the `unur_utdr_chg_pdfatmode` call is necessary to do this manually.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that derived parameters like the mode must also be (re-) set if the parameters or the domain has be changed. Moreover, if the PDF at the mode has been provided by a `unur_utdr_set_pdfatmode` call, additionally `unur_utdr_chg_pdfatmode` must be used (otherwise this call is not necessary since then this figure is computed directly from the PDF).

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_utdr_set_verify` and `unur_utdr_chg_verify`, respectively. Notice however that sampling is slower then.

Function reference

`UNUR_PAR* unur_utdr_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_utdr_set_pdfatmode` (*UNUR_PAR** `parameters`, *double* `fmode`)
Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters for the distribution.

Default: not set.

`int unur_utdr_set_cpfactor` (*UNUR_PAR** `parameters`, *double* `cp_factor`)
Set factor for position of left and right construction point. The *cp_factor* is used to find almost optimal construction points for the hat function. There is no need to change this factor in almost all situations.

Default is `0.664`.

`int unur_utdr_set_deltafactor` (*UNUR_PAR** `parameters`, *double* `delta`)
Set factor for replacing tangents by secants. higher factors increase the rejection constant but reduces the risk of serious round-off errors. There is no need to change this factor it almost all situations.

Default is `1.e-5`.

`int unur_utdr_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_utdr_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_utdr_chg_pdfatmode` (*UNUR_GEN** `generator`, *double* `fmode`)
Change PDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

5.4 Methods for continuous empirical univariate distributions

Overview of methods

Methods for **continuous empirical univariate distributions**
sample with `unur_sample_cont`

EMPK: Requires an observed sample.
EMPL: Requires an observed sample.

Example

```
/* ---------------------------------------------------------- */
/* File: example_emp.c                                        */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

/* Example how to sample from an empirial continuous univariate  */
/* distribution.                                              */

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;
  double x;

  /* data points                                              */
  double data[15] = { -0.1,  0.05, -0.5,   0.08,  0.13,\
      -0.21,-0.44, -0.43, -0.33, -0.3, \
       0.18, 0.2,  -0.37, -0.29, -0.9 };

  /* Declare the three UNURAN objects.                        */
  UNUR_DISTR *distr;    /* distribution object                */
  UNUR_PAR   *par;      /* parameter object                   */
  UNUR_GEN   *gen;      /* generator object                   */

  /* Create a distribution object and set empirical sample.   */
  distr = unur_distr_cemp_new();
  unur_distr_cemp_set_data(distr, data, 15);

  /* Choose a method: EMPK.                                   */
  par = unur_empk_new(distr);

  /* Set smooting factor.                                     */
  unur_empk_set_smoothing(par, 0.8);

  /* Create the generator object.                             */
  gen = unur_init(par);

  /* It is important to check if the creation of the generator   */
  /* object was successful. Otherwise 'gen' is the NULL pointer  */
  /* and would cause a segmentation fault if used for sampling.  */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }
```

```
  /* It is possible to reuse the distribution object to create   */
  /* another generator object. If you do not need it any more,   */
  /* it should be destroyed to free memory.                      */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------------- */
```

Example (String API)

```
/* ---------------------------------------------------------------- */
/* File: example_emp_str.c                                          */
/* ---------------------------------------------------------------- */
/* String API.                                                      */
/* ---------------------------------------------------------------- */

/* Include UNURAN header file.                                      */
#include <unuran.h>

/* ---------------------------------------------------------------- */

/* Example how to sample from an empirial continuous univariate  */
/* distribution.                                                    */

/* ---------------------------------------------------------------- */

int main(void)
{
  int    i;
  double x;

  /* Declare UNURAN generator object.                               */
  UNUR_GEN   *gen;      /* generator object                         */

  /* Create the generator object.                                   */
  gen = unur_str2gen("distr = cemp; \
                        data=(-0.10, 0.05,-0.50, 0.08, 0.13,   \
                              -0.21,-0.44,-0.43,-0.33,-0.30,   \
                               0.18, 0.20,-0.37,-0.29,-0.90) & \
                     method=empk; smoothing=0.8");

  /* It is important to check if the creation of the generator   */
  /* object was successful. Otherwise 'gen' is the NULL pointer  */
  /* and would cause a segmentation fault if used for sampling.  */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
```

```
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ------------------------------------------------------------ */
```

5.4.1 EMPK – EMPirical distribution with Kernel smoothing

Required: observed sample

Speed: Set-up: slow (as sample is sorted), Sampling: fast (depends on kernel)

Reinit: not implemented

Reference: [HLa00] [HLD04: Sect.12.1.2]

EMPK generates random variates from an empirical distribution that is given by an observed sample. The idea is that simply choosing a random point from the sample and to return it with some added noise results in a method that has very nice properties, as it can be seen as sampling from a kernel density estimate. If the underlying distribution is continuous, especially the fine structur of the resulting empirical distribution is much better than using only resampling without noise.

Clearly we have to decide about the density of the noise (called kernel) and about the standard deviation of the noise. The mathematical theory of kernel density estimation shows us that we are comparatively free in choosing the kernel. It also supplies us with a simple formula to compute the optimal standarddeviation of the noise, called bandwidth (or window width) of the kernel.

The variance of the estimated density is slightly larger than that of the observed sample. However, this can be easily corrected if required.

There is also a correction (mirroring technique) for distributions with non-negative support.

A simple robust reference method is implemented to find a good standard deviation of the noise (i.e. the bandwidth of kernel density estimation). For some cases (e.g. densities with two or more sharp distinct peaks) there kernel density estimation can be adjusted by changing the smoothness factor and the so called beta factor.

How To Use

EMPK uses empirical distributions. The main parameter is the choice if of kernel density. The most important kernels can be set by `unur_empk_set_kernel`. Additionally generators for other kernels can be used by using `unur_empk_set_kernelgen` instead. Additionally variance correction and a correction for non-negative variates can be switched on.

The two other parameters (smoothing factor and beta factor) are only useful for people knowing the theory of kernel density estimation. It is not necessary to change them if the true underlying distribution is somehow comparable with a bell-shaped curve, even skewed or with some not too sharp extra peaks. In all these cases the simple robust reference method implemented to find a good standard deviation of the noise (i.e. the bandwidth of kernel density estimation) should give sensible results. However, it might be necessary to overwrite this automatic method to find the bandwidth eg. when resampling from data with two or more sharp distinct peaks. Then the distribution has nearly discrete components as well and our automatic method may easily choose too large a bandwidth which results in an empirical distribution which is oversmoothed (i.e. it has lower peaks than the original distribution). Then it is recommended to decrease the bandwidth using the `unur_empk_set_smoothing` call. A smoothing factor of `1` is the default. A smoothing factor of `0` leads to naive resampling of the data. Thus an appropriate value between these extremes should be choosen. We recommend to consult a reference on kernel smoothing when doing so; but it is not a simple problem to determine an optimal bandwidth for distributions with sharp peaks.

In general, for most applications it is perfectly ok to use the default values offered. Unless you have some knowledge on density estimation we do not recommend to change anything. There are two exceptions:

A. In the case that the unknown underlying distribution is not continuous but discrete you should "turn off" the adding of the noise by setting:

```
unur_empk_set_smoothing(par, 0.)
```

B. In the case that you are especially interested in a fast sampling algorithm use the call

```
unur_empk_set_kernel(par, UNUR_DISTR_BOXCAR);
```

to change the used noise distribution from the default Gaussian distribution to the uniform distribution.

Function reference

`UNUR_PAR* unur_empk_new` (*const* ***UNUR_DISTR* distribution***)

Get default parameters for generator.

`int unur_empk_set_kernel` (*UNUR_PAR** ***parameters***, *unsigned* ***kernel***)

Select one of the supported kernel distributions. Currently the following kernels are supported:

`UNUR_DISTR_GAUSSIAN`
: Gaussian (normal) kernel

`UNUR_DISTR_EPANECHNIKOV`
: Epanechnikov kernel

`UNUR_DISTR_BOXCAR`
: Boxcar (uniform, rectangular) kernel

`UNUR_DISTR_STUDENT`
: t3 kernel (Student's distribution with 3 degrees of freedom)

`UNUR_DISTR_LOGISTIC`
: logistic kernel

For other kernels (including kernels with Student's distribution with other than 3 degrees of freedom) use the `unur_empk_set_kernelgen` call.

It is not possible to call `unur_empk_set_kernel` twice.

Default is the Gaussian kernel.

`int unur_empk_set_kernelgen` (*UNUR_PAR** ***parameters***, *const* ***UNUR_GEN* kernelgen***, *double* **alpha**, *double* ***kernelvar***)

Set generator for the kernel used for density estimation.

alpha is used to compute the optimal bandwidth from the point of view of minimizing the mean integrated square error (MISE). It depends on the kernel K and is given by

```
alpha(K) = Var(K)^(-2/5){ \int K(t)^2 dt}^(1/5)
```

For standard kernels (see above) alpha is computed by the algorithm.

kernvar is the variance of the used kernel. It is only required for the variance corrected version of density estimation (which is used by default); otherwise it is ignored. If *kernelvar* is nonpositive, variance correction is disabled. For standard kernels (see above) *kernvar* is computed by the algorithm.

It is not possible to call `unur_empk_set_kernelgen` after a standard kernel has been selected by a `unur_empk_set_kernel` call.

Notice that the uniform random number generator of the kernel generator is overwritten during the `unur_init` call and at each `unur_chg_urng` call with the uniform generator used for the empirical distribution.

Default is the Gaussian kernel.

`int unur_empk_set_beta (`*UNUR_PAR* parameters*, *double* `beta`)
beta is used to compute the optimal bandwidth from the point of view of minimizing the mean integrated square error (MISE). *beta* depends on the (unknown) distribution of the sampled data points. By default Gaussian distribution is assumed for the sample (*beta* = 1.3637439). There is no requirement to change *beta*.

Default: `1.3637439`

`int unur_empk_set_smoothing (`*UNUR_PAR* parameters*, *double* `smoothing`)
`int unur_empk_chg_smoothing (`*UNUR_GEN* generator*, *double* `smoothing`)
Set and change the smoothing factor. The smoothing factor controlles how "smooth" the resulting density estimation will be. A smoothing factor equal to `0` results in naive resampling. A very large smoothing factor (together with the variance correction) results in a density which is approximately equal to the kernel. Default is 1 which results in a smoothing parameter minimising the MISE (mean integrated squared error) if the data are not too far away from normal. If a large smoothing factor is used, then variance correction must be switched on.

Default: `1`

`int unur_empk_set_varcor (`*UNUR_PAR* parameters*, *int* `varcor`)
`int unur_empk_chg_varcor (`*UNUR_GEN* generator*, *int* `varcor`)
Switch variance correction in generator on/off. If *varcor* is `TRUE` then the variance of the used density estimation is the same as the sample variance. However this increases the MISE of the estimation a little bit.

Default is `FALSE`.

`int unur_empk_set_positive (`*UNUR_PAR* parameters*, *int* `positive`)
If *positive* is `TRUE` then only nonnegative random variates are generated. This is done by means of a mirroring technique.

Default is `FALSE`.

5.4.2 EMPL – EMPirical distribution with Linear interpolation

Required: observed sample

Speed: Set-up: slow (as sample is sorted), Sampling: very fast (inversion)

Reinit: not implemented

Reference: [HLa00] [HLD04: Sect.12.1.3]

EMPL generates random variates from an empirical distribution that is given by an observed sample. This is done by linear interpolation of the empirical CDF. Although this method is suggested in the books of Law and Kelton (2000) and Bratly, Fox, and Schrage (1987) we do not recommend this method at all since it has many theoretical drawbacks: The variance of empirical distribution function does not coincide with the variance of the given sample. Moreover, when the sample increases the empirical density function does not converge to the density of the underlying random variate. Notice that the range of the generated point set is always given by the range of the given sample.

This method is provided in UNU.RAN for the sake of completeness. We always recommend to use method EMPK (see Section 5.4.1 [EMPirical distribution with Kernel smoothing], page 142).

If the data seem to be far away from having a bell shaped histogram, then we think that naive resampling is still better than linear interpolation.

How To Use

EMPL creates and samples from an empiral distribution by linear interpolation of the empirical CDF. There are no parameters to set.

Important: We do not recommend to use this method! Use method EMPK (see Section 5.4.1 [EMPirical distribution with Kernel smoothing], page 142) instead.

Function reference

`UNUR_PAR* unur_empl_new (`*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

5.4.3 HIST – HISTogramm of empirical distribution

Required: histogram

Speed: Set-up: moderate, Sampling: fast

Reinit: not implemented

Method HIST generates random variates from an empirical distribution that is given as histogram. Sampling is done using the inversion method.

If observed (raw) data are provided we recommend method EMPK (see Section 5.4.1 [EMPirical distribution with Kernel smoothing], page 142) instead of compting a histogram as this reduces information.

How To Use

Method HIST uses empirical distributions that are given as a histgram. There are no optional parameters.

Function reference

`UNUR_PAR* unur_hist_new` (*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

5.5 Methods for continuous multivariate distributions

Overview of methods

Methods for **continuous multivariate distributions**
sample with `unur_sample_vec`

NORTA: Requires rank correlation matrix and marginal distributions.
VNROU: Requires the PDF.
MVSTD: Generator for built-in standard distributions.
MVSTD: Requires PDF and gradiant of PDF.

5.5.1 MVTDR – Multi-Variate Transformed Density Rejection

Required: log-concave (log)PDF, gradient of (log)PDF

Optional: mode

Speed: Set-up: slow, Sampling: depends on dimension

Reinit: not implemented

Reference: [HLD04: Sect.11.3.4; Alg.11.15.] [LJa98]

MVTDR a multivariate version of the Transformed Density Rection (see Section 5.3.15 [TDR], page 132) that works for log-concave densities. For this method the domain of the distribution is partitioned into cones with the mode (or the center) of the distribution as their (common) vertex. The hat function is then constructed as tangent planes of the transformed density in each of these cones. The respective construction points lie on the central lines in the cones through the vertex. The point is chosen such that the hat is minimal among all such points (see the given references for more details).

The cones are created by starting with the orthants of the reals space. These are then iteratively split when the volume below the hat in such cones is too large. Thus an increasing number of cones results in a better fitting hat function. Notice however, that the required number of cones increases exponentially with the number of dimension. Moreover, due to the construction the rejection does not converge to 1 and remains strictly larger than 1.

For distributions with bounded domains the cones are cut to pyramids that cover the domain.

How To Use

Create a multivariate generator object that contains the PDF and its gradient. This object also should contain the mode of the distribution (or a point nearby should be provided as center of the distribution).

The method has three parameter to adjust the method for the given distribution:

`stepsmin` Minimal number of iterations for splitting cones. Notice that we start with 2^dim initial cones and that we arrive at 2^(dim+stepsmin) cones after these splits. So this number must be set with care. It can be set by a `unur_mvtdr_set_stepsmin` call.

`boundsplitting`
Cones where the volume below the hat is relatively large (i.e. larger than the average volume over all cones times `boundsplitting` are further split. This parameter can set via a `unur_mvtdr_set_boundsplitting` call.

`maxcones` The maximum number of generated cones. When this number is reached, the initialization routine is stopped. Notice that the rejection constant can be still prohibitive large. This parameter can set via a `unur_mvtdr_set_maxcones` call.

Setting of these parameter can be quite tricky. The default settings lead to hat functions where the volume below the hat is similar in each cone. However, there might be some problems with distributions with higher correlations, since then too few cones are created. Then it might be necessary to increase the values for `stepsmin` and `maxcones` and to set `boundsplitting` to `0`.

The number of cones and the total volume below the hat can be controlled using the respective calls `unur_mvtdr_get_ncones` and `unur_mvtdr_get_hatvol`. Notice, that the rejection constant is bounded from below by some figure (larger than 1) that depends on the dimension.

Unfortunately, the algorithm cannot detect the quality of the constructed hat.

Function reference

`UNUR_PAR* unur_mvtdr_new` (*const* `UNUR_DISTR* distribution`)
Get parameters for generator.

`int unur_mvtdr_set_stepsmin` (*UNUR_PAR** `parameters`, *int* `stepsmin`)
Set minimum number of triangulation step for each starting cone. *stepsmin* must be nonnegative.

Default: `5`.

`int unur_mvtdr_set_boundsplitting` (*UNUR_PAR** `parameters`, *double* `boundsplitting`)
Set bound for splitting cones. All cones are split which have a volume below the hat that is greater than *bound_splitting* times the average over all volumes. However, the number given by the `unur_mvtdr_set_maxcones` is not exceeded. Notice that the later number is always reached if *bound_splitting* is less than 1.

Default: `1.5`

`int unur_mvtdr_set_maxcones` (*UNUR_PAR** `parameters`, *int* `maxcones`)
Set maximum number of cones.

Notice that this number is always increased to $2^{dim+stepsmin}$ where *dim* is the dimension of the distribution object and *stepsmin* the given mimimum number of triangulation steps.

Notice: For higher dimensions and/or higher correlations between the coordinates of the random vector the required number of cones can be very high. A too small maximum number of cones can lead to a very high rejection constant.

Default: `10000`.

`int unur_mvtdr_get_ncones` (*const* `UNUR_GEN* generator`)
Get the number of cones used for the hat function of the *generator*. (In case of an error `0` is returned.)

`double unur_mvtdr_get_hatvol` (*const* `UNUR_GEN* generator`)
Get the volume below the hat for the *generator*. (In case of an error `UNUR_INFINITY` is returned.)

`int unur_mvtdr_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_mvtdr_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

5.5.2 NORTA – NORmal To Anything

Required: rank correlation matrix, marginal distributions

Speed: Set-up: slow, Sampling: depends on dimension

Reinit: not implemented

Reference: [HLD04: Sect.12.5.2; Alg.12.11.]

NORTA (NORmal to anything) is a model to get random vectors with given marginal distributions and rank correlation.

Important: Notice that marginal distribution and (rank) correlation structure do not uniquely define a multivariate distribution. Thus there are many other (more or less sensible) models.

In the NORTA model multinormal random variates with the given (Spearman's) rank correlations are generated. In a second step the (standard normal distributed) marginal variates are transformed by means of the CDF of the normal distribution to get uniform marginals. The resulting random vectors have uniform marginals and the desired rank correlation between its components. Such a random vector is called 'copula'.

By means of the inverse CDF the uniform marginals are then transformed into the target marginal distributions. This transformation does not change the rank correlation.

For the generation of the multinormal distribution the (Spearman's) rank correlation matrix is transformed into the corresponding (Pearson) correlation matrix. Samples from the resulting multinormal distribution are generated by means of the Cholesky decomposition of the covariance matrix.

It can happen that the desired rank correlation matrix is not feasible, i.e., it cannot occur as rank correlation matrix of a multinormal distribution. The resulting "covariance" matrix is not positive definite. In this case an eigenvector correction method is used. Then all non-positive eigenvalues are set to a small positive value and hence the rank correlation matrix of the generated random vectors is "close" to the desired matrix.

How To Use

Create a multivariate generator object and set marginal distributions using `unur_distr_cvec_set_marginals`, `unur_distr_cvec_set_marginal_array` , or `unur_distr_cvec_set_marginal_list`. (Do not use the corresponding calls for the standard marginal distributions).

When the domain of the multivariate distribution is set by of a `unur_distr_cvec_set_domain_rect` call then the domain of each of the marginal distributions is truncated by the respective coordinates of the given rectangle.

If copulae are required (i.e. multivariate distributions with uniform marginals) such a generator object can be created by means of `unur_distr_copula` .

There are no optional parameters for this method.

Function reference

`UNUR_PAR* unur_norta_new (`*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

5.5.3 VNROU – Multivariate Naive Ratio-Of-Uniforms method

Required: PDF

Optional: mode, center, bounding rectangle for acceptance region

Speed: Set-up: fast or slow, Sampling: slow

Reinit: supported

Reference: [WGS91]

VNROU is an implementation of the multivariate ratio-of-uniforms method which uses a (minimal) bounding hyper-rectangle, see also Section A.4 [Ratio-of-Uniforms], page 232. It uses an additional parameter r that can be used for adjusting the algorithm to the given distribution to improve performance and/or to make this method applicable. Larger values of r increase the class of distributions for which the method works at the expense of higher rejection constants. Moreover, this implementation uses the center μ of the distribution (which is set to the mode or mean by default, see `unur_distr_cvec_get_center` for details of its default values).

