

UNU.RAN User Manual

Generating non-uniform random numbers
Version 0.4.2, 14 January 2003

Josef Leydold
Wolfgang Hörmann
Erich Janka
Günter Tirlir

Copyright © 2000–2003 Institut fuer Statistik, WU Wien.

Permission is granted to make and distribute verbatim copies of this manual provided the copyright notice and this permission notice are preserved on all copies.

Table of Contents

UNURAN – Universal Non-Uniform RANdom	
number generators	1
1 Introduction	3
1.1 Usage of this document	3
1.2 Installation	3
1.3 Using the library	4
1.4 Concepts of UNURAN	5
1.5 Contact the authors	8
2 Examples	9
2.1 As short as possible	9
2.2 As short as possible (String API)	10
2.3 Select a method	11
2.4 Select a method (String API)	12
2.5 Arbitrary distributions	14
2.6 Arbitrary distributions (String API)	16
2.7 Change parameters of the method	17
2.8 Change parameters of the method (String API)	19
2.9 Change uniform random generator	20
2.10 Change uniform random generator (String API)	22
2.11 Sample pairs of antithetic random variates	24
2.12 Sample pairs of antithetic random variates (String API)	27
2.13 More examples	28
3 String Interface	29
3.1 Syntax of String Interface	29
3.2 Distribution String	30
3.2.1 Keys for Distribution String	31
3.3 Function String	33
3.4 Method String	35
3.4.1 Keys for Method String	35
3.5 Uniform RNG String	39
4 Handling distribution objects	41
4.1 Functions for all kinds of distribution objects	41
4.2 Continuous univariate distributions	42
4.3 Continuous univariate order statistics	46
4.4 Continuous empirical univariate distributions	48
4.5 Continuous multivariate distributions	49
4.6 Continuous empirical multivariate distributions	52
4.7 Discrete univariate distributions	52

5	Methods for generating non-uniform random variates	57
5.1	Routines for all generator objects	57
5.2	AUTO – Select method automatically	57
5.3	Methods for continuous univariate distributions	58
5.3.1	AROU – Automatic Ratio-Of-Uniforms method	61
5.3.2	CSTD – Continuous STandarD distributions	63
5.3.3	HINV – Hermite interpolation based INVersion of CDF	65
5.3.4	HRB – Hazard Rate Bounded	67
5.3.5	HRD – Hazard Rate Decreasing	68
5.3.6	HRI – Hazard Rate Increasing	68
5.3.7	NINV – Numerical INVersion	69
5.3.8	SROU – Simple Ratio-Of-Uniforms method	71
5.3.9	SSR – Simple Setup Rejection	74
5.3.10	TABL – a TABLE method with piecewise constant hats	76
5.3.11	TDR – Transformed Density Rejection	79
5.3.12	UTDR – Universal Transformed Density Rejection	83
5.4	Methods for continuous empirical univariate distributions	85
5.4.1	EMPK – EMPirical distribution with Kernel smoothing	87
5.4.2	EMPL – EMPirical distribution with Linear interpolation	90
5.5	Methods for continuous multivariate distributions	90
5.5.1	VMT – Vector Matrix Transformation	90
5.6	Methods for continuous empirical multivariate distributions	91
5.6.1	VEMPK – (Vector) EMPirical distribution with Kernel smoothing	92
5.7	Methods for discrete univariate distributions	93
5.7.1	DARI – discrete automatic rejection inversion	96
5.7.2	DAU – (Discrete) Alias-Urn method	98
5.7.3	DGT – (Discrete) Guide Table method (indexed search)	98
5.7.4	DSROU – Discrete Simple Ratio-Of-Uniforms method	99
5.7.5	DSTD – Discrete STandarD distributions	101
5.8	Methods for uniform univariate distributions	102
5.8.1	UNIF – wrapper for UNIFORM random number generator	102
6	Using uniform random number generators	103
7	UNURAN Library of standard distributions	107
7.1	UNURAN Library of continuous univariate distributions	108
7.1.1	beta – Beta distribution	108
7.1.2	cauchy – Cauchy distribution	108
7.1.3	chi – Chi distribution	108
7.1.4	chisquare – Chisquare distribution	109
7.1.5	exponential – Exponential distribution	109
7.1.6	extremeI – Extreme value type I (Gumbel-type) distribution	109
7.1.7	extremeII – Extreme value type II (Frechet-type) distribution	109
7.1.8	gamma – Gamma distribution	110
7.1.9	laplace – Laplace distribution	110
7.1.10	logistic – Logistic distribution	110
7.1.11	lomax – Lomax distribution (Pareto distribution of second kind)	110
7.1.12	normal – Normal distribution	111