The minimal bounding has then the coordinates

$$v^+ = \sup_x (f(x))^{1/r\,d+1},$$
$$u_i^- = \inf_{x_i} (x_i - \mu_i)(f(x))^{r/r\,d+1},$$
$$u_i^+ = \sup_{x_i} (x_i - \mu_i)(f(x))^{r/r\,d+1},$$

where x_i is the i-th coordinate of point x; μ_i is the i-th coordinate of the center μ. d denotes the dimension of the distribution. These bounds can either be given directly, or are computed automatically by means of an numerical routine by Hooke and Jeeves [HJa61] called direct search (see '`src/utils/hooke.c`' for further references and details). Of course this algorithm can fail, especially when this rectangle is not bounded.

It is important to note that the algorithm works with $PDF(x - center)$ instead of $PDF(x)$, i.e. the bounding rectangle has to be provided for $PDF(x - center)$. This is important as otherwise the acceptance region can become a very long and skinny ellipsoid along a diagonal of the (huge) bounding rectangle.

VNROU is based on the rejection method (see Section A.2 [Rejection], page 230), and it is important to note that the acceptance probability decreases exponentially with dimension. Thus even for moderately many dimensions (e.g. 5) the number of repetitions to get one random vector can be prohibitively large and the algorithm seems to stay in an infinite loop.

How To Use

For using the VNROU method UNU.RAN needs the PDF of the distribution. Additionally, the parameter r can be set via a `unur_vnrou_set_r` call. Notice that the acceptance probability decreases when r is increased. On the other hand is is more unlikely that the bounding rectangle does not exist if r is small.

A bounding rectangle can be given by the `unur_vnrou_set_u` and `unur_vnrou_set_v` calls.

Important: The bounding rectangle has to be provided for the function $PDF(x - center)$! Notice that `center` is the center of the given distribution, see `unur_distr_cvec_set_center`. If in doubt or if this value is not optimal, it can be changed (overridden) by a `unur_distr_cvec_set_center` call.

If the coordinates of the bounding rectangle are not provided by the user then the minimal bounding rectangle is computed automatically.

By means of `unur_vnrou_set_verify` and `unur_vnrou_chg_verify` one can run the sampling algorithm in a checking mode, i.e., in every cycle of the rejection loop it is checked whether

the used rectangle indeed enclosed the acceptance region of the distribution. When in doubt (e.g., when it is not clear whether the numerical routine has worked correctly) this can be used to run a small Monte Carlo study.

Important: The rejection constant (i.e. the expected number of iterations for generationg one random vector) can be extremely high, in particular when the dimension is 4 or higher. Then the algorithm will perform almost infinite loops. Thus it is recommended to read the volume below the hat function by means of the `unur_vnrou_get_volumehat` call. The returned number divided by the volume below the PDF (which is 1 in case of a normalized PDF) gives the rejection constant.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that the coordinates of a bounding rectangle given by `unur_vnrou_set_u` and `unur_vnrou_set_v` calls are used also when the generator is reused. These can be changed by means of `unur_vnrou_chg_u` and `unur_vnrou_chg_v` calls. (If no such coordinates have been given, then they are computed numerically during the reinitialization proceedure.)

Function reference

`UNUR_PAR* unur_vnrou_new` (*const UNUR_DISTR* distribution*)
Get default parameters for generator.

`int unur_vnrou_set_u` (*UNUR_PAR* parameters*, *double* umin*, *double* umax*)
Sets left and right boundaries of bounding hyper-rectangle. If no values are given, the boundary of the minimal bounding hyper-rectangle is computed numerically.

Important: The boundaries are those of the density shifted by the center of the distribution, i.e., for the function $PDF(x - center)$!

Notice: Computing the minimal bounding rectangle may fail under some circumstances. Moreover, for multimodal distributions the bounds might be too small as only local extrema are computed. Nevertheless, for log-concave distributions it should work.

Default: not set (i.e. computed automatically)

`int unur_vnrou_chg_u` (*UNUR_GEN* generator*, *double* umin*, *double* umax*)
Change left and right boundaries of bounding hyper-rectangle.

`int unur_vnrou_set_v` (*UNUR_PAR* parameters*, *double vmax*)
Set upper boundary for bounding hyper-rectangle. If no values are given, the density at the mode is evaluated. If no mode is given for the distribution it is computed numerically (and might fail).

Default: not set (i.e. computed automatically)

`int unur_vnrou_chg_v` (*UNUR_GEN* generator*, *double vmax*)
Change upper boundary for bounding hyper-rectangle.

`int unur_vnrou_set_r` (*UNUR_PAR* parameters*, *double r*)
Sets the parameter r of the generalized multivariate ratio-of-uniforms method.

Notice: This parameter must satisfy $r>0$.

Default: `1`.

`int unur_vnrou_set_verify (`*UNUR_PAR* parameters*, *int verify*`)`
Turn verifying of algorithm while sampling on/off.

If the condition PDF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_vnrou_chg_verify (`*UNUR_GEN* generator*, *int verify*`)`
Change the verifying of algorithm while sampling on/off.

`double unur_vnrou_get_volumehat (`*const UNUR_GEN* generator*`)`
Get the volume of below the hat. For normalized densities, i.e. when the volume below PDF is 1, this value equals the rejection constant for the vnrou method.

In case of an error UNUR_INFINITY is returned.

5.6 Markov chain samplers for continuous multivariate distributions

Markov chain samplers generate sequences of random vectors which have the target distribution as stationary distribution. There generated vectors are (more or less) correlated and it might take a long time until the sequence has converged to the given target distribution.

Beware: MCMC sampling can be dangerous!

Overview of methods

Markov Chain Methods for **continuous multivariate distributions**
sample with `unur_sample_vec`

GIBBS: T-concave logPDF and derivatives of logPDF.
HITRO: Requires PDF.

5.6.1 GIBBS – Markov Chain - GIBBS sampler

Required: T-concave logPDF, derivatives of logPDF

Speed: Set-up: fast, Sampling: moderate

Reinit: not implemented

Reference: [HLD04: Sect.14.1.2]

Method GIBBS implements a Gibbs sampler for a multivariate distribution with given joint density and its gradient. When running such a Markov chain all coordinates are updated cyclically using full conditional distributions. After each step the state of the chain is returned (i.e., a random point is returned whenever a single coordinate has been updated). It is also possible to return only points after all coordinates have been updated by "thinning" the chain. Moreover, to reduce autocorrelation this thinning factor can be any integer. Notice, however, that the sampling time for a chain of given length is increased by the same factor, too.

GIBBS also provides a variant of the Gibbs sampler where in each step a point from the full conditional distribution along some random direction is sampled. This direction is chosen uniformly from the sphere in each step. This method is also known as Hit-and-Run algorithm for non-uniform distributions.

Our experiences shows that the original Gibbs sampler with sampling along coordinate axes is superior to random direction sampling as long as the correlations between the components of the random vector are not too high.

For both variants transformed density rejection (see methods see Section 5.3.15 [TDR], page 132 and see Section 5.3.2 [ARS], page 98) is used to sample from the full conditional distributions. In opposition to the univariate case, it is important that the factor `c` is as large as possible. I.e., for a log-concave density `c` must be set to `0.`, since otherwise numerical underflow might stop the algorithm.

Important: GIBBS does not generate independent random points. The starting point of the Gibbs chain must be in a "typical" region of the target distribution. If such a point is not known or would be too expensive, then the first part of the chain should be discarded (burn-in of the chain).

How To Use

For using the GIBBS method UNU.RAN needs the logarithm of the PDF of the multivariate joint distribution and its gradient or partial derivatives.

It provides two variants:

coordinate direction sampling (Gibbs sampling) [default]
: The coordinates are updated cyclically. It requires the partial derivatives of the (logarithm of the) PDF of the target distribution, see `unur_distr_cvec_set_pdlogpdf`. Otherwise, the gradient of the logPDF (see `unur_distr_cvec_set_dlogpdf`) is used, which is more expensive.

  This variant can be selected using `unur_gibbs_set_variant_coordinate`.

random direction sampling (nonuniform Hit-and-Run algorithm)
: In each step is a direction is sampled uniformly from the sphere and the next point in the chain is sampled from the full conditional distribution along this direction.

  It requires the gradient of the logPDF and thus each step is more expensive than each step for coordinate direction sampling.

  This variant can be selected using `unur_gibbs_set_variant_random_direction`.

It is important that the `c` parameter for the TDR method is as large as possible. For logconcave distribution it must be set to `0`, since otherwise numerical underflow can cause the algorithm to stop.

The starting point of the Gibbs chain must be "typical" for the target distribution. If such a point is not known or would be too expensive, then the first part of the chain should be discarded (burn-in of the chain). When using the `unur_gibbs_set_burnin` call this is done during the setup of the Gibbs sampler object.

In case of a fatal error in the generator for conditional distributions the methods generates points that contain UNUR_INFINITY.

Warning: The algorithm requires that all full conditionals for the given distribution object are T-concave. However, this property is not checked. If this property is not satisfied, then generation from the conditional distributions becomes (very) slow and might fail or (even worse) produces random vectors from an incorrect distribution. When using `unur_gibbs_set_burnin` then the setup already might fail. Thus when in doubt whether GIBBS can be used for the targent distribution it is a good idea to use a burn-in for checking.

Warning: Be carefull with debugging flags. If it contains flag `0x01000000u` it produces a lot of output for each step in the algorithm. (This flag is switched of in the default debugging flags).

Function reference

`UNUR_PAR* unur_gibbs_new` (*const* `UNUR_DISTR* distribution`)

.....

`int unur_gibbs_set_variant_coordinate` (*UNUR_PAR** `parameters`)

Coordinate Direction Sampling: Sampling along the coordinate directions (cyclic).

This is the default.

`int unur_gibbs_set_variant_random_direction` (*UNUR_PAR** `parameters`)

Random Direction Sampling: Sampling along the random directions.

`int unur_gibbs_set_c` (*UNUR_PAR** `parameters`, *double* `c`)

Set parameter c for transformation T of the transformed density rejection method. Currently only values between `0` and `-0.5` are allowed. If `c` is between `0` and `-0.5` it is set to `-0.5`.

For c `=0` (for logconcave densities) method ARS (see Section 5.3.2 [ARS], page 98) is used which is very robust against badly normalized PDFs. For other values method TDR (see Section 5.3.15 [TDR], page 132) is used.

The value for c should be as large as possible to avoid fatal numerical underflows. Thus for log-concave distributions c must be set to `0.`

Default is `0`.

`int unur_gibbs_set_startingpoint` (*UNUR_PAR** `parameters`, *const* `double* x0`)

Sets the starting point of the Gibbs sampler. *x0* must be a "typical" point of the given distribution. If such a "typical" point is not known and a starting point is merely guessed, the first part of the Gibbs chain should be discarded (*burn-in*), e.g.\ by mean of the `unur_gibbs_set_burnin` call.

Default is the result of `unur_distr_cvec_get_center` for the given distribution object.

`int unur_gibbs_set_thinning` (*UNUR_PAR** `parameters`, *int* `thinning`)
Sets the *thinning* parameter. When *thinning* is set to k then every k-th point from the iteration is returned by the sampling algorithm.

Notice: This parameter must satisfy *thinning*>=1.

Default: `1`.

`int unur_gibbs_set_burnin` (*UNUR_PAR** `parameters`, *int* `burnin`)
If a "typical" point for the target distribution is not known but merely guessed, the first part of the Gibbs chain should be discarded (*burn-in*). This can be done during the initialization of the generator object. The length of the burn-in can is then *burnin*.

When method GIBBS is not applicable for the target distribution then the initialization already might fail during the burn-in. Thus this reduces the risk of running a generator that returns UNUR_INFINITY cased by some fatal error during sampling.

The thinning factor set by a `unur_gibbs_set_thinning` call has no effect on the length of the burn-in, i.e., for the burn-in always a thinning factor `1` is used.

Notice: This parameter must satisfy *thinning*>=0.

Default: `0`.

`const double* unur_gibbs_get_state` (*UNUR_GEN** `generator`)
`int unur_gibbs_chg_state` (*UNUR_GEN** `generator`, *const* `double* state`)
Get and change the current state of the Gibbs chain.

`int unur_gibbs_reset_state` (*UNUR_GEN** `generator`)
Reset state of chain to starting point.

Notice: Currently this function does not reset the generators for conditional distributions. Thus it is not possible to get the same Gibbs chain even when the underlying uniform random number generator is reset.

5.6.2 HITRO – Markov Chain - HIT-and-run sampler with Ratio-Of-uniforms

Required: PDF

Optional: mode, center, bounding rectangle for acceptance region

Speed: Set-up: fast, Sampling: fast

Reinit: not implemented

HITRO is an implementation of a hit-and-run sampler that runs on the acceptance region of the multivariate ratio-of-uniforms method, see Section A.4 [Ratio-of-Uniforms], page 232.

The Ratio-of-Uniforms transforms the region below the density into some region that we call "region of acceptance" in the following. The minimal bounding hyperrectangle of this region is given by

$$v^+ = \sup_x (f(x))^{1/r\,d+1},$$
$$u_i^- = \inf_{x_i} (x_i - \mu_i)(f(x))^{r/r\,d+1},$$
$$u_i^+ = \sup_{x_i} (x_i - \mu_i)(f(x))^{r/r\,d+1},$$

where d denotes the dimension of the distribution; x_i is the i-th coordinate of point x; μ_i is the i-th coordinate of the center μ of the distribution, i.e., a point in the "main region" of the distribution. Using the center is important, since otherwise the acceptance region can become a very long and skinny ellipsoid along a diagonal of the (huge) bounding rectangle.

For each step of the Hit-and-Run algorithm we have to choose some direction. This direction together with the current point of the chain determines a straight line. Then a point is sampled uniformly on intersection of this line and the region of acceptance. This is done by rejection from a uniform distribution on a line segment that covers it. Depending of the chosen variant the endpoints of this covering line are computed either by means of a (not necessary minimal) bounding hyper-rectangle, or just the "covering plate" of the bounding hyper-rectangle.

The required bounds of the hyper-rectable can be given directly by the user. Otherwise, these are computed automatically by means of a numerical routine by Hooke and Jeeves [HJa61] called direct search (see '`src/utils/hooke.c`' for further references and details). However, this expensive computation can be avoided by determine these bounds "on the fly" by the following adaptive algorithm: Start with some (small) hyper-rectangle and enlarge it whenever the endpoints of the covering line segment are not contained in the acceptance region of the Ratio-of-Unfiorms method. This approach works reliable as long as the region of acceptance is convex.

The performance of the uniform sampling from the line segment is much improved if the covering line is adjusted (shortened) whenever a point is rejected (adaptive sampling). This technique reduces the expected number of iterations enormously.

Method HITRO requires that the region of acceptance of the Ratio-of-Uniforms method is bounded. The shape of this region can be controlled by a parameter r. Higher values of r result in larger classes of distributions with bounded region of acceptance. (A distribution that has such a bounded region for some r also has a bounded region for every r' greater than r.) On the other hand the acceptance probability decreases with increasing r. Moreover, round-off errors are more likely and (for large values of r) might result in a chain with a stationary distribution different from the target distribution.

Method HITRO works optimal for distributions whose region of acceptance is convex. This is in particular the case for all log-concave distributions when we set $r =$ `1`. For bounded but non-convex regions of acceptance convergence is yet not guarenteed by mathematical theory.

How To Use

Method HITRO requires the PDF of the target distribution (derivatives are not necessary).

The acceptance region of the Ratio-of-Uniforms transformation must be bounded. Its shape is controlled by parameter r. By default this parameter is set to `1` as this guarentees a convex region of acceptance when the PDF of the given distribution is log-concave. It should only be set to a different (higher!) value using `unur_vnrou_set_r` if otherwise $x_i\,(f(x))^{r/r\,d+1}$ were not bounded for each coordinate.

There are two variants of the HITRO sampler:

coordinate direction sampling. [default]
: The coordinates are updated cyclically. This can be seen as a Gibbs sampler running on the acceptance region of the Ratio-of-Uniforms method. This variant can be selected using `unur_hitro_set_variant_coordinate`.

random direction sampling.
: In each step is a direction is sampled uniformly from the sphere.

  This variant can be selected using `unur_hitro_set_variant_random_direction`.

Notice that each iteration of the coordinate direction sampler is cheaper than an iteration of the random direction sampler.

Sampling uniformly from the line segment can be adjusted in several ways:

Adaptive line sampling vs. simple rejection.
: When adaptive line sampling is switched on, the covering line is shortened whenever a point is rejected. However, when the region of acceptance is not convex the line segment from which we have to sample might not be connected. We found that the algorithm still works but at the time being there is no formal proof that the generated Markov chain has the required stationary distribution.

  Adaptive line sampling can switch on/off by means of the `unur_hitro_set_use_adaptiveline` call.

Bounding hyper-rectangle vs. "covering plate".
: For computing the covering line we can use the bounding hyper-rectangle or just its upper bound. The latter saves computing time during the setup and when computing the covering during at each iteration step at the expense of a longer covering line. When adaptive line sampling is used the total generation time for the entire chain is shorter when only the "covering plate" is used.

  Notice: When coordinate sampling is used the entire bounding rectangle is used.

  Using the entire bounding hyper-rectangle can be switched on/off by means of the `unur_hitro_set_use_boundingrectangle` call.

Deterministic vs. adaptive bounding hyper-rectangle.
: A bounding rectangle can be given by the `unur_vnrou_set_u` and `unur_vnrou_set_v` calls. Otherwise, the minimal bounding rectangle is computed automatically during the setup by means of a numerical algorithm. However, this is (very) slow especially in higher dimensions and it might happen that this algorithm (like any other numerical algorithm) does not return a correct result.

  Alternatively the bounding rectangle can be computed adaptively. In the latter case `unur_vnrou_set_u` and `unur_vnrou_set_v` can be used to provide a starting rectangle which must be sufficiently small. Then both endpoints of the covering line segment are always check whether they are outside the acceptance region of the Ratio-of-Uniforms method. If they are not, then the line segment and the

("bounding") rectangle are enlarged using a factor that can be given using the `unur_hitro_set_adaptive_multiplier` call.

Notice, that running this method in the adaptive rectangle mode requires that the region of acceptance is convex when random directions are used, or the given PDF is unimodal when coordinate direction sampling is used. Moreover, it requires two additional calls to the PDF in each iteration step of the chain.

Using addaptive bounding rectangles can be switched on/off by means of the `unur_hitro_set_use_adaptiverectangle` call.

The algorithm takes of a bounded rectangular domain given by a `unur_distr_cvec_set_domain_rect` call, i.e. the PDF is set to zero for every x outside the given domain. However, it is only the coordinate direction sampler where the boundary values are directly used to get the endpoins of the coverline line for the line sampling step.

Important: The bounding rectangle has to be provided for the function $PDF(x - center)$! Notice that `center` is the center of the given distribution, see `unur_distr_cvec_set_center`. If in doubt or if this value is not optimal, it can be changed (overridden) by a `unur_distr_cvec_set_center` call.

Function reference

`UNUR_PAR* unur_hitro_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_hitro_set_variant_coordinate` (*UNUR_PAR** `parameters`)
Coordinate Direction Sampling: Sampling along the coordinate directions (cyclic).

Notice: For this variant the entire bounding rectangle is always used independent of the `unur_hitro_set_use_boundingrectangle` call.

This is the default.

`int unur_hitro_set_variant_random_direction` (*UNUR_PAR** `parameters`)
Random Direction Sampling: Sampling along the random directions.

`int unur_hitro_set_use_adaptiveline` (*UNUR_PAR** `parameters`, *int* `adaptive`)
When *adaptive* is set to `TRUE` adaptive line sampling is applied, otherwise simple rejection is used.

Notice: When adaptive line sampling is switched off, the entire bounding rectangle must be used since otherwise the sampling time can be arbitrarily slow.

Warning: When adaptive line sampling is switched off, sampling can be arbitrarily slow. In particular this happens when random direction sampling is used for distributions with rectangular domains. Then the algorithm can be trapped into a vertex (or even edge).

Default is `TRUE`.

`int unur_hitro_set_use_boundingrectangle` (*UNUR_PAR** `parameters`, *int* `rectangle`)
When *rectangle* is set to `TRUE` the entire bounding rectangle is used for computing the covering line. Otherwise, only an upper bound for the acceptance region is used.

Notice: When coordinate sampling is used the entire bounding rectangle has is always used and this call has no effect.

Default: `FALSE` for random direction samplig, `TRUE` for coordinate direction sampling.

`int unur_hitro_set_use_adaptiverectangle (`*UNUR_PAR* parameters*, *int adaptive*`)`

When *adaptive* is set to `FALSE` the bounding rectangle is determined during the setup. Either, it is computed automatically by a (slow) numerical method, or it must be provided by `unur_vnrou_set_u` and `unur_vnrou_set_v` calls.

If *adaptive* is set to `TRUE` the bounding rectangle is computed adaptively. In this case the `unur_vnrou_set_u` and `unur_vnrou_set_v` calls can be used to provide a starting rectangle. This should be sufficiently small. If not given then we assume $v_{max} = 1$, $u_{min} = (-0.001, -0.001, \ldots, -0.001)$, and $u_{max} = (0.001, 0.001, \ldots, 0.001)$. Adaptive enlargements of the bounding hyperrectangle can be controlled set setting an enlargement factor given by a `unur_hitro_set_adaptive_multiplier` call.

Using adaptive computation of the bounding rectangle reduces the setup time significantly (when it is not given by the user) at the expense of two additional PDF evaluations during each iteration step.

Important: Using adaptive bounding rectangles requires that the region of acceptance is convex when random directions are used, or a unimodal PDF when coordinate direction sampling is used.

Default: `FALSE` for random direction samplig, `TRUE` for coordinate direction sampling.

`int unur_hitro_set_r (`*UNUR_PAR* parameters*, *double r*`)`

Sets the parameter r of the generalized multivariate ratio-of-uniforms method.

Notice: This parameter must satisfy $r>0$.

Default: `1`.

`int unur_hitro_set_v (`*UNUR_PAR* parameters*, *double vmax*`)`

Set upper boundary for bounding hyper-rectangle. If not set not set the mode of the distribution is used.

If adaptive bounding rectangles the value is used for the starting rectangle. If not given (and the mode of the distribution is not known) then *vmax*=`1e-3` is used.

If deterministic bounding rectangles these values are the given values are used for the rectangle. If no value is given (and the mode of the distribution is not known), the upper bound of the minimal bounding hyper-rectangle is computed numerically (slow).

Default: not set.

`int unur_hitro_set_u (`*UNUR_PAR* parameters*, *const double* umin*, *const double* umax*`)`

Sets left and right boundaries of bounding hyper-rectangle.

If adaptive bounding rectangles these values are used for the starting rectangle. If not given then *umin*=`{-b,-b,...,-b}` and *umax*=`{b,b,...,b}` with `b=1.e-3` is used.

If deterministic bounding rectangles these values are the given values are used for the rectangle. If no values are given, the boundary of the minimal bounding hyper-rectangle is computed numerically (slow).

Important: The boundaries are those of the density shifted by the center of the distribution, i.e., for the function $PDF(x - center)$!

Notice: Computing the minimal bounding rectangle may fail under some circumstances. Moreover, for multimodal distributions the bounds might be too small as only local extrema are computed. Nevertheless, for log-concave distributions it should work.

Default: not set.

`int unur_hitro_set_adaptive_multiplier (`*UNUR_PAR* parameters, double factor*`)`

Adaptive enlargements of the bounding hyperrectangle can be controlled set setting the enlargement *factor*. This must be greater than 1. Values close to 1 result in small adaptive steps and thus reduce the risk of too large bounding rectangles. On the other hand many adaptive steps might be necessary.

Notice: For practical reasons this call does not accept values for *factor* less than `1.0001`. If this value is UNUR_INFINITY this results in infinite loops.

Default: `1.1`

`int unur_hitro_set_startingpoint (`*UNUR_PAR* parameters, const double* x0*`)`

Sets the starting point of the HITRO sampler in the original scale. *x0* must be a "typical" point of the given distribution. If such a "typical" point is not known and a starting point is merely guessed, the first part of the HITRO chain should be discarded (*burn-in*), e.g.\ by mean of the `unur_hitro_set_burnin` call.

Important: The PDF of the distribution must not vanish at the given point *x0*.

Default is the result of `unur_distr_cvec_get_center` for the given distribution object.

`int unur_hitro_set_thinning (`*UNUR_PAR* parameters, int thinning*`)`

Sets the *thinning* parameter. When *thinning* is set to k then every k-th point from the iteration is returned by the sampling algorithm. If thinning has to be set such that each coordinate is updated when using coordinate direction sampling, then *thinning* should be `dim+1` (or any multiple of it) where `dim` is the dimension of the distribution object.

Notice: This parameter must satisfy *thinning*>=1.

Default: `1`.

`int unur_hitro_set_burnin (`*UNUR_PAR* parameters, int burnin*`)`

If a "typical" point for the target distribution is not known but merely guessed, the first part of the HITRO chain should be discarded (*burn-in*). This can be done during the initialization of the generator object. The length of the burn-in can is then *burnin*.

The thinning factor set by a `unur_hitro_set_thinning` call has no effect on the length of the burn-in, i.e., for the burn-in always a thinning factor `1` is used.

Notice: This parameter must satisfy *thinning*>=0.

Default: `0`.

`const double\* unur_hitro_get_state (`*UNUR_GEN* generator*`)`
`int unur_hitro_chg_state (`*UNUR_GEN* generator, const double* state*`)`

Get and change the current state of the HITRO chain.

Notice: The state variable contains the point in the `dim+1` dimensional point in the (tansformed) region of acceptance of the Ratio-of-Uniforms method. Its coordinate are stored in the following order: `state[] = {v, u1, u2, ..., udim}`.

If the state can only be changed if the given *state* is inside this region.

`int unur_hitro_reset_state (`*UNUR_GEN* generator*`)`

Reset state of chain to starting point.

Notice: Currently this function does not reset the generators for conditional distributions. Thus it is not possible to get the same HITRO chain even when the underlying uniform random number generator is reset.

5.7 Methods for continuous empirical multivariate distributions

Overview of methods

Methods for **continuous empirical multivariate distributions**
sample with `unur_sample_vec`

VEMPK: Requires an observed sample.

Example

```
/* ---------------------------------------------------------- */
/* File: example_vemp.c                                       */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

/* Example how to sample from an empirial continuous          */
/* multivariate distribution.                                 */

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;

  /* 4 data points of dimension 2                             */
  double data[] = { 1. ,1.,       /* 1st data point          */
    -1.,1.,       /* 2nd data point                 */
    1.,-1.,       /* 3rd data point                 */
    -1.,-1. };    /* 4th data point                 */

  double result[2];

  /* Declare the three UNURAN objects.                        */
  UNUR_DISTR *distr;    /* distribution object                */
  UNUR_PAR   *par;      /* parameter object                   */
  UNUR_GEN   *gen;      /* generator object                   */

  /* Create a distribution object with dimension 2.           */
  distr = unur_distr_cvemp_new( 2 );

  /* Set empirical sample.                                    */
  unur_distr_cvemp_set_data(distr, data, 4);

  /* Choose a method: VEMPK.                                  */
  par = unur_vempk_new(distr);

  /* Use variance correction.                                 */
  unur_vempk_set_varcor( par, 1 );

  /* Create the generator object.                             */
  gen = unur_init(par);

  /* It is important to check if the creation of the generator */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
```

```
    exit (EXIT_FAILURE);
  }

  /* It is possible to reuse the distribution object to create   */
  /* another generator object. If you do not need it any more,   */
  /* it should be destroyed to free memory.                      */
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from   */
  /* the distribution. Eg.:                                      */
  for (i=0; i<10; i++) {
    unur_sample_vec(gen, result);
    printf("(%f,%f)\n", result[0], result[1]);
  }

  /* When you do not need the generator object any more, you     */
  /* can destroy it.                                             */
  unur_free(gen);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------------- */
```

Example (String API)

(not implemented)

5.7.1 VEMPK – (Vector) EMPirical distribution with Kernel smoothing

Required: observed sample

Speed: Set-up: slow, Sampling: slow (depends on dimension)

Reinit: not implemented

Reference: [HLa00] [HLD04: Sect.12.2.1]

VEMPK generates random variates from a multivariate empirical distribution that is given by an observed sample. The idea is that simply choosing a random point from the sample and to return it with some added noise results in a method that has very nice properties, as it can be seen as sampling from a kernel density estimate. Clearly we have to decide about the density of the noise (called kernel) and about the covariance matrix of the noise. The mathematical theory of kernel density estimation shows us that we are comparatively free in choosing the kernel. It also supplies us with a simple formula to compute the optimal standarddeviation of the noise, called bandwidth (or window width) of the kernel.

Currently only a Gaussian kernel with the same covariance matrix as the given sample is implemented. However it is possible to choose between a variance corrected version or those with optimal MISE. Additionally a smoothing factor can be set to adjust the estimated density to non-bell-shaped data densities.

How To Use

VEMPK uses empirical distributions. The main parameter would be the choice if of kernel density. However, currently only Gaussian kernels are supported. The parameters for the density are computed by a simple but robust method. However, it is possible to control its behavior by changing the smoothing factor. Additionally, variance correction can be swithed on (at the price of suboptimal MISE).

Function reference

`UNUR_PAR* unur_vempk_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_vempk_set_smoothing` (*UNUR_PAR** `parameters`, *double* `smoothing`)
`int unur_vempk_chg_smoothing` (*UNUR_GEN** `generator`, *double* `smoothing`)
Set and change the smoothing factor. The smoothing factor controlles how "smooth" the resulting density estimation will be. A smoothing factor equal to 0 results in naive resampling. A very large smoothing factor (together with the variance correction) results in a density which is approximately equal to the kernel. Default is 1 which results in a smoothing parameter minimising the MISE (mean integrated squared error) if the data are not too far away from normal. If a large smoothing factor is used, then variance correction must be switched on.

Default: `1`

`int unur_vempk_set_varcor` (*UNUR_PAR** `parameters`, *int* `varcor`)
`int unur_vempk_chg_varcor` (*UNUR_GEN** `generator`, *int* `varcor`)
Switch variance correction in generator on/off. If *varcor* is `TRUE` then the variance of the used density estimation is the same as the sample variance. However this increases the MISE of the estimation a little bit.

Default is `FALSE`.

5.8 Methods for discrete univariate distributions

Overview of methods

Methods for **discrete univariate distributions**
sample with `unur_sample_discr`

method	PMF	PV	mode	sum	other
DARI	x		x	~	T-concave
DAU	[x]	x			
DGT	[x]	x			
DSROU	x		x	x	T-concave
DSS	[x]	x		x	
DSTD					build-in standard distribution
CEXT					wrapper for external generator

Example

```
/* ------------------------------------------------------------ */
/* File: example_discr.c                                        */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

/* Example how to sample from a discrete univariate distribution.*/

/* ------------------------------------------------------------ */

int main(void)
{
  int    i;
  double param = 0.3;

  double probvec[10] = {1.0, 2.0, 3.0, 4.0, 5.0,\
                        6.0, 7.0, 8.0, 4.0, 3.0};

  /* Declare the three UNURAN objects.                          */
  UNUR_DISTR *distr1, *distr2;    /* distribution objects       */
  UNUR_PAR   *par1, *par2;        /* parameter objects          */
  UNUR_GEN   *gen1, *gen2;        /* generator objects          */

  /* First distribution: defined by PMF.                        */
  distr1 = unur_distr_geometric(&param, 1);
  unur_distr_discr_set_mode(distr1, 0);

  /* Choose a method: DARI.                                     */
  par1 = unur_dari_new(distr1);
  gen1 = unur_init(par1);

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen1 == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }
```

```
  /* Second distribution: defined by (finite) PV.                */
  distr2 = unur_distr_discr_new();
  unur_distr_discr_set_pv(distr2, probvec, 10);

  /* Choose a method: DGT.                                       */
  par2 = unur_dgt_new(distr2);
  gen2 = unur_init(par2);
  if (gen2 == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* print some random integers                                  */
  for (i=0; i<10; i++){
    printf("number %d: %d\n", i*2,   unur_sample_discr(gen1) );
    printf("number %d: %d\n", i*2+1, unur_sample_discr(gen2) );
  }

  /* Destroy all objects.                                        */
  unur_distr_free(distr1);
  unur_distr_free(distr2);
  unur_free(gen1);
  unur_free(gen2);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------- */
```

Example (String API)

```
/* ---------------------------------------------------------- */
/* File: example_discr_str.c                                  */
/* ---------------------------------------------------------- */
/* String API.                                                */
/* ---------------------------------------------------------- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ---------------------------------------------------------- */

/* Example how to sample from a discrete univariate distribution.*/

/* ---------------------------------------------------------- */

int main(void)
{
  int    i;     /* loop variable                              */

  /* Declare UNURAN generator objects.                        */
  UNUR_GEN   *gen1, *gen2;        /* generator objects       */


  /* First distribution: defined by PMF.                      */
  gen1 = unur_str2gen("geometric(0.3); mode=0 & method=dari");

  /* It is important to check if the creation of the generator  */
  /* object was successful. Otherwise 'gen' is the NULL pointer */
  /* and would cause a segmentation fault if used for sampling. */
  if (gen1 == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }
```

```
  /* Second distribution: defined by (finite) PV.                */
  gen2 = unur_str2gen(
             "distr=discr; pv=(1,2,3,4,5,6,7,8,4,3) & method=dgt");
  if (gen2 == NULL) {
     fprintf(stderr, "ERROR: cannot create generator object\n");
     exit (EXIT_FAILURE);
  }

  /* print some random integers                                  */
  for (i=0; i<10; i++){
    printf("number %d: %d\n", i*2,   unur_sample_discr(gen1) );
    printf("number %d: %d\n", i*2+1, unur_sample_discr(gen2) );
  }

  /* Destroy all objects.                                        */
  unur_free(gen1);
  unur_free(gen2);

  exit (EXIT_SUCCESS);

} /* end of main() */

/* ---------------------------------------------------------------- */
```

5.8.1 DARI – Discrete Automatic Rejection Inversion

Required: T-concave PMF, mode, approximate area

Speed: Set-up: moderate, Sampling: fast

Reinit: supported

Reference: [HDa96] [HLD04: Sect.10.2; Alg.10.4]

DARI is based on rejection inversion, which can be seen as an adaptation of transformed density rejection to discrete distributions. The used transformation is $-1/\sqrt{x}$.

DARI uses three almost optimal points for constructing the (continuous) hat. Rejection is then done in horizontal direction. Rejection inversion uses only one uniform random variate per trial.

DARI has moderate set-up times (the PMF is evaluated nine times), and good marginal speed, especially if an auxiliary array is used to store values during generation.

DARI works for all $T_{-1/2}$ -concave distributions. It requires the PMF and the location of the mode. Moreover the approximate sum over the PMF is used. (If no sum is given for the distribution the algorithm assumes that it is approximately 1.) The rejection constant is bounded from above by 4 for all T-concave distributions.

How To Use

DARI works for discrete distribution object with given PMF. The sum over probabilities should be approximately one. Otherwise it must be set by a `unur_distr_discr_set_pmfsum` call to its (approximate) value.

The size of an auxiliary table can be set by `unur_dari_set_tablesize`. The expected number of evaluations can be reduced by switching the use of squeezes by means of `unur_dari_set_squeeze`. It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. Notice, that derived parameters like the mode must also be (re-) set if the parameters or the domain has be changed.

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_dari_set_verify` and `unur_dari_chg_verify`, respectively. Notice however that sampling is (much) slower then.

Function reference

`UNUR_PAR* unur_dari_new` (*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

`int unur_dari_set_squeeze` (*UNUR_PAR** `parameters`, *int* `squeeze`)

Turn utilization of the squeeze of the algorithm on/off. This squeeze does not resamble the squeeze of the continuous TDR method. It was especially designed for rejection inversion.

The squeeze is not necessary if the size of the auxiliary table is big enough (for the given distribution). Using a squeeze is suggested to speed up the algorithm if the domain of the distribution is very big or if only small samples are produced.

Default: no squeeze.

`int unur_dari_set_tablesize` (*UNUR_PAR** `parameters`, *int* `size`)

Set the size for the auxiliary table, that stores constants computed during generation. If *size* is set to `0` no table is used. The speed-up can be impressive if the PMF is expensive to evaluate and the "main part of the distribution" is concentrated in an interval shorter than the size of the table.

Default is `100`.

`int unur_dari_set_cpfactor` (*UNUR_PAR** `parameters`, *double* `cp_factor`)
Set factor for position of the left and right construction point, resp. The *cp_factor* is used to find almost optimal construction points for the hat function. The *cp_factor* must be positive and should not exceed 2. There is no need to change this factor in almost all situations.

Default is `0.664`.

`int unur_dari_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_dari_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the condition is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

5.8.2 DAU – (Discrete) Alias-Urn method

Required: probability vector (PV)

Speed: Set-up: slow (linear with the vector-length), Sampling: very fast

Reinit: supported

Reference: [WAa77] [HLD04: Sect.3.2]

DAU samples from distributions with arbitrary but finite probability vectors (PV) of length *N*. The algorithmus is based on an ingeneous method by A.J. Walker and requires a table of size (at least) *N*. It needs one random numbers and only one comparison for each generated random variate. The setup time for constructing the tables is *O(N)*.

By default the probability vector is indexed starting at `0`. However this can be changed in the distribution object by a `unur_distr_discr_set_domain` call.

The method also works when no probability vector but a PMF is given. However then additionally a bounded (not too large) domain must be given or the sum over the PMF (see `unur_distr_discr_make_pv` for details).

How To Use

Create an object for a discrete distribution either by setting a probability vector or a PMF. The performance can be slightly influenced by setting the size of the used table which can be changed by `unur_dau_set_urnfactor`. It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_dau_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_dau_set_urnfactor` (*UNUR_PAR** `parameters`, *double* `factor`)
Set size of urn table relative to length of the probability vector. It must not be less than 1. Larger tables result in (slightly) faster generation times but require a more expensive setup. However sizes larger than 2 are not recommended.

Default is `1`.

5.8.3 DEXT – wrapper for Discrete EXTernal generators

Required: routine for sampling discrete random variates

Speed: depends on external generator

Reinit: supported

Method DEXT is a wrapper for external generators for discrete univariate distributions. It allows the usage of external random variate generators within the UNU.RAN framework.

How To Use

The following steps are required to use some external generator within the UNU.RAN framework (some of these are optional):

1. Make an empty generator object using a `unur_dext_new` call. The argument *distribution* is optional and can be replaced by `NULL`. However, it is required if you want to pass parameters of the generated distribution to the external generator or for running some validation tests provided by UNU.RAN.
2. Create an initialization routine of type `int (*init)(UNUR_GEN *gen)` and plug it into the generator object using the `unur_dext_set_init` call. Notice that the *init* routine must return `UNUR_SUCCESS` when it has been executed successfully and `UNUR_FAILURE` otherwise. It is possible to get the size of and the pointer to the array of parameters of the underlying distribution object by the respective calls `unur_dext_get_ndistrparams` and `unur_dext_get_distrparams`. Parameters for the external generator that are computed in the *init* routine can be stored in a single array or structure which is available by the `unur_dext_get_params` call.

   Using an *init* routine is optional and can be omitted.
3. Create a sampling routine of type `int (*sample)(UNUR_GEN *gen)` and plug it into the generator object using the `unur_dext_set_sample` call.

   Uniform random numbers are provided by the `unur_sample_urng` call. Do not use your own implementation of a uniform random number generator directly. If you want to use your own random number generator we recommend to use the UNU.RAN interface (see see Chapter 6 [Using uniform random number generators], page 189).

   The array or structure that contains parameters for the external generator that are computed in the *init* routine are available using the `unur_dext_get_params` call.

   Using a *sample* routine is of course obligatory.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object. The *init* routine is then called again.