7.1.13	<code>pareto</code> – Pareto distribution (of first kind)	111
7.1.14	<code>powerexponential</code> – Powerexponential (Subbotin) distribution	111
7.1.15	<code>rayleigh</code> – Rayleigh distribution	112
7.1.16	<code>student</code> – Student’s t distribution	112
7.1.17	<code>triangular</code> – Triangular distribution	112
7.1.18	<code>uniform</code> – Uniform distribution	112
7.1.19	<code>weibull</code> – Weibull distribution	113
7.2	UNURAN Library of continuous multivariate distributions . . .	113
7.3	UNURAN Library of discrete univariate distributions	113
7.3.1	<code>binomial</code> – Binomial distribution	113
7.3.2	<code>geometric</code> – Geometric distribution	113
7.3.3	<code>hypergeometric</code> – Hypergeometric distribution	114
7.3.4	<code>logarithmic</code> – Logarithmic distribution	114
7.3.5	<code>negativebinomial</code> – Negative Binomial distribution	114
7.3.6	<code>poisson</code> – Poisson distribution	115
8	Error handling	117
8.1	Error reporting	117
8.2	Output streams	119
9	Debugging	121
10	Testing	123
11	Miscellaneous	127
11.1	Mathematics	127
Appendix A	Glossary	129
Appendix B	Bibliography	131
Appendix C	Function Index	133

UNURAN – Universal Non-Uniform RANdom number generators

UNURAN (Universal Non-Uniform RANdom Number generator) is a collection of algorithms for generating non-uniform pseudorandom variates as a library of C functions designed and implemented by the ARVAG (Automatic Random VARIate Generation) project group in Vienna, and released under the GNU Public License (GPL). It is especially designed for such situations where

- a non-standard distribution or a truncated distribution is needed.
- experiments with different types of distributions are made.
- random variates for variance reduction techniques are used.
- fast generators of predictable quality are necessary.

Of course it is also well suited for standard distributions. However due to its more sophisticated programming interface it might not be as easy to use if you only look for a generator for the standard normal distribution. (Although UNURAN provides generators that are superior in many aspects to those found in quite a number of other libraries.)

UNURAN implements several methods for generating random numbers. The choice depends primary on the information about the distribution can be provided and – if the user is familiar with the different methods – on the preferences of the user.

The design goals of UNURAN are to provide *reliable*, *portable* and *robust* (as far as this is possible) functions with a consistent and easy to use interface. It is suitable for all situation where experiments with different distributions including non-standard distributions. For example it is no problem to replace the normal distribution by an empirical distribution in a model.

Since originally designed as a library for so called black-box or universal algorithms its interface is different from other libraries. (Nevertheless it also contains special generators for standard distributions.) It does not provide subroutines for random variate generation for particular distributions. Instead it uses an object-oriented interface. Distributions and generators are treated as independent objects. This approach allows one not only to have different methods for generating non-uniform random variates. It is also possible to choose the method which is optimal for a given situation (e.g. speed, quality of random numbers, using for variance reduction techniques, etc.). It also allows to sample from non-standard distribution or even from distributions that arise in a model and can only be computed in a complicated subroutine.