Here is a short example that demonstrates the application of this method by means of the geometric distribution:

```
/* ------------------------------------------------------------ */
/* File: example_dext.c                                         */
/* ------------------------------------------------------------ */

/* Include UNURAN header file.                                  */
#include <unuran.h>

/* ------------------------------------------------------------ */

/* This example shows how an external generator for the         */
/* geometric distribution can be used within the UNURAN         */
/* framework.                                                   */
/*                                                              */
```

```
/* Notice, that this example does not provide the simplest       */
/* solution.                                                     */

/* ------------------------------------------------------------- */
/* Initialization routine.                                       */
/*                                                               */
/*   Here we simply read the parameter of the geometric          */
/*   distribution and store it in an array for parameters of     */
/*   the external generator.                                     */
/*   [ Of course we could do this in the sampling routine as     */
/*   and avoid the necessity of this initialization routine. ]   */

int geometric_init (UNUR_GEN *gen)
{
  /* Get pointer to parameters of geometric distribution         */
  double *params = unur_dext_get_distrparams(gen);

  /* The parameter is the first entry (see manual)               */
  double p = params[0];

  /* Get array to store this parameter for external generator    */
  double *genpar = unur_dext_get_params(gen, sizeof(double));
  genpar[0] = p;

  /* Executed successfully                                       */
  return UNUR_SUCCESS;
}

/* ------------------------------------------------------------- */
/* Sampling routine.                                             */
/*                                                               */
/*   Contains the code for the external generator.               */

int geometric_sample (UNUR_GEN *gen)
{
  /* Get scale parameter                                         */
  double *genpar = unur_dext_get_params(gen,0);
  double p = genpar[0];

  /* Sample a uniformly distributed random number                */
  double U = unur_sample_urng(gen);

  /* Transform into geometrically distributed random variate     */
  return ( (int) (log(U) / log(1.-p)) );
}

/* ------------------------------------------------------------- */

int main(void)
{
  int i;     /* loop variable                                    */
  int K;     /* will hold the random number                      */

  /* Declare the three UNURAN objects.                           */
  UNUR_DISTR *distr;    /* distribution object                   */
  UNUR_PAR   *par;      /* parameter object                      */
  UNUR_GEN   *gen;      /* generator object                      */

  /* Use predefined geometric distribution with parameter 1/10   */
  double fpar[1] = { 0.1 };
  distr = unur_distr_geometric(fpar, 1);

  /* Use method DEXT                                             */
  par = unur_dext_new(distr);
```

```
    /* Set initialization and sampling routines.                  */
    unur_dext_set_init(par, geometric_init);
    unur_dext_set_sample(par, geometric_sample);

    /* Create the generator object.                               */
    gen = unur_init(par);

    /* It is important to check if the creation of the generator  */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
       fprintf(stderr, "ERROR: cannot create generator object\n");
       exit (EXIT_FAILURE);
    }

    /* It is possible to reuse the distribution object to create  */
    /* another generator object. If you do not need it any more,  */
    /* it should be destroyed to free memory.                     */
    unur_distr_free(distr);

    /* Now you can use the generator object 'gen' to sample from  */
    /* the standard Gaussian distribution.                        */
    /* Eg.:                                                       */
    for (i=0; i<10; i++) {
      K = unur_sample_discr(gen);
      printf("%d\n",K);
    }

    /* When you do not need the generator object any more, you    */
    /* can destroy it.                                            */
    unur_free(gen);

    exit (EXIT_SUCCESS);

  } /* end of main() */

  /* ---------------------------------------------------------- */
```

Function reference

`UNUR_PAR* unur_dext_new` (*const UNUR_DISTR* distribution*)
Get default parameters for new generator.

`int unur_dext_set_init` (*UNUR_PAR* parameters*, *int* (* *init*)(*UNUR_GEN* *gen*))
Set initialization routine for external generator. Inside the

Important: The routine *init* must return `UNUR_SUCCESS` when the generator was initialized successfully and `UNUR_FAILURE` otherwise.

Parameters that are computed in the *init* routine can be stored in an array or structure that is avaiable by means of the `unur_dext_get_params` call. Parameters of the underlying distribution object can be obtained by the `unur_dext_get_distrparams` call.

`int unur_dext_set_sample` (*UNUR_PAR* parameters*, *int* (* *sample*)(*UNUR_GEN* *gen*))
Set sampling routine for external generator.

Important: Use `unur_sample_urng(gen)` to get a uniform random number. The pointer to the array or structure that contains the parameters that are precomputed in the *init* routine are available by `unur_dext_get_params(gen,0)`. Additionally one can use the `unur_dext_get_distrparams` call.

`void* unur_dext_get_params` (*UNUR_GEN* **generator**, *size_t* **size**)
Get pointer to memory block for storing parameters of external generator. A memory block of size *size* is automatically (re-) allocated if necessary and the pointer to this block is stored in the *generator* object. If one only needs the pointer to this memory block set *size* to `0`.

Notice, that *size* is the size of the memory block and not the length of an array.

Important: This rountine should only be used in the initialization and sampling routine of the external generator.

`double* unur_dext_get_distrparams` (*UNUR_GEN* **generator**)
`int unur_dext_get_ndistrparams` (*UNUR_GEN* **generator**)
Get size of and pointer to array of parameters of underlying distribution in *generator* object.

Important: These rountines should only be used in the initialization and sampling routine of the external generator.

5.8.4 DGT – (Discrete) Guide Table method (indexed search)

Required: probability vector (PV)

Speed: Set-up: slow (linear with the vector-length), Sampling: very fast

Reinit: supported

Reference: [CAa74] [HLD04: Sect.3.1.2]

DGT samples from arbitrary but finite probability vectors. Random numbers are generated by the inversion method, i.e.,

1. Generate a random number U ~ U(0,1).
2. Find largest integer I such that F(I) = P(X<=I) <= U.

Step (2) is the crucial step. Using sequential search requires *O(E(X))* comparisons, where *E(X)* is the expectation of the distribution. Indexed search, however, uses a guide table to jump to some *I'* <= *I* near *I* to find *X* in constant time. Indeed the expected number of comparisons is reduced to 2, when the guide table has the same size as the probability vector (this is the default). For larger guide tables this number becomes smaller (but is always larger than 1), for smaller tables it becomes larger. For the limit case of table size 1 the algorithm simply does sequential search (but uses a more expensive setup then method DSS (see Section 5.8.6 [DSS], page 180). On the other hand the setup time for guide table is *O(N)*, where *N* denotes the length of the probability vector (for size 1 no preprocessing is required). Moreover, for very large guide tables memory effects might even reduce the speed of the algorithm. So we do not recommend to use guide tables that are more than three times larger than the given probability vector. If only a few random numbers have to be generated, (much) smaller table sizes are better. The size of the guide table relative to the length of the given probability vector can be set by a `unur_dgt_set_guidefactor` call.

There exist two variants for the setup step which can be set by a `unur_dgt_set_variant` call: Variants 1 and 2. Variant 2 is faster but more sensitive to roundoff errors when the guide table is large. By default variant 2 is used for short probability vectors (*N*<1000) and variant 1 otherwise.

By default the probability vector is indexed starting at `0`. However this can be changed in the distribution object by a `unur_distr_discr_set_domain` call.

The method also works when no probability vector but a PMF is given. However, then additionally a bounded (not too large) domain must be given or the sum over the PMF. In the latter case the domain of the distribution is trucated (see `unur_distr_discr_make_pv` for details).

How To Use

Create an object for a discrete distribution either by setting a probability vector or a PMF. The performance can be slightly influenced by setting the size of the used table which can be changed by `unur_dgt_set_guidefactor`. It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_dgt_new` (*const* `UNUR_DISTR* distribution`)

Get default parameters for generator.

`int unur_dgt_set_guidefactor` (*UNUR_PAR** `parameters`, *double* `factor`)

Set size of guide table relative to length of PV. Larger guide tables result in faster generation time but require a more expensive setup. Sizes larger than 3 are not recommended. If the

relative size is set to 0, sequential search is used. However, this is not recommended, except in exceptional cases, since method DSS (see Section 5.8.6 [DSS], page 180) is has almost no setup and is thus faster (but requires the sum over the PV as input parameter).

Default is `1`.

`int unur_dgt_set_variant (`*UNUR_PAR** `parameters`, *unsigned* `variant`)

Set variant for setup step. Possible values are `1` or `2`. Variant `2` is faster but more sensitive to roundoff errors when the guide table is large. By default variant `2` is used for short probability vectors (N<1000) and variant `1` otherwise.

5.8.5 DSROU – Discrete Simple Ratio-Of-Uniforms method

Required: T-concave PMF, mode, sum over PMF

Speed: Set-up: fast, Sampling: slow

Reinit: supported

Reference: [LJa01] [HLD04: Sect.10.3.2; Alg.10.6]

DSROU is based on the ratio-of-uniforms method (see Section A.4 [Ratio-of-Uniforms], page 232) but uses universal inequalities for constructing a (universal) bounding rectangle. It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

The method requires the PMF, the (exact) location of the mode and the sum over the given PDF. The rejection constant is 4 for all T-concave distributions. Optionally the CDF at the mode can be given to increase the performance of the algorithm. Then the rejection constant is reduced to 2.

How To Use

The method works for T-concave discrete distributions with given PMF. The sum over of the PMF or an upper bound of this sum must be known.

Optionally the CDF at the mode can be given to increase the performance using `unur_dsrou_set_cdfatmode`. However, this **must not** be called if the sum over the PMF is replaced by an upper bound.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

If any of mode, CDF at mode, or the sum over the PMF has been changed, then `unur_reinit` must be executed. (Otherwise the generator produces garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on or off by calling `unur_dsrou_set_verify` and `unur_dsrou_chg_verify`, respectively. Notice however that sampling is (a little bit) slower then.

Function reference

`UNUR_PAR* unur_dsrou_new` (*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_dsrou_set_cdfatmode` (*UNUR_PAR** `parameters`, *double* `Fmode`)
Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_dsrou_chg_cdfatmode` has to be used to update this value. Notice that the algorithm detects a mode at the left boundary of the domain automatically and it is not necessary to use this call for a monotonically decreasing PMF.

Default: not set.

`int unur_dsrou_set_verify` (*UNUR_PAR** `parameters`, *int* `verify`)
`int unur_dsrou_chg_verify` (*UNUR_GEN** `generator`, *int* `verify`)
Turn verifying of algorithm while sampling on/off. If the condition squeeze(x) <= PMF(x) <= hat(x) is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).

Default is `FALSE`.

`int unur_dsrou_chg_cdfatmode` (*UNUR_GEN* generator*, *double Fmode*)
Change CDF at mode of distribution. `unur_reinit` must be executed before sampling from the generator again.

5.8.6 DSS – (Discrete) Sequential Search method

Required: probability vector (PV) and sum over PV; or probability mass function(PMF), sum over PV and domain; or or cumulative distribution function (CDF)

Speed: Set-up: fast, Sampling: very slow (linear in expectation)

Reinit: supported

Reference: [HLD04: Sect.3.1.1; Alg.3.1]

DSS samples from arbitrary discrete distributions. Random numbers are generated by the inversion method, i.e.,

1. Generate a random number U ~ U(0,1).
2. Find largest integer I such that F(I) = P(X<=I) <= U.

Step (2) is the crucial step. Using sequential search requires *O(E(X))* comparisons, where *E(X)* is the expectation of the distribution. Thus this method is only recommended when only a few random variates from the given distribution are required. Otherwise, table methods like DGT (see Section 5.8.4 [DGT], page 176) or DAU (see Section 5.8.2 [DAU], page 171) are much faster. These methods also need not the sum over the PMF (or PV) as input. On the other hand, however, these methods always compute a table.

DSS runs with the PV, the PMF, or the CDF of the distribution. It uses actually uses the first one in this list (in this ordering) that could be found.

How To Use

It works with a discrete distribution object with contains at least the PV, the PMF, or the CDF.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_dss_new (`*`const UNUR_DISTR* distribution`*`)`

Get default parameters for generator.

5.8.7 DSTD – Discrete STandarD distributions

Required: standard distribution from UNU.RAN library (see Chapter 7 [Standard distributions], page 201).

Speed: Set-up: fast, Sampling: depends on distribution and generator

Reinit: supported

DSTD is a wrapper for special generators for discrete univariate standard distributions. It only works for distributions in the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201). If a distribution object is provided that is build from scratch, or no special generator for the given standard distribution is provided, the `NULL` pointer is returned.

For some distributions more than one special generator is possible.

How To Use

Create a distribution object for a standard distribution from the UNU.RAN library (see Chapter 7 [Standard distributions], page 201). For some distributions more than one special generator (*variants*) is possible. These can be choosen by a `unur_dstd_set_variant` call. For possible variants See Chapter 7 [Standard distributions], page 201. However the following are common to all distributions:

`UNUR_STDGEN_DEFAULT`
: the default generator.

`UNUR_STDGEN_FAST`
: the fasted available special generator.

`UNUR_STDGEN_INVERSION`
: the inversion method (if available).

Notice that the variant `UNUR_STDGEN_FAST` for a special generator might be slower than one of the universal algorithms! Additional variants may exist for particular distributions.

Sampling from truncated distributions (which can be constructed by changing the default domain of a distribution by means of `unur_distr_discr_set_domain` call) is possible but requires the inversion method.

It is possible to change the parameters and the domain of the chosen distribution and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_dstd_new` (*const UNUR_DISTR* distribution*)
: Get default parameters for new generator. It requires a distribution object for a discrete univariant distribution from the UNU.RAN library of standard distributions (see Chapter 7 [Standard distributions], page 201).

  Using a truncated distribution is allowed only if the inversion method is available and selected by the `unur_dstd_set_variant` call immediately after creating the parameter object. Use a `unur_distr_discr_set_domain` call to get a truncated distribution.

`int unur_dstd_set_variant` (*UNUR_PAR* parameters*, *unsigned variant*)
: Set variant (special generator) for sampling from a given distribution. For possible variants see Chapter 7 [Standard distributions], page 201.

  Common variants are `UNUR_STDGEN_DEFAULT` for the default generator, `UNUR_STDGEN_FAST` for (one of the) fastest implemented special generators, and `UNUR_STDGEN_INVERSION` for the

inversion method (if available). If the selected variant number is not implemented, this call has no effect.

5.9 Methods for random matrices

Overview of methods

Methods for **matrix distributions**
sample with `unur_sample_matr`

MCORR: Distribution object for random correlation matrix.

5.9.1 MCORR – Random CORRelation matrix

Required: Distribution object for random correlation matrix

Speed: Set-up: fast, Sampling: depends on dimension

Reinit: supported

Reference: [DLa86: Sect.6.1; p.605] [MOa84]

MCORR generates a random correlation matrix (Pearson's correlation). Two methods are used:

1. When a random correlation matrix having given eigenvalues is sought, the method of Marsaglia and Olkin [MOa84] is used. In this case, the correlation matrix R is given as $R = PDP'$ where D is a diagonal matrix containing the eigenvalues and P is a random orthonormal matrix. In higher dimensions, the rounding-errors introduced in the previous matrix multiplications could lead to a non-symmetric correlation matrix. Therefore the symmetric correlation matrix is computed as $R = (PDP' + P'DP)/2$.
2. A matrix H is generated where all rows are independent random vectors of unit length uniformly on a sphere. Then HH' is a correlation matrix (and vice versa if HH' is a correlation matrix then the rows of H are random vectors on a sphere).

Notice that due to round-off errors the generated matrices might not be positive definite in extremely rare cases (especially when the given eigenvalues are amost 0).

There are many other possibilites (distributions) of sampling the random rows from a sphere. The chosen methods are simple but does not result in a uniform distriubution of the random correlation matrices.

It only works with distribution objects of random correlation matrices (see Section 7.4.1 [Random Correlation Matrix], page 213).

How To Use

Create a distibution object for random correlation matrices by a `unur_distr_correlation` call (see Section 7.4.1 [Random Correlation Matrix], page 213).

When a correlation matrix with given eigenvalues should be generated, these eigenvalues can be set by a `unur_mcorr_set_eigenvalues` call.

Otherwise, a faster algorithm is used that generates correlation matrices with random eigenstructure.

Notice that due to round-off errors, there is a (small) chance that the resulting matrix is not positive definite for a Cholesky decomposition algorithm, especially when the dimension of the distribution is high.

It is possible to change the given eigenvalues using `unur_mcorr_chg_eigenvalues` and run `unur_reinit` to reinitialize the generator object.

Function reference

`UNUR_PAR* unur_mcorr_new (`*const* `UNUR_DISTR* distribution`)
Get default parameters for generator.

`int unur_mcorr_set_eigenvalues (`*UNUR_PAR** `par`, *const* `double* eigenvalues`)
Sets the (optional) eigenvalues of the correlation matrix. If set, then the Marsaglia and Olkin algorithm will be used to generate random correlation matrices with given eigenvalues.

Important: the given eigenvalues of the correlation matrix must be strictly positive and sum to the dimension of the matrix. If non-positive eigenvalues are attempted, no eigenvalues are set and an error code is returned. In case, that their sum is different from the dimension, an implicit scaling to give the correct sum is performed.

`int unur_mcorr_chg_eigenvalues (`*UNUR_GEN* gen, const double* eigenvalues*`)`

Change the eigenvalues of the correlation matrix. One must run `unur_reinit` to reinitialize the generator object then.

5.10 Methods for uniform univariate distributions

5.10.1 UNIF – wrapper for UNIForm random number generator

UNIF is a simple wrapper that makes it possible to use a uniform random number generator as a UNU.RAN generator. There are no parameters for this method.

How To Use

Create a generator object with `NULL` as argument. The created generator object returns raw random numbers from the underlying uniform random number generator.

Function reference

`UNUR_PAR* unur_unif_new` (*const* `UNUR_DISTR* dummy`)

Get default parameters for generator. UNIF does not need a distribution object. *dummy* is not used and can (should) be set to `NULL`. It is used to keep the API consistent.

6 Using uniform random number generators

UNU.RAN is designed to work with many sources of (pseudo-) random numbers or low discrepancy numbers (so called quasi-random numbers) for almost all tasks in discrete event simulation, (quasi-) Monte Carlo integration or any other stochastic methods. Hence UNU.RAN uses pointers to access uniform (pseudo-) random number generators (URNG).

Each UNU.RAN (non-uniform random variate) generator object has a pointer to a URNG object. Thus each UNU.RAN generator object may have its own (independent) URNG or several generator objects can share the same URNG.

If no URNG is provided for a parameter or generator object a default generator is used which is the same for all generators. This URNG is defined in '`unuran_config.h`' at compile time and can be changed at runtime.

UNU.RAN uses a unified interface for all sources of random numbers. Unfortunately, the API for random number generators, like the '`GSL`' (GNU Scientific Library), Otmar Lendl's '`prng`' (Pseudo random number generators), or a single function implemented by the user herself, are quite different. Hence an object of type `UNUR_URNG` is introduced to store the URNG. Thus it is possible to handle different sources of such URNGs with the unified API. It is inspired from similar to Pierre L'Ecuyers '`RngStreams`' library:

- seed the random number generator;
- get a uniform random number;
- reset the URNG;
- skip to the begining next substream;
- sample antithetic numbers;
- delete the URNG object.

The routine to create a URNG depends on the chosen random number generator (i.e. library). Nevertheless, there exist wrapper functions to simplify this task.

Currently the following sources of uniform random numbers are directly supported (i.e., there exist wrapper functions). Of course other random number generation libraries can be used.

1. `FVOID`

   URNGs of type `double uniform(void *state)`. The argument *state* can be simply ignored in the implementation of `uniform` when a global state variable is used. UNU.RAN contains some build-in URNGs of this type in directory '`src/uniform/`'.

2. `PRNG`

   URNGs from Otmar Lendl's `prng` library. It provides a very flexible way to sample form arbitrary URNGs by means of an object oriented programing paradigma. Similarly to the UNU.RAN library independent generator objects can be build and used.

   This library has been developed by the pLab group at the university of Salzburg (Austria, EU) and implemented by Otmar Lendl. It is available from http://statistik.wu-wien.ac.at/prng/ or from the pLab site at http://random.mat.sbg.ac.at/.

   This interface must be compiled into UNU.RAN using the configure flag `--with-urng-prng`.

3. `RNGSTREAM`

   Pierre L'Ecuyer's `RngStream` library for multiple independent streams of pseudo-random numbers. A GNU-style package is available from http://statistik.wu-wien.ac.at/software/RngStreams/.

   This interface must be compiled into UNU.RAN using the configure flag `--with-urng-rngstream`.

4. `GSL`
   URNG from the GNU Scientific Library (GSL). It is available from http://www.gnu.org/software/gsl/.
   This interface must be compiled into UNU.RAN using the configure flag `--with-urng-gsl`.

How To Use

Each UNU.RAN generator object has a pointer to a uniform (pseudo-) random number generator (URNG). It can be set via the `unur_set_urng` call. It is also possible to read this pointer via `unur_get_urng` or change the URNG for an existing generator object by means of `unur_chg_urng`. It is important to note that these calls only copy the pointer to the URNG object into the generator object.

If no URNG is provided for a parameter or generator object a default URNG is used which is the same for all generators. This URNG is defined in '`unuran_config.h`' at compile time. A pointer to this default URNG can be obtained via `unur_get_default_urng`. Nevertheless, it is also possible to change this default URNG by another one at runtime by means of the `unur_set_default_urng` call. However, this only takes effect for new parameter objects.

Some generating methods provide the possibility of correlation induction. For this feature a second auxiliary URNG is required. It can be set and changed by `unur_set_urng_aux` and `unur_chg_urng_aux` calls, respectively. Since the auxiliary URNG is by default the same as the main URNG, the auxiliary URNG must be set after any `unur_set_urng` or `unur_chg_urng` call! Since in special cases mixing of two URNG might cause problems, we supply a default auxiliary generator that can be used by a `unur_use_urng_aux_default` call (after the main URNG has been set). This default auxiliary generator can be changed with analogous calls as the (main) default uniform generator.

Uniform random number generators form different sources have different programming interfaces. Thus UNU.RAN stores all information about a particular uniform random number generator in a structure of type `UNUR_URNG`. Before a URNG can be used with UNU.RAN an appropriate object has to be created ba a `unur_urng_new` call. This call takes two arguments: the pointer to the sampling routine of the generator and a pointer to a possible argument that stores the state of the generator. The function must be of type `double (*sampleunif)(void *params)`, but functions without any argument also work. Additionally one can set pointers to functions for reseting or jumping the streams generated by the URNG by the corresponding `set` calls.

UNU.RAN provides a unified API to all sources of random numbers. Notice, however, that not all functions work for all random number generators (as the respective library has not implemented the corresponding feature).

There are wrapper functions for some libraries of uniform random number generators to simplify the task of creating a UNU.RAN object for URNGs. These functions must be compiled into UNU.RAN using the corresponding configure flags (see description of the respective interface below).

Function reference

Set and get default uniform RNGs

`UNUR_URNG* unur_get_default_urng` (*void*)
Get the pointer to the default URNG. The default URNG is used by all generators where no URNG was set explicitly by a `unur_set_urng` call.

`UNUR_URNG* unur_set_default_urng` (*UNUR_URNG* `urng_new`)
Change the default URNG that is used for new parameter objects. It returns the pointer to the old default URNG that has been used.

`UNUR_URNG* unur_set_default_urng_aux` (*UNUR_URNG* urng_new*)
`UNUR_URNG* unur_get_default_urng_aux` (*void*)
Analogous calls for default auxiliary generator.

Set, change and get uniform RNGs in generator objects

`int unur_set_urng` (*UNUR_PAR* parameters*, *UNUR_URNG* urng*)
Use the URNG `urng` for the new generator. This overrides the default URNG. It also sets the auxiliary URNG to `urng`.

Important: For multivariate distributions that use marginal distributions this call does not work properly. It is then better first to create the generator object (by a `unur_init` call) and then change the URNG by means of `unur_chg_urng`.

`UNUR_URNG* unur_chg_urng` (*UNUR_GEN* generator*, *UNUR_URNG* urng*)
Change the URNG for the given generator. It returns the pointer to the old URNG that has been used by the generator. It also changes the auxiliary URNG to `urng` and thus it overrides the last `unur_chg_urng_aux` call.

`UNUR_URNG* unur_get_urng` (*UNUR_GEN* generator*)
Get the pointer to the URNG that is used by the *generator*. This is usefull if two generators should share the same URNG.

`int unur_set_urng_aux` (*UNUR_PAR* parameters*, *UNUR_URNG* urng_aux*)
Use the auxiliary URNG `urng_aux` for the new generator. (Default is the default URNG or the URNG from the last `unur_set_urng` call. Thus if the auxiliary generator should be different to the main URNG, `unur_set_urng_aux` must be called after `unur_set_urng`. The auxiliary URNG is used as second stream of uniform random number for correlation induction. It is not possible to set an auxiliary URNG for a method that does not need one. In this case an error code is returned.

`int unur_use_urng_aux_default` (*UNUR_PAR* parameters*)
Use the default auxiliary URNG. (It must be set after `unur_get_urng`.) It is not possible to set an auxiliary URNG for a method that does not use one (i.e. the call returns an error code).

`int unur_chgto_urng_aux_default` (*UNUR_GEN* generator*)
Switch to default auxiliary URNG. (It must be set after `unur_get_urng`.) It is not possible to set an auxiliary URNG for a method that does not use one (i.e. the call returns an error code).

`UNUR_URNG* unur_chg_urng_aux` (*UNUR_GEN* generator*, *UNUR_URNG* urng_aux*)
Change the auxiliary URNG for the given *generator*. It returns the pointer to the old auxiliary URNG that has been used by the generator. It has to be called after each `unur_chg_urng` when the auxiliary URNG should be different from the main URNG. It is not possible to change the auxiliary URNG for a method that does not use one (i.e. the call `NULL`).

`UNUR_URNG* unur_get_urng_aux` (*UNUR_GEN* generator*)
Get the pointer to the auxiliary URNG that is used by the *generator*. This is usefull if two generators should share the same URNG.

Handle uniform RNGs

Notice: Some of the below function calls do not work for every source of random numbers since not every library has implemented these features.

`double unur_urng_sample (`*UNUR_URNG* urng*`)`
Get a uniform random number from *urng*. If the `NULL` pointer is given, the default uniform generator is used.

`double unur_sample_urng (`*UNUR_GEN* gen*`)`
Get a uniform random number from the underlying uniform random number generator of generator *gen*. If the `NULL` pointer is given, the default uniform generator is used.

`int unur_urng_sample_array (`*UNUR_URNG* urng, double* X, int dim*`)`
Set array X of length *dim* with uniform random numbers sampled from generator *urng*. If *urng* is the `NULL` pointer, the default uniform generator is used.

Important: If *urng* is based on a point set generator (this is the case for generators of low discrepance point sets as used in quasi-Monte Carlo methods) it has a "natural dimension" s. In this case either only the first s entries of X are filled (if $s <$ *dim*), or the first *dim* coordinates of the generated point are filled.

The called returns the actual number of entries filled. In case of an error `0` is returned.

`int unur_urng_reset (`*UNUR_URNG* urng*`)`
Reset *urng* object. The routine tries two ways to reset the generator (in this order):

1. It uses the reset function given by an `unur_urng_set_reset` call.
2. It uses the seed given by the last `unur_urng_seed` call (which requires a seeding function given by a `unur_urng_set_seed` call).

If neither of the two methods work resetting of the generator is not possible and an error code is returned.

If the `NULL` pointer is given, the default uniform generator is reset.

`int unur_urng_sync (`*UNUR_URNG* urng*`)`
Jump into defined state (`"sync"`) of the generator. This is useful when point generators are used where the coordinates are sampled via `unur_urng_sample`. Then this call can be used to jump to the first coordinate of the next generated point.

`int unur_urng_seed (`*UNUR_URNG* urng, unsigned long seed*`)`
Set *seed* for generator *urng*. It returns an error code if this is not possible for the given URNG. If the `NULL` pointer is given, the default uniform generator is seeded (if possible).

Notice: Seeding should be done only once for a particular generator (except for resetting it to the initial state). Expertise is required when multiple seeds are used to get independent streams. Thus we recommend appropriate libraries for this task, e.g. Pierre L'Ecuyer's '`RngStreams`' package. For this library only a package seed can be set and thus the `unur_urng_seed` call will not have any effect to generators of this type. Use `unur_urng_reset` or `unur_urng_rngstream_new` instead, depending whether one wants to reset the stream or get a new stream that is independent from the previous ones.

`int unur_urng_anti (`*UNUR_URNG* urng, int anti*`)`
Switch to antithetic random numbers in *urng*. It returns an error code if this is not possible for the given URNG.

If the `NULL` pointer is given, the antithetic flag of the default uniform generator is switched (if possible).

`int unur_urng_nextsub (`*UNUR_URNG* urng*`)`
Jump to start of the next substream of *urng*. It returns an error code if this is not possible for the given URNG.

If the `NULL` pointer is given, the default uniform generator is set to the start of the next substream (if possible).

`int unur_urng_resetsub (`*UNUR_URNG* urng*`)`
Jump to start of the current substream of *urng*. It returns an error code if this is not possible for the given URNG.

If the `NULL` pointer is given, the default uniform generator is set to the start of the current substream (if possible).

`int unur_gen_sync (`*UNUR_GEN* generator*`)`
`int unur_gen_seed (`*UNUR_GEN* generator*, *unsigned long seed*`)`
`int unur_gen_anti (`*UNUR_GEN* generator*, *int anti*`)`
`int unur_gen_reset (`*UNUR_GEN* generator*`)`
`int unur_gen_nextsub (`*UNUR_GEN* generator*`)`
`int unur_gen_resetsub (`*UNUR_GEN* generator*`)`
Analogous to `unur_urng_sync`, `unur_urng_seed` , `unur_urng_anti`, `unur_urng_reset` , `unur_urng_nextsub`, and `unur_urng_resetsub`, but act on the URNG object used by the *generator* object.

Warning: These calls should be used with care as it influences all generator objects that share the same URNG object!

API to create a new URNG object

Notice: These functions are provided to built a UNUR_URNG object for a particular external random number generator from scratch. For some libraries that contain random number generators (like the GSL) there are special calls, e.g. `unur_urng_gsl_new`, to get such an object. Then there is no need to change the UNUR_URNG object as it already contains all available features.

If you have a particular library for random number generators you can either write wrapper function like those in '`src/uniform/urng_gsl.c`' or write an email to the authors of UNU.RAN to write it for you.

`UNUR_URNG* unur_urng_new (`*double (* sampleunif*`)(`*void* state* `), `*void* state*`)`
Get a new URNG object. *sampleunif* is a function to the uniform sampling routine, *state* a pointer to its arguments which usually contains the state variables of the generator.

Functions *sampleunif* with a different type for *p* or without an argument at all also work. A typecast might be necessary to avoid compiler warnings or error messages.

For functions *sampleunif* that does not have any argument should use `NULL` for *state*.

Important: *sampleunif* must not be the `NULL` pointer.

There are appropriate calls that simplifies the task of creating URNG objects for some libraries with uniform random number generators, see below.

`void unur_urng_free (`*UNUR_URNG* urng*`)`
Destroy *urng* object. It returns an error code if this is not possible.

If the `NULL` is given, this function does nothing.

Warning: This call must be used with care. The *urng* object must not be used by any existing generator object! It is designed to work in conjunction with the wrapper functions to create URNG objects for generators of a particular library. Thus an object created by an `unur_urng_prng_new` call can be simply destroyed by an `unur_urng_free` call.

`int unur_urng_set_sample_array` (*UNUR_URNG* **urng***, *unsigned int*(* ***samplearray***)(*void* state, double* **X**, int **dim** *))
Set function to fill array X of length *dim* with random numbers generated by generator *urng* (if available).

`int unur_urng_set_sync` (*UNUR_URNG* **urng***, *void* (* ***sync***)(*void* state*))
Set function for jumping into a defined state ("sync").

`int unur_urng_set_seed` (*UNUR_URNG* **urng***, *void* (* ***setseed***)(*void* state, unsigned* ***long seed***))
Set function to seed generator *urng* (if available).

`int unur_urng_set_anti` (*UNUR_URNG* **urng***, *void* (* ***setanti***)(*void* state, int* ***anti***))
Set function to switch the antithetic flag of generator *urng* (if available).

`int unur_urng_set_reset` (*UNUR_URNG* **urng***, *void* (* ***reset***)(*void* state*))
Set function for reseting the uniform random number generator *urng* (if available).

`int unur_urng_set_nextsub` (*UNUR_URNG* **urng***, *void* (* ***nextsub***)(*void* state*))
Set function that allows jumping to start of the next substream of *urng* (if available).

`int unur_urng_set_resetsub` (*UNUR_URNG* **urng***, *void* (* ***resetsub***)(*void* state*))
Set function that allows jumping to start of the current substream of *urng* (if available).

`int unur_urng_set_delete` (*UNUR_URNG* **urng***, *void* (* ***fpdelete***)(*void* state*))
Set function for destroying *urng* (if available).

6.1 Simple interface for uniform random number generators

Simple interface for URNGs of type `double uniform(void *state)`.

UNU.RAN contains some build-in URNGs of this type:

`unur_urng_MRG31k3p`
: Combined multiple recursive generator by Pierre L'Ecuyer and Renee Touzin.

`unur_urng_fish`
: Linear congruential generator by Fishman and Moore.

`unur_urng_mstd`
: Linear congruential generator "Minimal Standard" by Park and Miller.

Notice, however, that these generators are provided as a fallback for the case that no state-of-the-art uniform random number generators (e.g. see Section 6.5 [Pierre L'Ecuyer's 'Rngstream' library], page 198) are used.

How To Use

Create an URNG object using `unur_urng_fvoid_new`. By this call a pointer to the sampling routine and (optional) a pointer to a reset routine are copied into the URNG object. Other functions, like seeding the URNG, switching to antithetic random number, or jumping to next substream, can be added to the URNG object by the respective calls, e.g. by `unur_urng_set_seed`. The following routines are supported for URNG objects of this type:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_seed` [optional]
- `unur_urng_reset` [optional]
- `unur_urng_free`

Function reference

`UNUR_URNG* unur_urng_fvoid_new` (*double* (* ***`random`***)(*void* state), *void* (* ***`reset`***)(*void* state))

Make a URNG object for a genertor that consists of a single function call.

If there is no reset function use `NULL` for the second argument.

6.2 Interface to GSL uniform random number generators

Interface to the uniform random number generators from the GNU Scientific Library (GSL). Documentation and source code of this library is available from http://www.gnu.org/software/gsl/.

The interface to the GSL must be compiled into UNU.RAN using the configure flag `--with-urng-gsl`. Notice that the GSL has to be installed before running `./configure`.

How To Use

When using this interface '`unuran_urng_gsl.h`' must be included in the corresponding C file, i.e., one must add the line

```
#include <unuran_urng_gsl.h>
```

Moreover, one must not forget to link the executable against '`libgsl`'.

The following routines are supported for URNG objects of type GSL:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_seed`
- `unur_urng_reset`
- `unur_urng_free`

```
/* ------------------------------------------------------------ */
/* File: example_gsl.c                                          */
/* ------------------------------------------------------------ */
#ifdef UNURAN_SUPPORTS_GSL
/* ------------------------------------------------------------ */
/* This example makes use of the GSL library for generating     */
/* uniform random numbers.                                      */
/* (see http://www.gnu.org/software/gsl/)                       */
/* To compile this example you must have set                    */
/*   ./configure --with-urng-gsl                                */
/* (Of course the executable has to be linked against the       */
/* GSL library.)                                                */
/* ------------------------------------------------------------ */

/* Include UNURAN header files.                                 */
#include <unuran.h>
#include <unuran_urng_gsl.h>

/* ------------------------------------------------------------ */

int main(void)
{
```

```
  int    i;          /* loop variable                          */
  double x;          /* will hold the random number            */

  /* Declare the three UNURAN objects.                         */
  UNUR_DISTR *distr;    /* distribution object                 */
  UNUR_PAR   *par;      /* parameter object                    */
  UNUR_GEN   *gen;      /* generator object                    */

  /* Declare objects for uniform random number generators.     */
  UNUR_URNG  *urng;     /* uniform generator objects           */

  /* GNU Scientific Library only:                              */
  /* Make a object for uniform random number generator.        */
  urng = unur_urng_gsl_new(gsl_rng_mt19937);
  if (urng == NULL) exit (EXIT_FAILURE);

  /* Create a generator object using this URNG */
  distr = unur_distr_normal( NULL, 0 );
  par = unur_tdr_new(distr);
  unur_set_urng( par, urng );
  gen = unur_init(par);
  if (gen == NULL)  exit (EXIT_FAILURE);
  unur_distr_free(distr);

  /* Now you can use the generator object 'gen' to sample from */
  /* the distribution. Eg.:                                    */
  for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
  }

  /* Destroy objects */
  unur_free(gen);
  unur_urng_free(urng);

  exit (EXIT_SUCCESS);
} /* end of main() */

/* ---------------------------------------------------------- */
#else
#include <stdio.h>
#include <stdlib.h>
int main(void) {
  printf("You must enable the GSL to run this example!\n\n");
  exit (77);    /* exit code for automake check routines */
}
#endif
/* ---------------------------------------------------------- */
```

Function reference

`UNUR_URNG* unur_urng_gsl_new` (*const gsl_rng_type* urngtype*)

Make object for URNGs from the '`GSL`' (GNU Scientific Library). *urngtype* is the type of the chosen generator as described in the GSL manual (see Section Random Number Generation). This library is available from http://www.gnu.org/software/gsl/.

`UNUR_URNG* unur_urng_gslptr_new` (*gsl_rng* urng*)

Similar to `unur_urng_gsl_new` but it uses a pointer to a generator object as returned by `gsl_rng_alloc(rng_type)`; see '`GSL`' manual for details.

Notice: There is a subtle but important difference between these two calls. When a generator object is created by a `unur_urng_gsl_new` call, then resetting of the generator works. When

a generator object is created by a `unur_urng_gslptr_new` call, then resetting only works after a `unur_urng_seed(urng,myseed)` call.

6.3 Interface to GSL generators for quasi-random points

Interface to the generators for quasi-random points (also called low discrepancy point sets) from the GNU Scientific Library (GSL). Documentation and source code of this library is available from http://www.gnu.org/software/gsl/.

The interface to the GSL must be compiled into UNU.RAN using the configure flag `--with-urng-gsl`. Notice that the GSL has to be installed before running `./configure`.

How To Use

When using this interface '`unuran_urng_gsl.h`' must be included in the corresponding C file, i.e., one must add the line

```
#include <unuran_urng_gsl.h>
```

Moreover, one must not forget to link the executable against '`libgsl`'.

The following routines are supported for URNG objects of this type:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_reset`
- `unur_urng_sync`
- `unur_urng_free`

`unur_urng_sync` is used to jump to the first coordinate of the next point generated by the generator.

Function reference

`UNUR_URNG* unur_urng_gslqrng_new` (*const gsl_qrng_type* qrngtype, unsigned int dim*)

Make object for quasi-random point generators for dimension *dim* from the '`GSL`' (GNU Scientific Library). *qrngtype* is the type of the chosen generator as described in the GSL manual (see section Quasi-Random Sequences). This library is available from http://www.gnu.org/software/gsl/.

6.4 Interface to Otmar Lendl's pseudo-random number generators

URNGs from Otmar Lendl's `prng` library. It provides a very flexible way to sample form arbitrary URNGs by means of an object oriented programing paradigma. Similarly to the UNU.RAN library independent generator objects can be build and used.

This library has been developed by the pLab group at the university of Salzburg (Austria, EU) and implemented by Otmar Lendl. It is available from http://statistik.wu-wien.ac.at/prng/ or from the pLab site at http://random.mat.sbg.ac.at/.

The interface to the PRNG library must be compiled into UNU.RAN using the configure flag `--with-urng-prng`. Notice that the PRNG library has to be installed before running `./configure`.

How To Use

When using this interface '`unuran_urng_prng.h`' must be included in the corresponding C file, i.e., one must add the line

```
#include <unuran_urng_prng.h>
```

Moreover, one must not forget to link the executable against '`libprng`'.

The following routines are supported for URNG objects of type PRNG:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_reset`
- `unur_urng_free`

Function reference

`UNUR_URNG* unur_urng_prng_new` (*const char* prngstr*)

Make object for URNGs from Otmar Lendl's '`prng`' package. *prngstr* is a string that contains the necessary information to create a uniform random number generator. For the format of this string see the '`prng`' user manual.

The '`prng`' library provides a very flexible way to sample form arbitrary URNGs by means of an object oriented programing paradigma. Similarly to the UNU.RAN library independent generator objects can be build and used. The library has been developed and implemented by Otmar Lendl as member of the pLab group at the university of Salzburg (Austria, EU).

It is available via anonymous ftp from http://statistik.wu-wien.ac.at/prng/ or from the pLab site at http://random.mat.sbg.ac.at/.

`UNUR_URNG* unur_urng_prngptr_new` (*struct prng* urng*)

Similar to `unur_urng_prng_new` but it uses a pointer to a generator object as returned by `prng_new(prngstr)`; see '`prng`' manual for details.

6.5 Interface to L'Ecuyer's RNGSTREAM random number generators

URNGs from Pierre L'Ecuyer's '`RngStream`' library for multiple independent streams of pseudo-random numbers. This library provides multiple independent streams of pseudo-random numbers which itselves can be splitted into many substreams. It is available from http://www.iro.umontreal.ca/~lecuyer/myftp/streams00/c/. A GNU-style package is available from http://statistik.wu-wien.ac.at/software/RngStreams/.