Sampling from a particular distribution requires the following steps:

1. Create a distribution object. (Objects for standard distributions are available in the library)
2. Choose a method.
3. Initialize the generator, i.e., create the generator object. If the chosen method is not suitable for the given distribution (or if the distribution object contains too little information about the distribution) the initialization routine fails and produces an error message. Thus the generator object does (probably) not produce false results (random variates of a different distribution).
4. Use this generator object to sample from the distribution.

There are four types of objects that can be manipulated independently:

- **Distribution objects:** hold all information about the random variates that should be generated. The following types of distributions are available:
 - Continuous and Discrete distributions
 - Empirical distributions
 - Multivariate distributions

Of course a library of standard distributions is included (and these can be further modified to get, e.g., truncated distributions). Moreover the library provides subroutines to build almost arbitrary distributions.

- **Generator objects:** hold the generators for the given distributions. It is possible to build independent generator objects for the same distribution object which might use the same or different methods for generation. (If the chosen method is not suitable for the given method, a NULL pointer is returned in the initialization step).

- **Parameter objects:** Each transformation method requires several parameters to adjust the generator to a given distribution. The parameter object holds all this information. When created it contains all necessary default settings. It is only used to create a generator object and destroyed immediately. Although there is no need to change these parameters or even know about their existence for “usual distributions”, they allow a fine tuning of the generator to work with distributions with some awkward properties. The library provides all necessary functions to change these default parameters.
- **Uniform Random Number Generators:** All generator objects need one (or more) streams of uniform random numbers that are transformed into random variates of the given distribution. These are given as pointers to appropriate functions or structures (objects). Two generator objects may have their own uniform random number generators or share a common one. Any functions that produce uniform (pseudo-) random numbers can be used. We suggest Otmar Lendl’s PRNG library.

1 Introduction

1.1 Usage of this document

We designed this document in a way such that one can use UNURAN with reading as little as necessary. Read [Section 1.2 \[Installation\], page 3](#) for the instructions to install the library. [Section 1.4 \[Concepts of UNURAN\], page 5](#), describes the basics of UNURAN. It also has a short guideline for choosing an appropriate method. In [Chapter 2 \[Examples\], page 9](#) examples are given that can be copied and modified. They also can be found in the directory ‘examples’ in the source tree.

Further information are given in consecutive chapters. [Chapter 4 \[Handling distribution objects\], page 41](#), describes how to create and manipulate distribution objects. [Chapter 7 \[standard distributions\], page 107](#), describes predefined distribution objects that are ready to use. [Chapter 5 \[Methods\], page 57](#) describes the various methods in detail. For each of possible distribution classes (continuous, discrete, empirical, multivariate) there exists a short overview section that can be used to choose an appropriate method followed by sections that describe each of the particular methods in detail. These are merely for users with some knowledge about the methods who want to change method-specific parameters and can be ignored by others.

Abbreviations and explanation of some basic terms can be found in [Appendix A \[Glossary\], page 129](#).

1.2 Installation

UNURAN was developed on an Intel architecture under Linux with the GNU C compiler.

Uniform random number generator

It can be used with any uniform random number generator but (at the moment) some features work best with Otmar Lendl’s *prng* library (see <http://statistik.wu-wien.ac.at/prng/> for description and downloading). For more details on using uniform random number in UNURAN see [Chapter 6 \[Using uniform random number generators\], page 103](#).

UNURAN

1. First unzip and untar the package and change to the directory:

```
tar zxvf unuran-0.4.2.tar.gz
cd unuran-0.4.2
```

2. Edit the file ‘src/unuran_config.h’. Set the appropriate source of uniform random numbers: #define UNUR_URNG_TYPE (see [Chapter 6 \[URNG\], page 103](#) for details).

Important: If neither UNUR_URNG_FVOID nor UNUR_URNG_GENERIC is used, check if the corresponding library is installed.