The interface to the RngStream library must be compiled into UNU.RAN using the configure flag `--with-urng-rngstream`. Notice that the RngStream library has to be installed before running `./configure`.

How To Use

When using this interface '`unuran_urng_rngstream.h`' must be included in the corresponding C file, i.e., one must add the line

```
#include <unuran_urng_rngstream.h>
```

Moreover, one must not forget to link the executable against the '`RngStream`' library (i.e., when using the GNU-style package in UNIX like environments one has to add `-lrngstreams` when linking an executable).

Notice that the '`rngstream`' library uses a package seed, that means one should seed the uniform random number generator only once in an application using the routine RngStream_SetPackageSeed:

```
unsigned long seed[] = {111u, 222u, 333u, 444u, 555u, 666u};
RngStream_SetPackageSeed(seed);
```

The following routines are supported for URNG objects of this type:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_reset`
- `unur_urng_nextsub`
- `unur_urng_resetsub`
- `unur_urng_anti`
- `unur_urng_free`

Function reference

`UNUR_URNG* unur_urng_rngstream_new` (*const* `char* urngstr`)
Make object for URNGs from Pierre L'Ecuyer's '`RngStream`' library. *urngstr* is an arbitrary string to label a stream. It need not be unique.

`UNUR_URNG* unur_urng_rngstreamptr_new` (*RngStream* `rngstream`)
Similar to `unur_urng_rngstream_new` but it uses a pointer to a generator object as returned by `RngStream_CreateStream()`.

6.6 Combine point set generator with random shifts

Generators of type RANDOMSHIFT combine a point set generator with generators to apply random shifts as proposed in [CPa76] :

1. Sample and store a random vector S.
2. Run a QMC simulation where S is added to each point of the generated quasi-random point (mod 1).
3. Repeat steps 1 and 2.

How To Use

Create a URNG object for a point set generator and a URNG object for a generator to create shift vectors at random. The meta URNG object can then be created using `unur_urng_randomshift_new`. Notice that only pointers to the two underlying URNG generator objects are copied into the newly created meta generator. Thus manipulating the meta URNG also changes the underlying URNGs and vice versa.

The following routines are supported for URNG objects of type RANDOMSHIFT:

- `unur_urng_sample`
- `unur_urng_sample_array`
- `unur_urng_reset`
- `unur_urng_sync`
- `unur_urng_randomshift_nextshift`
- `unur_urng_free`

`unur_urng_sync` is used to jump to the first coordinate of the next point generated by the generator. `unur_urng_randomshift_nextshift` allows to replace the shift vector by another randomly chosen shift vector.

Important: `unur_urng_sync` is only available if it is if it is implemented for the underlying point set generator.

Important: `unur_urng_reset` is only available if it is available for both underlying generators.

Function reference

`UNUR_URNG* unur_urng_randomshift_new` (*UNUR_URNG* **qrng**, *UNUR_URNG* **srng**, *int* **dim**)

Make object for URNG with randomly shifted point sets. *qrng* is a generated that generates point sets of dimension *dim*. *srng* is a generated that generates random numbers or vectors.

Notice: Only pointers to the respective objects *qrng* and *srng* are copied into the created meta generator. Thus manipulating the meta URNG also changes the underlying URNGs and vice versa.

`int unur_urng_randomshift_nextshift` (*UNUR_URNG* **urng**)

Get the next (randomly chosen) vector for shifting the points set, and the underlying point generator *qrng* is reset.

7 UNU.RAN Library of standard distributions

Although it is not its primary target, many distributions are already implemented in UNU.RAN. This section presents these available distributions and their parameters.

The syntax to get a distribuion object for distributions `<dname>` is:

`UNUR_DISTR* unur_distr_<dname>` (*double** ***params***, *int* ***n_params***) [–]

params is an array of doubles of size *n_params* holding the parameters.

E.g. to get an object for the gamma distribution (with shape parameter) use

```
unur_distr_gamma( params, 1 );
```

Distributions may have default parameters with need not be given explicitely. E.g. The gamma distribution has three parameters: the shape, scale and location parameter. Only the (first) shape parameter is required. The others can be omitted and are then set by default values.

```
/* alpha = 5; default: beta = 1, gamma = 0 */
double fpar[] = {5.};
unur_distr_gamma( fpar, 1 );

/* alpha = 5, beta = 3; default: gamma = 0 */
double fpar[] = {5., 3.};
unur_distr_gamma( fpar, 2 );

/* alpha = 5, beta = 3, gamma = -2
double fpar[] = {5., 3., -2.};
unur_distr_gamma( fpar, 3 );
```

Important: Naturally the computational accuracy limits the possible parameters. There shouldn't be problems when the parameters of a distribution are in a "reasonable" range but e.g. the normal distribution N(10^15,1) won't yield the desired results. (In this case it would be better generating N(0,1) and *then* transform the results.)
Of course computational inaccuracy is not specific to UNU.RAN and should always be kept in mind when working with computers.

Important: The routines of the standard library are included for non-uniform random variate generation and not to provide special functions for statistical computations.

Remark

The following keywords are used in the tables:

PDF probability density function, with variable x.

PMF probability mass function, with variable k.

constant normalization constant for given PDF and PMF, resp. They must be multiplied by *constant* to get the "real" PDF and PMF.

CDF gives information whether the CDF is implemented in UNU.RAN.

domain domain PDF and PMF, resp.

parameters *n_std (n_total):* list
list of parameters for distribution, where *n_std* is the number of parameters for the standard form of the distribution and *n_total* the total number for the (non-standard form of the) distribution. *list* is the list of parameters in the order as they are stored in the array of parameters. Optional parameter that can be omitted are enclosed in square brackets `[...]`.

A detailed list of these parameters gives then the range of valid parameters and defaults for optional parameters that are used when these are omitted.

reference gives reference for distribution (see Appendix C [Bibliography], page 237).

special generators
lists available special generators for the distribution. The first number is the variant that to be set by `unur_cstd_set_variant` and `unur_dstd_set_variant` call, respectively. If no variant is set the default variant `DEF` is used. In the table the respective abbreviations `DEF` and `INV` are used for `UNUR_STDGEN_DEFAULT` and `UNUR_STDGEN_INVERSION`. Also the references for these methods are given (see Appendix C [Bibliography], page 237).

Notice that these generators might be slower than universal methods.

If `DEF` is ommited, the first entry is the default generator.

7.1 UNU.RAN Library of continuous univariate distributions

7.1.1 `F` – F-distribution

PDF: $(x^{\nu_1/2-1})/(1+\nu_1/\nu_2 x)^{(\nu_1+\nu_2)/2}$

constant: $(\nu_1/\nu_2)^{\nu_1/2}/Beta(\nu_1/2, \nu_2/2)$

domain: $0 < x < \infty$

parameters 2 (2): nu_1, nu_2

No.	name		default	
`[0]`	nu_1	> 0		(*scale*)
`[1]`	nu_2	> 0		(*scale*)

reference: [JKBc95: Ch.27; p.322]

7.1.2 `beta` – Beta distribution

PDF: $(x-a)^{p-1}\,(b-x)^{q-1}$

constant: $1/(Beta(p,q)\,(b-a)^{p+q-1})$

domain: $a < x < b$

parameters 2 (4): p, q [, a, b]

No.	name		default	
`[0]`	p	> 0		(*scale*)
`[1]`	q	> 0		(*scale*)
`[2]`	a		0	(*location, scale*)
`[3]`	b	$> a$	1	(*location, scale*)

reference: [JKBc95: Ch.25; p.210]

7.1.3 `cauchy` – Cauchy distribution

PDF: $\frac{1}{1+((x-\theta)/\lambda)^2}$

constant: $\frac{1}{\pi\,\lambda}$

domain: $-\infty < x < \infty$

parameters 0 (2): [theta [, lambda]]

No.	name		default	
`[0]`	θ		0	(*location*)
`[1]`	λ	> 0	1	(*scale*)

reference: [JKBb94: Ch.16; p.299]

special generators:

`INV`	Inversion method

7.1.4 `chi` – Chi distribution

PDF: $x^{\nu-1}\,\exp(-x^2/2)$

constant: $1/(2^{(\nu/2)-1}\,\Gamma(\nu/2))$

domain: $0 \le x < \infty$

parameters 1 (1): nu

No.	name		default	
[0]	ν	> 0		(*shape*)

reference: [JKBb94: Ch.18; p.417]

special generators:

DEF	Ratio of Uniforms with shift (only for $\nu \geq 1$) [MJa87]

7.1.5 chisquare – Chisquare distribution

PDF: $x^{(\nu/2)-1} \exp(-x/2)$

constant: $1/(2^{\nu/2}\,\Gamma(\nu/2))$

domain: $0 \leq x < \infty$

parameters 1 (1): nu

No.	name		default	
[0]	ν	> 0		(*shape (degrees of freedom)*)

reference: [JKBb94: Ch.18; p.416]

7.1.6 exponential – Exponential distribution

PDF: $\exp(-\frac{x-\theta}{\sigma})$

constant: $\frac{1}{\sigma}$

domain: $\theta \leq x < \infty$

parameters 0 (2): [sigma [, theta]]

No.	name		default	
[0]	σ	> 0	1	(*scale*)
[1]	θ		0	(*location*)

reference: [JKBb94: Ch.19; p.494]

special generators:

INV	Inversion method

7.1.7 extremeI – Extreme value type I (Gumbel-type) distribution

PDF: $\exp(-\exp(-\frac{x-\zeta}{\theta}) - \frac{x-\zeta}{\theta})$

constant: $\frac{1}{\theta}$

domain: $-\infty < x < \infty$

parameters 0 (2): [zeta [, theta]]

No.	name		default	
[0]	ζ		0	(*location*)
[1]	θ	> 0	1	(*scale*)

reference: [JKBc95: Ch.22; p.2]

special generators:

INV	Inversion method

7.1.8 `extremeII` – Extreme value type II (Frechet-type) distribution

PDF: $\exp(-(\frac{x-\zeta}{\theta})^{-k})\,(\frac{x-\zeta}{\theta})^{-k-1}$

constant: $\frac{k}{\theta}$

domain: $\zeta < x < \infty$

parameters 1 (3): k [, zeta [, theta]]

No.	name		default	
`[0]`	k	> 0		(*shape*)
`[1]`	ζ		0	(*location*)
`[2]`	θ	> 0	1	(*scale*)

reference: [JKBc95: Ch.22; p.2]

special generators:

`INV`	Inversion method

7.1.9 `gamma` – Gamma distribution

PDF: $(\frac{x-\gamma}{\beta})^{\alpha-1}\,\exp(-\frac{x-\gamma}{\beta})$

constant: $1/(\beta\,\Gamma(\alpha))$

domain: $\gamma < x < \infty$

parameters 1 (3): alpha [, beta [, gamma]]

No.	name		default	
`[0]`	α	> 0		(*shape*)
`[1]`	β	> 0	1	(*scale*)
`[2]`	γ		0	(*location*)

reference: [JKBb94: Ch.17; p.337]

special generators:

`DEF`	Acceptance Rejection combined with Acceptance Complement [ADa74] [ADa82]
`2`	Rejection from log-logistic envelopes [CHa77]

7.1.10 `laplace` – Laplace distribution

PDF: $\exp(-\frac{|x-\theta|}{\phi})$

constant: $\frac{1}{2\,\phi}$

domain: $-\infty < x < \infty$

parameters 0 (2): [theta [, phi]]

No.	name		default	
`[0]`	θ		0	(*location*)
`[1]`	ϕ	> 0	1	(*scale*)

reference: [JKBc95: Ch.24; p.164]

special generators:

`INV`	Inversion method

7.1.11 `logistic` – Logistic distribution

PDF: $\exp(-\frac{x-\alpha}{\beta})\,(1+\exp(-\frac{x-\alpha}{\beta}))^{-2}$

constant: $\frac{1}{\beta}$

domain: $-\infty < x < \infty$

parameters 0 (2): [alpha [, beta]]

No.	name		default	
`[0]`	α		0	(*location*)
`[1]`	β	> 0	1	(*scale*)

reference: [JKBc95: Ch.23; p.115]

special generators:

`INV`	Inversion method

7.1.12 `lomax` – Lomax distribution (Pareto distribution of second kind)

PDF: $(x+C)^{-(a+1)}$

constant: $a\,C^a$

domain: $0 \le x < \infty$

parameters 1 (2): a [, C]

No.	name		default	
`[0]`	a	> 0		(*shape*)
`[1]`	C	> 0	1	(*scale*)

reference: [JKBb94: Ch.20; p.575]

special generators:

`INV`	Inversion method

7.1.13 `normal` – Normal distribution

PDF: $\exp(-\frac{1}{2}\,(\frac{x-\mu}{\sigma})^2)$

constant: $\frac{1}{\sigma\sqrt{2\pi}}$

domain: $-\infty < x < \infty$

parameters 0 (2): [mu [, sigma]]

No.	name		default	
`[0]`	μ		0	(*location*)
`[1]`	σ	> 0	1	(*scale*)

reference: [JKBb94: Ch.13; p.80]

special generators:

`DEF`	ACR method (Acceptance-Complement Ratio) [HDa90]
`1`	Box-Muller method [BMa58]
`2`	Polar method with rejection [MGa62]
`3`	Kindermann-Ramage method [KRa76]
`INV`	Inversion method (slow)

7.1.14 `pareto` – Pareto distribution (of first kind)

PDF: $x^{-(a+1)}$

constant: $a\,k^a$

domain: $k < x < \infty$

parameters 2 (2): k, a

No.	name		default	
[0]	k	> 0		(*shape, location*)
[1]	a	> 0		(*shape*)

reference: [JKBb94: Ch.20; p.574]

special generators:

`INV` Inversion method

7.1.15 `powerexponential` – Powerexponential (Subbotin) distribution

PDF: $\exp(-|x|^\tau)$

constant: $1/(2\,\Gamma(1+1/\tau))$

domain: $-\infty < x < \infty$

parameters 1 (1): tau

No.	name		default	
[0]	τ	> 0		(*shape*)

reference: [JKBc95: Ch.24; p.195]

special generators:

`DEF` Transformed density rejection (only for $\tau \geq 1$) [DLa86]

7.1.16 `rayleigh` – Rayleigh distribution

PDF: $x\,\exp(-1/2\,(\frac{x}{\sigma})^2)$

constant: $\frac{1}{\sigma^2}$

domain: $0 \leq x < \infty$

parameters 1 (1): sigma

No.	name		default	
[0]	σ	> 0		(*scale*)

reference: [JKBb94: Ch.18; p.456]

7.1.17 `student` – Student's t distribution

PDF: $(1+\frac{t^2}{\nu})^{-(\nu+1)/2}$

constant: $\frac{1}{\sqrt{\nu}\,B(1/2,\nu/2)}$

CDF: not implemented!

domain: $-\infty < x < \infty$

parameters 1 (1): nu

No.	name		default	
[0]	ν	> 0		(*shape*)

reference: [JKBc95: Ch.28; p.362]

7.1.18 `triangular` – Triangular distribution

PDF: $2\,x/H$, for $0 \le x \le H$
$2\,(1-x)/(1-H)$, for $H \le x \le 1$

constant: 1

domain: $0 \le x \le 1$

parameters 0 (1): [H]

No.	name		default	
[0]	H	$0 \le H \le 1$	1/2	(*shape*)

reference: [JKBc95: Ch.26; p.297]

special generators:

`INV`	Inversion method

7.1.19 `uniform` – Uniform distribution

PDF: $\frac{1}{b-a}$

constant: 1

domain: $a < x < b$

parameters 0 (2): [a, b]

No.	name		default	
[0]	a		0	(*location*)
[1]	b	$> a$	1	(*location*)

reference: [JKBc95: Ch.26; p.276]

special generators:

`INV`	Inversion method

7.1.20 `weibull` – Weibull distribution

PDF: $(\frac{x-\zeta}{\alpha})^{c-1} \exp(-(\frac{x-\zeta}{\alpha})^c)$

constant: $\frac{c}{\alpha}$

domain: $\zeta < x < \infty$

parameters 1 (3): c [, alpha [, zeta]]

No.	name		default	
[0]	c	> 0		(*shape*)
[1]	α	> 0	1	(*scale*)
[2]	ζ		0	(*location*)

reference: [JKBb94: Ch.21; p.628]

special generators:

`INV`	Inversion method

7.2 UNU.RAN Library of continuous multivariate distributions

7.2.1 `copula` – Copula (distribution with uniform marginals)

`UNUR_DISTR *unur_distr_copula(int dim, const double *rankcorr)` creates a distribution object for a copula with *dim* components. *rankcorr* is an array of size *dim*x*dim* and holds the rank correlation matrix (Spearman's correlation), where the rows of the matrix are stored consecutively in this array. The `NULL` pointer can be used instead the identity matrix.

If *covar* is not a valid rank correlation matrix (i.e., not positive definite) then no distribution object is created and `NULL` is returned.

7.2.2 `multicauchy` – Multicauchy distribution

PDF: $f(x) = 1/(1 + (x-\mu)^t.\Sigma^{-1}.(x-\mu))^{(dim+1)/2}$

constant: $\Gamma((dim+1)/2)/(\pi^{(dim+1)/2}\sqrt{det(\Sigma)})$

domain: $-\infty^{dim} < x < \infty^{dim}$

parameters 0 (2): [mu, Sigma]

No.	name		default	
`[0]`	μ		(0,. . .,0)	(*location*)
`[1]`	$Sigma$	$Symm, Pos.def.$	I	(*shape*)

special generators:

`DEF`	Cholesky factor

7.2.3 `multiexponential` – Multiexponential distribution

PDF: $f(x) = Prod_{i=0}^{i=dim-1} \exp(-(dim-i)(x_i - x_{i-1} - (\theta_i - \theta_{i-1}))/\sigma_i); with x_{-1} = 0 and \theta_{i-1} = 0$

constant: $Prod_{i=0}^{i=dim-1} 1/\sigma_i$

domain: $0^{dim} \le x < \infty^{dim}$

parameters 0 (2): [sigma, theta]

No.	name	default	
`[0]`	σ	(1,. . .,1)	(*shape*)
`[1]`	θ	(0,. . .,0)	(*location*)

7.2.4 `multinormal` – Multinormal distribution

PDF: $f(x) = \exp(-1/2\,(x-\mu)^t.\Sigma^{-1}.(x-\mu))$

constant: $1/((2\pi)^{dim/2}\sqrt{det(\Sigma)})$

domain: $-\infty^{dim} < x < \infty^{dim}$

parameters 0 (2): [mu, Sigma]

No.	name		default	
`[0]`	μ		(0,. . .,0)	(*location*)
`[1]`	$Sigma$	$Symm, Pos.def.$	I	(*shape*)

reference: [KBJe00: Ch.45; p.105]

7.2.5 `multistudent` – Multistudent distribution

PDF: $f(x) = 1/(1 + (x-\mu)^t.\Sigma^{-1}.(x-\mu)/m)^{(dim+m)/2})$

constant: $\Gamma((dim+m)/2)/(\Gamma(m/2)(m\,\pi)^{dim/2}\,\sqrt{det(\Sigma)})$

domain: $-\infty^{dim} < x < \infty^{dim}$

parameters 0 (3): [m, mu, Sigma]

No.	name		default	
`[0]`	m	$m > 0$	1	(*location*)
`[1]`	μ		(0,. . .,0)	(*location*)
`[2]`	$Sigma$	$Symm, Pos.def.$	I	(*shape*)

7.3 UNU.RAN Library of discrete univariate distributions

At the moment there are no CDFs implemented for discrete distribution. Thus `unur_distr_discr_upd_pmfsum` does not work properly for truncated distribution.