3. Run a configuration script:

```
sh ./configure --prefix=<prefix>
```

where <prefix> is the root of the installation tree. When omitted ‘/usr/local’ is used.

Use `configure --help` to get a list of other options.

Important: You must install PRNG *before* `configure` is executed.

4. Compile and install the library:

```
make
make install
```

This installs the following files:

```
$(prefix)/include/unuran.h
$(prefix)/include/unuran_config.h
$(prefix)/include/unuran_tests.h
$(prefix)/info/unuran.info
```

```
$(prefix)/lib/libunuran.a
$(prefix)/lib/libunuran.so
```

(However, the names of the libraries may vary with your OS.)

Obviously `$(prefix)/include` and `$(prefix)/lib` must be in the search path of your compiler. You can use environment variables to add these directories to the search path. If you are using the bash type (or add to your profile):

```
export LIBRARY_PATH="<prefix>/lib"
export C_INCLUDE_PATH="<prefix>/include"
```

If you want to link against the shared library make sure that it can be found when executing the binary that links to the library. If it is not installed in the usual path, then the easiest way is to set the `LD_LIBRARY_PATH` environment variable. See any operating system documentation about shared libraries for more information, such as the `ld(1)` and `ld.so(8)` manual pages.

If you do not want to make a shared library, than making such a library can be disabled using

```
sh ./configure --disable-shared
```

5. Documentation in various formats (PS, PDF, info, dvi, HTML, plain text) can be found in the directory `'doc'`.
6. You can run some tests my

```
make check
```

However, this test suite requires the usage of `prng`. It might happen that some of the tests might fail due to roundoff errors or the mysteries of floating point arithmetic, since we have used some extreme settings to test the library.

1.3 Using the library

ANSI C Compliance

The library is written in ANSI C and is intended to conform to the ANSI C standard. It should be portable to any system with a working ANSI C compiler.

The library does not rely on any non-ANSI extensions in the interface it exports to the user. Programs you write using UNURAN can be ANSI compliant. Extensions which can be used in a way compatible with pure ANSI C are supported, however, via conditional compilation. This allows the library to take advantage of compiler extensions on those platforms which support them.

To avoid namespace conflicts all exported function names and variables have the prefix `unur_`, while exported macros have the prefix `UNUR_`.

Compiling and Linking

If you want to use the library you must include the UNURAN header file

```
#include <unuran.h>
```

If you also need the test routines then also add

```
#include <unuran_tests.h>
```

If these header files are not installed on the standard search path of your compiler you will also need to provide its location to the preprocessor as a command line flag. The default location of the `'unuran.h'` is `'/usr/local/include'`. A typical compilation command for a source file `'app.c'` with the GNU C compiler `gcc` is,

```
gcc -I/usr/local/include -c app.c
```

This results in an object file `'app.o'`. The default include path for `gcc` searches `'/usr/local/include'` automatically so the `-I` option can be omitted when UNURAN is installed in its default location.

The library is installed as a single file, 'libunuran.a'. A shared version of the library is also installed on systems that support shared libraries. The default location of these files is '/usr/local/lib'. To link against the library you need to specify the main library. The following example shows how to link an application with the library (and the the PRNG library if you decide to use this source of uniform pseudo-random numbers),

```
gcc app.o -lunuran -lprng -lm
```

Shared Libraries

To run a program linked with the shared version of the library it may be necessary to define the shell variable `LD_LIBRARY_PATH` to include the directory where the library is installed. For example,

```
LD_LIBRARY_PATH=/usr/local/lib:$LD_LIBRARY_PATH ./app
```

To compile a statically linked version of the program instead, use the `-static` flag in `gcc`,

```
gcc -static app.o -lunuran -lprng -lm
```

Compatibility with C++

The library header files automatically define functions to have `extern "C"` linkage when included in C++ programs.

1.4 Concepts of UNURAN

UNURAN is a C library for generating non-uniformly distributed random variates. Its emphasis is on the generation of non-standard distribution and on streams of random variates of special purposes. It is designed to provide a consistent tool to sample from distributions with various properties. Since there is no universal method that fits for all situations, various methods for sampling are implemented.

UNURAN solves this complex task by means of an object oriented programming interface. Three basic objects are used:

- distribution object `UNUR_DISTR`
Hold all information about the random variates that should be generated.
- generator object `UNUR_GEN`
Hold the generators for the given distributions. Two generator objects are completely independent of each other. They may share a common uniform random number generator or have their owns.
- parameter object `UNUR_PAR`
Hold all information for creating a generator object. It is necessary due to various parameters and switches for each of these generation methods.

For programming notice that the parameter objects only hold pointer to arrays but do not have their own copy of such an array. Especially, if a dynamically allocated array is used it *must not* be freed until the generator object has been created!

The idea behind these structures is to make distributions, choosing a generation method and sampling to orthogonal (ie. independent) functions of the library. The parameter object is only introduced due to the necessity to deal with various parameters and switches for each of these generation methods which are required to adjust the algorithms to unusual distributions with extreme properties but have default values that are suitable for most applications. These parameters and the data for distributions are set by various functions.

Once a generator object has been created sampling (from the univariate continuous distribution) can be done by the following command:

```
double x = unur_sample_cont(generator);
```

Analogous commands exist for discrete and multivariate distributions. For detailed examples that can be copied and modified see [Chapter 2 \[Examples\]](#), page 9.

Distribution objects

All information about a distribution are stored in objects (structures) of type `UNUR_DISTR`. UNURAN has five different types of distribution objects:

<code>cont</code>	Continuous univariate distributions.
<code>cvec</code>	Continuous multivariate distributions.
<code>discr</code>	Discrete univariate distributions.
<code>cemp</code>	Continuous empirical univariate distribution, ie. given by sample.
<code>cvemp</code>	Continuous empirical multivariate distribution, ie. given by sample.

Distribution objects can be created from scratch by the following call

```
distr = unur_distr_<type>_new();
```

where `<type>` is one of the five possible types from the above table. Notice that these commands only create an *empty* object which still must be filled by means of calls for each type of distribution object (see [Chapter 4 \[Handling distribution objects\], page 41](#)). The naming scheme of these functions is designed to indicate the corresponding type of the distribution object and the task to be performed. It is demonstrated on the following example.

```
unur_distr_cont_set_pdf(distr, mypdf);
```

This command stores a PDF named `mypdf` in the distribution object `distr` which must have the type `cont`.

Of course UNURAN provides an easier way to use standard distribution. Instead of using `unur_distr_<type>_new` calls and functions `unur_distr_<type>_set_<...>` for setting data objects for standard distribution can be created by a single call. Eg. to get an object for the normal distribution with mean 2 and standard deviation 5 use

```
double parameter[2] = {2.0 ,5.0};
UNUR_DISTR *distr = unur_distr_normal(parameter, 2);
```

For a list of standard distributions see [Chapter 7 \[Standard distributions\], page 107](#).

Generation methods

The information a distribution object must contain depends heavily on the method chosen for sampling random variates.

Brackets indicate optional information while a tilde indicates that only an approximation must be provided. See [Appendix A \[Glossary\], page 129](#), for unfamiliar terms.

Methods for **continuous univariate distributions**

sample with `unur_sample_cont`

method	PDF	dPDF	mode	area	other
AROU	x	x	[x]		T-concave
CSTD					build-in standard distribution
HINV	[x]	[x]			CDF
NINV	[x]				CDF
SROU			x	x	T-concave
SSR			x	x	T-concave
TABLE	x		x	[~]	all local extrema
TDR	x	x			T-concave
UTDR	x		x	~	T-concave

Methods for **continuous empirical univariate distributions**sample with `unur_sample_cont`

EMPK: Requires an observed sample. EMPL: Requires an observed sample.