7.3.1 `binomial` – Binomial distribution

PMF: $\binom{n}{k} p^k (1-p)^{n-k}$

constant: 1

domain: $0 \le k \le n$

parameters 2 (2): n, p

No.	name		default	
[0]	n	≥ 1		(*no. of elements*)
[1]	p	$0 < p < 1$		(*shape*)

reference: [JKKa92: Ch.3; p.105]

special generators:

`DEF` Ratio of Uniforms/Inversion [STa89]

7.3.2 `geometric` – Geometric distribution

PMF: $p\,(1-p)^k$

constant: 1

domain: $0 \le k < \infty$

parameters 1 (1): p

No.	name		default	
[0]	p	$0 < p < 1$		(*shape*)

reference: [JKKa92: Ch.5.2; p.201]

special generators:

`INV` Inversion method

7.3.3 `hypergeometric` – Hypergeometric distribution

PMF: $\binom{M}{k}\binom{N-M}{n-k}/\binom{N}{n}$

constant: 1

domain: $\max(0, n-N+M) \le k \le \min(n, M)$

parameters 3 (3): N, M, n

No.	name		default	
[0]	N	≥ 1		(*no. of elements*)
[1]	M	$1 \le M \le N$		(*shape*)
[2]	n	$1 \le n \le N$		(*shape*)

reference: [JKKa92: Ch.6; p.237]

special generators:

`DEF` Ratio of Uniforms/Inversion [STa89]

7.3.4 `logarithmic` – Logarithmic distribution

PMF: θ^k/k

constant: $-log(1.-\theta)$;

domain: $1 \le k < \infty$

parameters 1 (1): theta

No.	name		default	
`[0]`	θ	$0 < \theta < 1$		(*shape*)

reference: [JKKa92: Ch.7; p.285]

special generators:

`DEF`	Inversion/Transformation [KAa81]

7.3.5 `negativebinomial` – Negative Binomial distribution

PMF: $\binom{k+r-1}{r-1} p^r (1-p)^k$

constant: 1

domain: $0 \le k < \infty$

parameters 2 (2): p, r

No.	name		default	
`[0]`	p	$0 < p < 1$		(*shape*)
`[1]`	r	> 0		(*shape*)

reference: [JKKa92: Ch.5.1; p.200]

7.3.6 `poisson` – Poisson distribution

PMF: $\theta^k/k!$

constant: $\exp(\theta)$

domain: $0 \le k < \infty$

parameters 1 (1): theta

No.	name		default	
`[0]`	θ	> 0		(*shape*)

reference: [JKKa92: Ch.4; p.151]

special generators:

`DEF`	Tabulated Inversion combined with Acceptance Complement [ADb82]
`2`	Tabulated Inversion combined with Patchwork Rejection [ZHa94]

7.4 UNU.RAN Library of random matrices

7.4.1 `correlation` – Random correlation matrix

`UNUR_DISTR *unur_distr_correlation( int n )` creates a distribution object for a random correlation matrix of n rows and columns. It can be used with method MCORR (see Section 5.9.1 [Random Correlation Matrix], page 184) to generate random correlation matrices of the given size.

8 Error handling and Debugging

UNU.RAN routines report an error whenever they cannot perform the requested task. Additionally it is possible to get information about the generated distribution of generator objects for debugging purposes. However, the latter must be enabled when compiling and installing the library. (It is disabled by default.) This chapter describes all necessary details:

- Choose an output stream to for writing the requested information.
- Select a debugging level.
- Select an error handler.
- Write your own error handler.
- Get more information for a particular error code.

8.1 Output streams

UNU.RAN uses a logfile for writing all error messages, warnings, and debugging information onto an output stream. This stream can be set at runtime by the `unur_set_stream` call. If no such stream is given by the user a default stream is used by the library: all messages are written into the file '`unuran.log`' in the current working directory. The name of this logfile is defined by the macro `UNUR_LOG_FILE` in '`unuran_config.h`'. (If UNU.RAN fails to open this file for writing, '`stderr`' is used instead.)

To destinguish between messages for different objects each of these has its own identifier which is composed by the name of the distribution obejct and generator type, resp., followed by a dot and three digits. (If there are more than 999 generators then the identifiers are not unique.)

Remark: Writting debugging information must be switched on at compile time using the configure flag `--enable-logging`, see Section 8.2 [Debugging], page 215.

Function reference

`FILE* unur_set_stream` (*FILE* new_stream*)

This function sets a new file handler for the output stream, *new_stream*, for the UNU.RAN library routines. The previous handler is returned (so that you can restore it later). Note that the pointer to a user defined file handler is stored in a static variable, so there can be only one output stream handler per program. This function should be not be used in multi-threaded programs except to set up a program-wide error handler from a master thread.

The `NULL` pointer is not allowed. (If you want to disable logging of debugging information use unur_set_default_debug(UNUR_DEBUG_OFF) instead. If you want to disable error messages at all use `unur_set_error_handler_off`.)

`FILE* unur_get_stream` (*void*)

Get the file handle for the current output stream. It can be used to allow applications to write additional information into the logfile.

8.2 Debugging

The UNU.RAN library has several debugging levels which can be switched on/off by debugging flags. This debugging feature must be enabled when building the library using the `--enable-logging` configure flag.

The debugging levels range from print a short description of the created generator object to a detailed description of hat functions and tracing the sampling routines. The output is printed onto the debugging output stream (see Section 8.1 [Output streams], page 215).

The debugging flags can be set or changed by the respective calls `unur_set_debug` and `unur_chg_debug` independently for each generator.

By default flag `UNUR_DEBUG_INIT` (see below) is used. This default flags is set by the macro `UNUR_DEBUGFLAG_DEFAULT` in '`unuran_config.h`' and can be changed at runtime by a `unur_set_default_debug` call.

Off course these debugging flags depend on the chosen method. Since most of these are merely for debugging the library itself, a description of the flags are given in the corresponding source files of the method. Nevertheless, the following flags can be used with all methods.

Common debug flags:

`UNUR_DEBUG_OFF`
: switch off all debuging information

`UNUR_DEBUG_ALL`
: all avaivable information

`UNUR_DEBUG_INIT`
: parameters of generator object after initialization

`UNUR_DEBUG_SETUP`
: data created at setup

`UNUR_DEBUG_ADAPT`
: data created during adaptive steps

`UNUR_DEBUG_SAMPLE`
: trace sampling

Notice that these are flags which could be combined using the `|` operator.

Almost all routines check a given pointer before they read from or write to the given address. This does not hold for time-critical routines like all sampling routines. Thus you are responsible for checking a pointer that is returned from a `unur_init` call. However, it is possible to turn on checking for invalid `NULL` pointers even in such time-critical routines by building the library using the `--enable-check-struct` configure flag.

Another debugging tool used in the library are magic cookies that validate a given pointer. It produces an error whenever a given pointer points to an object that is invalid in the context. The usage of magic cookies is also switched on by the `--enable-check-struct` configure flag.

Function reference

`int unur_set_debug` (*UNUR_PAR** `parameters`, *unsigned* `debug`)
Set debugging flags for generator.

`int unur_chg_debug` (*UNUR_GEN** `generator`, *unsigned* `debug`)
Change debugging flags for generator.

`int unur_set_default_debug` (*unsigned* `debug`)
Change default debugging flag.

8.3 Error reporting

UNU.RAN routines report an error whenever they cannot perform the requested task. For example, applying transformed density rejection to a distribution that violates the T-concavity condition, or trying to set a parameter that is out of range, result in an error message. It might also happen that the setup fails for transformed density rejection for a T-concave distribution with some extreme density function simply because of round-off errors that makes the generation of a hat function numerically impossible. Situations like this may happen when using black box algorithms and you should check the return values of all routines.

All `..._set_...`, and `..._chg_...` calls return `UNUR_SUCCESS` if they could be executed successfully. Otherwise, some error codes are returned if it was not possible to set or change the desired parameters, e.g. because the given values are out of range, or simply because the set call does not work for the chosen method.

All routines that return a pointer to the requested object will return a `NULL` pointer in case of error. (Thus you should always check the pointer to avoid possible segmentation faults. Sampling routines usually do not check the given pointer to the generator object.)

The library distinguishes between two major classes of error:

(fatal) errors:
: The library was not able to construct the requested object.

warnings:
: Some problems encounters while constructing a generator object. The routine has tried to solve the problem but the resulting object might not be what you want. For example, chosing a special variant of a method does not work and the initialization routine might switch to another variant. Then the generator produces random variates of the requested distribution but correlation induction is not possible. However, it also might happen that changing the domain of a distribution has failed. Then the generator produced random variates with too large/too small range, i.e. their distribution is not correct.

It is obvious from the example that this distinction between errors and warning is rather crude and sometimes arbitrary.

UNU.RAN routines use the global variable *unur_errno* to report errors, completely analogously to *errno* in the ANSI C standard library. (However this approach is not thread-safe. There can be only one instance of a global variable per program. Different threads of execution may overwrite *unur_errno* simultaneously). Thus when an error occurs the caller of the routine can examine the error code in *unur_errno* to get more details about the reason why a routine failed. You get a short description of the error by a `unur_get_strerror` call. All the error code numbers have prefix `UNUR_ERR_` and expand to non-zero constant unsigned integer values. Error codes are divided into six main groups, see Section 8.4 [Error codes], page 218.

Alternatively, the variable *unur_errno* can also read by a `unur_get_errno` call and can be reset by the `unur_reset_errno` call (this is in particular required for the Windows version of the library).

Additionally, there exists a error handler (see Section 8.5 [Error handlers], page 220) that is invoked in case of an error.

In addition to reporting errors by setting error codes in *unur_errno*, the library also has an error handler function. This function is called by other library functions when they report an error, just before they return to the caller (see Section 8.5 [Error handlers], page 220). The default behavior of the error handler is to print a short message:

```
AROU.004: [error] arou.c:1500 - (generator) condition for method violated:
AROU.004: ..>  PDF not unimodal
```

The purpose of the error handler is to provide a function where a breakpoint can be set that will catch library errors when running under the debugger. It is not intended for use in production programs, which should handle any errors using the return codes.

Function reference

`extern int unur_errno` [Variable]
Global variable for reporting diagnostics of error.

`int unur_get_errno` (*void*)
Get current value of global variable *unur_errno*.

`void unur_reset_errno` (*void*)
Reset global variable *unur_errno* to `UNUR_SUCCESS` (i.e., no errors occured).

`const char* unur_get_strerror` (*const int unur_errno*)
Get a short description for error code value.

8.4 Error codes

List of error codes

- Procedure executed successfully (no error)

  `UNUR_SUCCESS (0x0u)`
  success (no error)
- Errors that occurred while handling distribution objects.

  `UNUR_ERR_DISTR_SET`
  set failed (invalid parameter).

  `UNUR_ERR_DISTR_GET`
  get failed (parameter not set).

  `UNUR_ERR_DISTR_NPARAMS`
  invalid number of parameters.

  `UNUR_ERR_DISTR_DOMAIN`
  parameter(s) out of domain.

  `UNUR_ERR_DISTR_GEN`
  invalid variant for special generator.

  `UNUR_ERR_DISTR_REQUIRED`
  incomplete distribution object, entry missing.

  `UNUR_ERR_DISTR_UNKNOWN`
  unknown distribution, cannot handle.

  `UNUR_ERR_DISTR_INVALID`
  invalid distribution object.

  `UNUR_ERR_DISTR_DATA`
  data are missing.

  `UNUR_ERR_DISTR_PROP`
  desired property does not exist
- Errors that occurred while handling parameter objects.

`UNUR_ERR_PAR_SET`
: set failed (invalid parameter)

`UNUR_ERR_PAR_VARIANT`
: invalid variant -> using default

`UNUR_ERR_PAR_INVALID`
: invalid parameter object

- Errors that occurred while handling generator objects.

`UNUR_ERR_GEN`
: error with generator object.

`UNUR_ERR_GEN_DATA`
: (possibly) invalid data.

`UNUR_ERR_GEN_CONDITION`
: condition for method violated.

`UNUR_ERR_GEN_INVALID`
: invalid generator object.

`UNUR_ERR_GEN_SAMPLING`
: sampling error.

`UNUR_ERR_NO_REINIT`
: reinit routine not implemented.

- Errors that occurred while handling URNG objects.

`UNUR_ERR_URNG`
: generic error with URNG object.

`UNUR_ERR_URNG_MISS`
: missing functionality.

- Errors that occurred while parsing strings.

`UNUR_ERR_STR`
: error in string.

`UNUR_ERR_STR_UNKNOWN`
: unknown keyword.

`UNUR_ERR_STR_SYNTAX`
: syntax error.

`UNUR_ERR_STR_INVALID`
: invalid parameter.

`UNUR_ERR_FSTR_SYNTAX`
: syntax error in function string.

`UNUR_ERR_FSTR_DERIV`
: cannot derivate function.

- Other run time errors.

`UNUR_ERR_DOMAIN`
: argument out of domain.

`UNUR_ERR_ROUNDOFF`
: (serious) round-off error.

`UNUR_ERR_MALLOC`
: virtual memory exhausted.

`UNUR_ERR_NULL`
: invalid `NULL` pointer.

`UNUR_ERR_COOKIE`
: invalid cookie.

`UNUR_ERR_GENERIC`
: generic error.

`UNUR_ERR_SILENT`
: silent error (no error message).

`UNUR_ERR_INF`
: infinity occured.

`UNUR_ERR_NAN`
: NaN occured.

`UNUR_ERR_COMPILE`
: Requested routine requires different compilation switches. Recompilation of library necessary.

`UNUR_ERR_SHOULD_NOT_HAPPEN`
: Internal error, that should not happen. Please report this bug!

8.5 Error handlers

The default behavior of the UNU.RAN error handler is to print a short message onto the output stream, usually a logfile (see Section 8.1 [Output streams], page 215), e.g.,

```
AROU.004: [error] arou.c:1500 - (generator) condition for method violated:
AROU.004: ..>  PDF not unimodal
```

This error handler can be switched off using the `unur_set_error_handler_off` call, or replace it by a new one. Thus it allows to set a breakpoint that will catch library errors when running under the debugger. It also can be used to redirect error messages when UNU.RAN is included in general purpose libraries or in interactive programming environments.

`UNUR_ERROR_HANDLER` [Data Type]

This is the type of UNU.RAN error handler functions. An error handler will be passed six arguments which specify the identifier of the object where the error occured (a string), the name of the source file in which it occurred (also a string), the line number in that file (an integer), the type of error (a string: `"error"` or `"warning"`), the error number (an integert), and the reason for the error (a string). The source file and line number are set at compile time using the `__FILE__` and `__LINE__` directives in the preprocessor. The error number can be translated into a short description using a `unur_get_strerror` call. An error handler function returns type `void`.

Error handler functions should be defined like this,

```
void my_handler(
           const char *objid,
           const char *file,
           int line,
           const char *errortype,
           int unur_errno,
           const char *reason )
```

To request the use of your own error handler you need the call `unur_set_error_handler`.

Function reference

`UNUR_ERROR_HANDLER* unur_set_error_handler (`*UNUR_ERROR_HANDLER* `new_handler`*`)`*

This function sets a new error handler, *new_handler*, for the UNU.RAN library routines. The previous handler is returned (so that you can restore it later). Note that the pointer to a user defined error handler function is stored in a static variable, so there can be only one error handler per program. This function should be not be used in multi-threaded programs except to set up a program-wide error handler from a master thread.

To use the default behavior set the error handler to `NULL`.

`UNUR_ERROR_HANDLER* unur_set_error_handler_off (`*void*`)`

This function turns off the error handler by defining an error handler which does nothing (except of setting *unur_errno*. The previous handler is returned (so that you can restore it later).

9 Testing

The following routines can be used to test the performance of the implemented generators and can be used to verify the implementions. They are declared in '`unuran_tests.h`' which has to be included.

Function reference

`void unur_run_tests (`*UNUR_PAR** `parameters`, *unsigned* `tests`, *FILE** `out`)

Run a battery of tests. The following tests are available (use `|` to combine these tests):

`UNUR_TEST_ALL`
: run all possible tests.

`UNUR_TEST_TIME`
: estimate generation times.

`UNUR_TEST_N_URNG`
: count number of uniform random numbers

`UNUR_TEST_N_PDF`
: count number of PDF calls

`UNUR_TEST_CHI2`
: run chi^2 test for goodness of fit

`UNUR_TEST_SAMPLE`
: print a small sample.

All these tests can be started individually (see below).

`void unur_test_printsample (`*UNUR_GEN** `generator`, *int* `n_rows`, *int* `n_cols`, *FILE** `out`)

Print a small sample with *n_rows* rows and *n_cols* columns. *out* is the output stream to which all results are written.

`UNUR_GEN* unur_test_timing (`*UNUR_PAR** `parameters`, *int* `log_samplesize`, *double** `time_setup`, *double** `time_sample`, *int* `verbosity`, *FILE** `out`)

Timing. *parameters* is an parameter object for which setup time and marginal generation times have to be measured. The results are written into *time_setup* and *time_sample*, respectively. *log_samplesize* is the common logarithm of the sample size that is used for timing.

If *verbosity* is `TRUE` then a small table is printed to output stream *out* with setup time, marginal generation time and average generation times for generating 10, 100, ... random variates. All times are given in micro seconds and relative to the generation times for the underlying uniform random number (using the UNIF interface) and an exponential distributed random variate using the inversion method.

The created generator object is returned. If a generator object could not be created successfully, then `NULL` is returned.

If *verbosity* is `TRUE` the result is written to the output stream *out*.

Notice: All timing results are subject to heavy changes. Reruning timings usually results in different results. Minor changes in the source code can cause changes in such timings up to 25 percent.

`double unur_test_timing_uniform` (*const* `UNUR_PAR* parameters`, *int* `log_samplesize`)
`double unur_test_timing_exponential` (*const* `UNUR_PAR* parameters`, *int* `log_samplesize`)

Marginal generation times for the underlying uniform random number (using the UNIF interface) and an exponential distributed random variate using the inversion method. These times are used in `unur_test_timing` to compute the relative timings results.

`double unur_test_timing_total` (*const* `UNUR_PAR* parameters`, *int* `samplesize`, *double* `avg_duration`)

Timing. *parameters* is an parameter object for which average times a sample of size *samplesize* (including setup) are estimated. Thus sampling is repeated and the median of these timings is returned (in micro seconds). The number of iterations is computed automatically such that the total amount of time necessary for the test ist approximately *avg_duration* (given in seconds). However, for very slow generator with expensive setup time the time necessary for this test may be (much) larger.

If an error occurs then `-1` is returned.

Notice: All timing results are subject to heavy changes. Reruning timings usually results in different results. Minor changes in the source code can cause changes in such timings up to 25 percent.

`int unur_test_count_urn` (*UNUR_GEN** `generator`, *int* `samplesize`, *int* `verbosity`, *FILE** `out`)

Count used uniform random numbers. It returns the total number of uniform random numbers required for a sample of non-uniform random variates of size *samplesize*. In case of an error `-1` is returned.

If *verbosity* is `TRUE` the result is written to the output stream *out*.

Notice: This test uses global variables to store counters. Thus it is not thread save.

`int unur_test_count_pdf` (*UNUR_GEN** `generator`, *int* `samplesize`, *int* `verbosity`, *FILE** `out`)

Count evaluations of PDF and similar functions. It returns the total number of evaluations of all such functions required for a sample of non-uniform random variates of size *samplesize*. If *verbosity* is `TRUE` then a more detailed report is printed to the output stream *out*. In case of an error `-1` is returned. This test is run on a copy of the given generator object.

Notice: The printed numbers of evaluation should be interpreted with care. For example, methods either use the PDF or the logPDF; if only the logPDF is given, but a method needs the PDF then both the logPDF and the PDF (a wrapper around the logPDF) are called and thus one call to the PDF is counted twice.

Notice: This test uses global variables to store function pointers and counters. Thus it is not thread save.

`int unur_test_par_count_pdf` (*UNUR_PAR** `parameters`, *int* `samplesize`, *int* `verbosity`, *FILE** `out`)

Same as `unur_test_count_pdf` except that it is run on a parameter object. Thus it also prints the number of function evaluations for the setup. The temporary created generator object is destroyed before the results are returned.

`double unur_test_chi2 (`*UNUR_GEN* generator*, *int intervals*, *int samplesize*, *int classmin*, *int verbosity*, *FILE* out*`)`

Run a Chi^2 test with the *generator*. The resulting p-value is returned.

It works with discrete und continuous univariate distributions. For the latter the CDF of the distribution is required.

intervals is the number of intervals that is used for continuous univariate distributions. *samplesize* is the size of the sample that is used for testing. If it is set to `0` then a sample of size *intervals*^2 is used (bounded to some upper bound).

classmin is the minimum number of expected entries per class. If a class has to few entries then some classes are joined.

verbosity controls the output of the routine. If it is set to `1` then the result is written to the output stream *out*. If it is set to `2` additionally the list of expected and observed data is printed. If it is set to `3` then all generated numbers are printed. There is no output when it is set to `0`.

Notice: For multivariate distributions the generated points are transformed by the inverse of the Cholesky factor of the covariance matrix and the mean vectors (if given for the underlying distribution). The marginal distributions of the transformed vectors are then tested against the marginal distribution given by a `unur_distr_cvec_set_marginals` or `unur_distr_cvec_set_marginal_array` call. (Notice that these marginal distributions are never set by default for any of the distributions provided by UNU.RAN.) Then the Bonferroni corrected p-value of all these tests is returned. However, the test may not be performed correctly if the domain of the underlying distribution is truncated by a `unur_distr_cvec_set_domain_rect` call and the components of the distribution are correlated (i.e. `unur_distr_cvec_set_covar` is called with the non-NULL argument). Then it almost surely will fail.

`int unur_test_moments (`*UNUR_GEN* generator*, *double* moments*, *int n_moments*, *int samplesize*, *int verbosity*, *FILE* out*`)`

Computes the first *n_moments* central moments for a sample of size *samplesize*. The result is stored into the array *moments*. *n_moments* must be an integer between `1` and `4`. For multivariate distributions the moments are stored consecutively for each dimension and the provided *moments*-array must have a length of at least (*n_moments*+1) * *dim*, where *dim* is the dimension of the multivariate distribution. The m'th moment for the d'th dimension ($0 <= d < dim$) is thus stored in the array element *moments*[d**n_moments*+m]

If *verbosity* is `TRUE` the result is written to the output stream *out*.

`double unur_test_correlation (`*UNUR_GEN* generator1*, *UNUR_GEN* generator2*, *int samplesize*, *int verbosity*, *FILE* out*`)`

Compute the correlation coefficient between streams from *generator1* and *generator2* for two samples of size *samplesize*. The resultung correlation is returned.

If *verbosity* is `TRUE` the result is written to the output stream *out*.

`int unur_test_quartiles (`*UNUR_GEN* generator*, *double* q0*, *double* q1*, *double* q2*, *double* q3*, *double* q4*, *int samplesize*, *int verbosity*, *FILE* out*`)`

Estimate quartiles of sample of size *samplesize*. The resulting quantiles are stored in the variables q:

q0	minimum
q1	25%
q2	median (50%)

q3	75%
q4	maximum

If *verbosity* is `TRUE` the result is written to the output stream *out*.

10 Miscelleanous

10.1 Mathematics

The following macros have been defined

`UNUR_INFINITY`
: indicates infinity for floating point numbers (of type `double`). Internally `HUGE_VAL` is used.

`INT_MAX`
`INT_MIN`
: indicate infinity and minus infinity, resp., for integers (defined by ISO C standard).

`TRUE`
`FALSE`
: bolean expression for return values of `set` functions.

Appendix A A Short Introduction to Random Variate Generation

Random variate generation is the small field of research that deals with algorithms to generate random variates from various distributions. It is common to assume that a uniform random number generator is available. This is a program that produces a sequence of independent and identically distributed continuous $U(0,1)$ random variates (i.e. uniform random variates on the interval $(0,1)$). Of course real world computers can never generate ideal random numbers and they cannot produce numbers of arbitrary precision but state-of-the-art uniform random number generators come close to this aim. Thus random variate generation deals with the problem of transforming such a sequence of $U(0,1)$ random numbers into non-uniform random variates.

Here we shortly explain the basic ideas of the *inversion*, *rejection*, and the *ratio of uniforms* method. How these ideas can be used to design a particular automatic random variate generation algorithms that can be applied to large classes of distributions is shortly explained in the description of the different methods included in this manual.

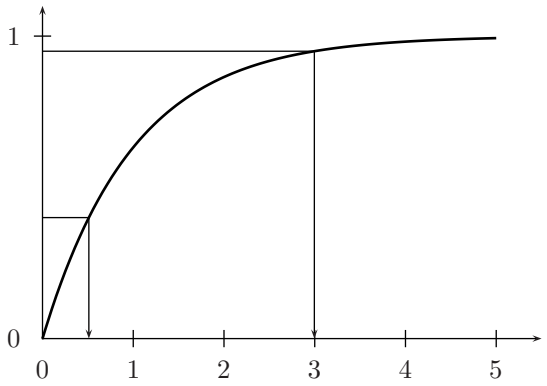
For a deeper treatment of the ideas presented here, for other basic methods and for automatic generators we refer the interested reader to our book [HLD04].

A.1 The Inversion Method

When the inverse F^{-1} of the cumulative distribution function is known, then random variate generation is easy. We just generate a uniformly $U(0,1)$ distributed random number U and return

$$X = F^{-1}(U).$$

The following figure shows how the inversion method works for the exponential distribution.



This algorithm is so simple that inversion is certainly the method of choice if the inverse CDF is available in closed form. This is the case e.g. for the exponential and the Cauchy distribution.

The inversion method also has other special advantages that make it even more attractive for simulation purposes. It preserves the structural properties of the underlying uniform pseudo-random number generator. Consequently it can be used, e.g., for variance reduction techniques, it is easy to sample from truncated distributions, from marginal distributions, and from order statistics. Moreover, the quality of the generated random variables depends only on the underlying uniform (pseudo-) random number generator. Another important advantage of the inversion method is that we can easily characterize its performance. To generate one random variate we always need exactly one uniform variate and one evaluation of the inverse CDF. So its speed

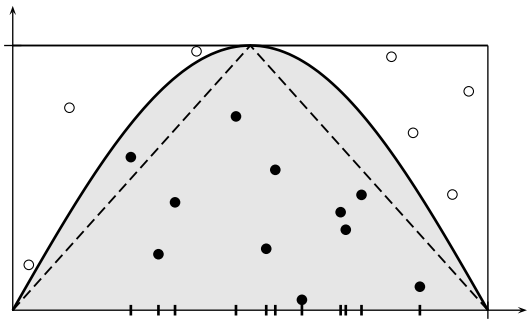
mainly depends on the costs for evaluating the inverse CDF. Hence inversion is often considered as the method of choice in the simulation literature.

Unfortunately computing the inverse CDF is, for many important standard distributions (e.g. for normal, student, gamma, and beta-distributions), comparatively difficult and slow. Often no such routines are available in standard programming libraries. Then numerical methods for inverting the CDF are necessary, e.g. Newton's method. Such procedures, however, have the disadvantage that they may be slow or not exact, i.e. they compute approximate values. The methods NINV (see Section 5.3.10 [NINV], page 117) and HINV (see Section 5.3.5 [HINV], page 107) of UNU.RAN are numerical inversion methods. Both require the CDF of the desired distribution. As the CDF is quite complicated and slow for many distributions this implies either that the generation is very slow (NINV) or a very slow setup step and large tables are necessary (HINV). Sometimes the CDF of a distribution is not available and alternative methods like the rejection method (see Section A.2 [Rejection], page 230) must be used.

A.2 The Rejection Method

The rejection method, often called *acceptance-rejection method*, has been suggested by John von Neumann in 1951. Since then it has proven to be the most flexible and most efficient method to generate variates from continuous distributions.

We explain the rejection principle first for the density $f(x) = sin(x)/2$ on the interval $(0, \pi)$. To generate random variates from this distribution we also can sample random points that are uniformly distributed in the region between the graph of $f(x)$ and the x-axis, i.e., the shaded region in the below figure.



In general this is not a trivial task but in this example we can easily use the rejection trick: Sample a random point (X, Y) uniformly in the bounding rectangle $(0, \pi) \times (0, 0.5)$. This is easy since each coordinate can be sampled independently from the respective uniform distributions $U(0, \pi)$ and $U(0, 0.5)$. Whenever the point falls into the shaded region below the graph (indicated by dots in the figure), i.e., when $Y < sin(X)/2$, we accept it and return X as a random variate from the distribution with density $f(x)$. Otherwise we have to reject the point (indicated by small circles in the figure), and try again.

It is quite clear that this idea works for every distribution with a bounded density on a bounded domain. Moreover, we can use this procedure with any multiple of the density, i.e., with any positive bounded function with bounded integral and it is not necessary to know the integral of this function. So we use the term density in the sequel for any positive function with bounded integral.

From the figure we can conclude that the performance of a rejection algorithm depends heavily on the area of the enveloping rectangle. Moreover, the method does not work if the target distribution has infinite tails (or is unbounded). Hence non-rectangular shaped regions

for the envelopes are important and we have to solve the problem of sampling points uniformly from such domains. Looking again at the example above we notice that the x-coordinate of the random point (X, Y) was sampled by inversion from the uniform distribution on the domain of the given density. This motivates us to replace the density of the uniform distribution by the (multiple of a) density $h(x)$ of some other appropriate distribution. We only have to take care that it is chosen such that it is always an upper bound, i.e., $h(x) \geq f(x)$ for all x in the domain of the distribution. To generate the pair (X, Y) we generate X from the distribution with density proportional to $h(x)$ and Y uniformly between 0 and $h(X)$. The first step (generate X) is usually done by inversion (see Section A.1 [Inversion], page 229).

Thus the general rejection algorithm for a hat $h(x)$ with inverse CDF H^{-1} consists of the following steps:

1. Generate a $U(0, 1)$ random number U.
2. Set X to $H^{-1}(U)$.
3. Generate a $U(0, 1)$ random number V.
4. Set Y to $Vh(X)$.
5. If $Y \leq f(X)$ accept X as the random variate.
6. Else try again.

If the evaluation of the density $f(x)$ is expensive (i.e., time consuming) it is possible to use a simple lower bound of the density as so called *squeeze function* $s(x)$ (the triangular shaped function in the above figure is an example for such a squeeze). We can then accept X when $Y \leq s(X)$ and can thus often save the evaluation of the density.

We have seen so far that the rejection principle leads to short and simple generation algorithms. The main practical problem to apply the rejection algorithm is the search for a good fitting hat function and for squeezes. We do not discuss these topics here as they are the heart of the different automatic algorithms implemented in UNU.RAN. Information about the construction of hat and squeeze can therefore be found in the descriptions of the methods.

The performance characteristics of rejection algorithms mainly depend on the fit of the hat and the squeeze. It is not difficult to prove that:

- The expected number of trials to generate one variate is the ratio between the area below the hat and the area below the density.
- The expected number of evaluations of the density necessary to generate one variate is equal to the ratio between the area below the hat and the area below the density, when no squeeze is used. Otherwise, when a squeeze is given it is equal to the ratio between the area between hat and squeeze and the area below the hat.
- The `sqhratio` (i.e., the ratio between the area below the squeeze and the area below the hat) used in some of the UNU.RAN methods is easy to compute. It is useful as its reciprocal is an upper bound for the expected number of trials of the rejection algoritm. The expected number of evaluations of the density is bounded by $(1/sqhratio) - 1$.

A.3 The Composition Method

The composition method is an important principle to facilitate and speed up random variate generation. The basic idea is simple. To generate random variates with a given density we first split the domain of the density into subintervals. Then we select one of these randomly with probabilities given by the area below the density in the respective subintervals. Finally we generate a random variate from the density of the selected part by inversion and return it as random variate of the full distribution.

Composition can be combined with rejection. Thus it is possible to decompose the domain of the distribution into subintervals and to construct hat and squeeze functions seperatly in

every subinterval. The area below the hat must be determined in every subinterval. Then the Composition rejection algorithm contains the following steps:

1. Generate the index J of the subinterval as the realisation of a discrete random variate with probabilities proportional to the area below the hat.
2. Generate a random variate X proportional to the hat in interval J.
3. Generate the $U(0, f(X))$ random number Y.
4. If $Y \leq f(X)$ accept X as random variate.
5. Else start again with generating the index J.

The first step can be done in constant time (i.e., independent of the number of chosen subintervals) by means of the indexed search method (see Section A.6 [IndexedSearch], page 234).

It is possible to reduce the number of uniform random numbers required in the above algorithm by recycling the random numbers used in Step 1 and additionally by applying the principle of *immediate acceptance*. For details see [HLD04: Sect. 3.1] .

A.4 The Ratio-of-Uniforms Method

The construction of an appropriate hat function for the given density is the crucial step for constructing rejection algorithms. Equivalently we can try to find an appropriate envelope for the region between the graph of the density and the x-axis, such that we can easily sample uniformly distributed random points. This task could become easier if we can find transformations that map the region between the density and the axis into a region of more suitable shape (for example into a bounded region).

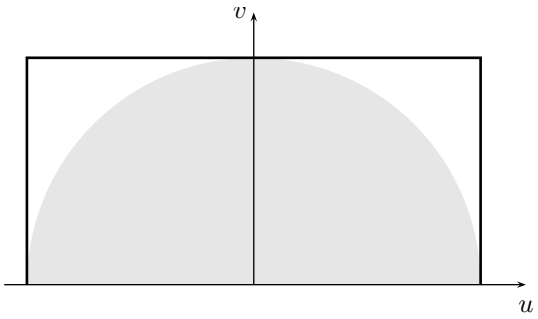
As a first example we consider the following simple algorithm for the Cauchy distribution.

1. Generate a $U(-1, 1)$ random number U and a $U(0, 1)$ random number V.
2. If $U^2 + V^2 \leq 1$ accept $X = U/V$ as a Cauchy random variate.
3. Else try again.

It is possible to prove that the above algorithm indeed generates Cauchy random variates. The fundamental principle behind this algorithm is the fact that the region below the density is mapped by the transformation

$$(X, Y) \mapsto (U, V) = (2\,X\sqrt{Y}, 2\,\sqrt{Y})$$

into a half-disc in such a way that the ratio between the area of the image to the area of the preimage is constant. This is due to the fact that that the Jacobian of this transformation is constant.



The above example is a special case of a more general principle, called the *Ratio-of-uniforms (RoU) method*. It is based on the fact that for a random variable X with density $f(x)$ and some

constant μ we can generate X from the desired density by calculating $X = U/V + \mu$ for a pair (U, V) uniformly distributed in the set

$$A_f = \{(u, v): 0 < v \le \sqrt{f(u/v + \mu)}\}.$$

For most distributions it is best to set the constant μ equal to the mode of the distribution. For sampling random points uniformly distributed in A_f rejection from a convenient enveloping region is used, usually the minimal bounding rectangle, i.e., the smallest possible rectangle that contains A_f (see the above figure). It is given by $(u^-, u^+) \times (0, v^+)$ where

$$v^+ = \sup_{b_l < x < b_r} \sqrt{f(x)},$$
$$u^- = \inf_{b_l < x < b_r} (x - \mu)\sqrt{f(x)},$$
$$u^+ = \sup_{b_l < x < b_r} (x - \mu)\sqrt{f(x)}.$$

Then the ratio-of-uniforms method consists of the following simple steps:

1. Generate a $U(u^-, u^+)$ random number U and a $U(0, v^+)$ random number V.
2. Set X to $U/V + \mu$.
3. If $V^2 \le f(X)$ accept X as the random variate.
4. Else try again.

To apply the ratio-of-uniforms algorithm to a certain density we have to solve the simple optimization problems in the definitions above to obtain the design constants u^-, u^+, and v^+. This simple algorithm works for all distributions with bounded densities that have subquadratic tails (i.e., tails like $1/x^2$ or lower). For most standard distributions it has quite good rejection constants. (E.g. 1.3688 for the normal and 1.4715 for the exponential distribution.)

Nevertheless, we use more sophisticated method that construct better fitting envelopes, like method AROU (see Section 5.3.1 [AROU], page 95), or even avoid the computation of these design constants and thus have almost no setup, like method SROU (see Section 5.3.12 [SROU], page 122).

The Generalized Ratio-of-Uniforms Method

The Ratio-of-Uniforms method can be generalized: If a point (U, V) is uniformly distributed in the set

$$A_f = \{(u, v): 0 < v \le (f(u/v^r + \mu))^{1/(r+1)}\}$$

then $X = U/V^r + \mu$ has the denity $f(x)$. The minimal bounding rectangle of this region is given by $(u^-, u^+) \times (0, v^+)$ where

$$v^+ = \sup_{b_l < x < b_r} (f(x))^{1/(r+1)},$$
$$u^- = \inf_{b_l < x < b_r} (x - \mu)(f(x))^{r/(r+1)},$$
$$u^+ = \sup_{b_l < x < b_r} (x - \mu)(f(x))^{r/(r+1)}.$$

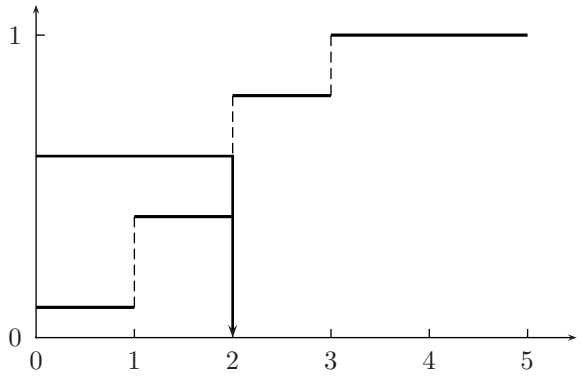
The above algorithm has then to be adjusted accordingly. Notice that the original Ratio-of-Uniforms method is the special case with $r = 1$.

A.5 Inversion for Discrete Distributions

We have already presented the idea of the inversion method to generate from continuous random variables (see Section A.1 [Inversion], page 229). For a discrete random variable X we can write it mathematically in the same way:

$$X = F^{-1}(U),$$

where F is the CDF of the desired distribution and U is a uniform $U(0, 1)$ random number. The difference compared to the continuous case is that F is now a step-function. The following figure illustrates the idea of discrete inversion for a simple distribution.



To realize this idea on a computer we have to use a search algorithm. For the simplest version called *Sequential Search* the CDF is computed on-the-fly as sum of the probabilities $p(k)$, since this is usually much cheaper than computing the CDF directly. It is obvious that the basic form of the search algorithm only works for discrete random variables with probability mass functions $p(k)$ for nonnegative k. The sequential search algorithm consists of the following basic steps:

1. Generate a $U(0,1)$ random number U.
2. Set X to 0 and P to $p(0)$.
3. Do while $U > P$
4. Set X to $X+1$ and P to $P+p(X)$.
5. Return X.

With the exception of some very simple discrete distributions, sequential search algorithms become very slow as the while-loop has to be repeated very often. The expected number of iterations, i.e., the number of comparisons in the while condition, is equal to the expectation of the distribution plus 1. It can therefore become arbitrary large or even infinity if the tail of the distribution is very heavy. Another serious problem can be critical round-off errors due to summing up many probabilities $p(k)$. To speed up the search procedure it is best to use indexed search.

A.6 Indexed Search (Guide Table Method)

The idea to speed up the sequential search algorithm is easy to understand. Instead of starting always at 0 we store a table of size C with starting points for our search. For this table we compute $F^{-1}(U)$ for C equidistributed values of U, i.e., for $u_i = i/C$, $i = 0, ..., C-1$. Such a table is called *guide table* or *hash table*. Then it is easy to prove that for every U in $(0,1)$ the guide table entry for $k = floor(UC)$ is bounded by $F^{-1}(U)$. This shows that we can really start our sequential search procedure from the table entry for k and the index k of the correct table entry can be found rapidly by means of the truncation operation.

The two main differences between *indexed search* and *sequential search* are that we start searching at the number determined by the guide table, and that we have to compute and store the cumulative probabilities in the setup as we have to know the cumulative probability for the starting point of the search algorithm. The rounding problems that can occur in the sequential search algorithm can occur here as well. Compared to sequential search we have now the obvious drawback of a slow setup. The computation of the cumulative probabilities grows linear with the size of the domain of the distribution L. What we gain is really high speed as the marginal execution time of the sampling algorithm becomes very small. The expected number of comparisons is bounded by $1 + L/C$. This shows that there is a trade-off between speed and the size of the guide table. Cache-effects in modern computers will however slow down the speed-up for really large table sizes. Thus we recommend to use a guide table that is about two times larger than the probability vector to obtain optimal speed.

Appendix B Glossary

CDF cumulative distribution function

HR hazard rate (or failure rate)

inverse local concavity
local concavity of inverse PDF $f^{-1}(y)$ expressed in term of $x = f^{-1}(y)$. Is is given by $ilc_f(x) = 1 + x\, f''(x)/f'(x)$

local concavity
maximum value of c such that PDF *f(x)* is T_c. Is is given by $lc_f(x) = 1 - f''(x)\, f(x)/f'(x)^2$

PDF probability density function

dPDF derivative (gradient) of probability density function

PMF probability mass function

PV (finite) probability vector

URNG uniform random number generator

$U(a, b)$ continuous uniform distribution on the interval (a, b)

T-concave a function *f(x)* is called T-convace if the transformed function *T(f(x))* is concave. We only deal with transformations T_c, where

`c = 0`	$T(x) = \log(x)$
`c = -0.5`	$T(x) = -1/\sqrt{x}$
`c != 0`	$T(x) = sign(x) \cdot x^c$

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