Methods for **continuous multivariate distributions**sample with `unur_sample_vec`

VMT: Requires the mean vector and the covariance matrix.

Methods for **continuous empirical multivariate distributions**sample with `unur_sample_vec`

VEMPK: Requires an observed sample.

Methods for **discrete univariate distributions**sample with `unur_sample_discr`

method	PMF	PV	mode	sum	other
DARI	x		x	~	T-concave
DAU	[x]	x			
DGT	[x]	x			
DSTD					build-in standard distribution

Because of tremendous variety of possible problems, UNURAN provides many methods. All information for creating an generator object have to collected in a parameter first. For example if the task is to sample from a continuous distribution the method AROU might be a good choice. Then the call

```
UNUR_PAR *par = unur_arou_new(distribution);
```

creates an parameter object `par` with a pointer to the distribution object and default values for all necessary parameters for method AROU. Other methods can be used by replacing `arou` with the name of the desired methods (in lower case letters):

```
UNUR_PAR *par = unur_<method>_new(distribution);
```

This sets the default values for all necessary parameters for the chosen methods. These are suitable for almost all applications. Nevertheless it is possible to control the behaviour of the method using corresponding `set` calls for each method. This might be necessary to adjust the algorithm for an unusual distribution with extreme properties, or just for fine tuning the perforce of the algorithm. The following example demonstrates how to change the maximum number of iterations for method NINV to the value 50:

```
unur_ninv_set_max_iteration(par, 50);
```

All available methods are described in details in [Chapter 5 \[Methods\]](#), page 57.

Creating a generator object

Now it is possible to create a generator object:

```
UNUR_GEN *generator = unur_init(par);
if (generator == NULL) exit(EXIT_FAILURE);
```

Important: You must always check whether `unur_init` has been executed successfully. Otherwise the NULL pointer is returned which causes a segmentation fault when used for sampling.

Important: The call of `unur_init` **destroys** the parameter object! Moreover it is recommended to call `unur_init` immediately after the parameter object `par` has created and modified.

An existing generator object is a rather static construct. Nevertheless some of the parameters can still be modified by `chg` calls, e.g.

```
unur_ninv_chg_max_iteration(gen, 30);
```

Notice that it is important *when* parameters are changed because different functions must be used:

To change the parameters *before* creating the generator object, the function name includes the term `set` and the first argument must be of type `UNUR_PAR`.

To change the parameters for an *existing* generator object, the function name includes the term `chg` and the first argument must be of type `UNUR_GEN`.

For details see [Chapter 5 \[Methods\]](#), page 57.

Sampling

You can now use your generator object in any place of your program to sample from your distribution. You only have take about the type of number it computes: `double`, `int` or a vector (array of `doubles`). Notice that at this point it does not matter whether you are sampling from a gamma distribution, a truncated normal distribution or even an empirical distribution.

Destroy

When you do not need your generator object any more, you should destroy it:

```
unur_free(generator);
```

Uniform random numbers

Each generator object can have its own uniform random number generator or share one with others. When created a parameter object the pointer for the uniform random number generator is set to the default generator. However it can be changed at any time to any other generator:

```
unur_set_urng(par, urng);
```

or

```
unur_chg_urng(generator, urng);
```

respectively. See [Chapter 6 \[Using uniform random number generators\]](#), page 103, for details.

1.5 Contact the authors

If you have any problems with UNURAN, suggestions how to improve the library or find a bug, please contact us via email unuran@statistik.wu-wien.ac.at.

For news please visit our homepage at <http://statistik.wu-wien.ac.at/unuran/>.

2 Examples

The examples in this chapter should compile cleanly and can be found in the directory ‘examples’ of the source tree of UNURAN. Assuming that UNURAN as well as the PRNG libraries have been installed properly (see [Section 1.2 \[Installation\], page 3](#)) each of these can be compiled (using the GCC in this example) with

```
gcc -Wall -O2 -o example example.c -lunuran -lprng -lm
```

Remark: `-lprng` must be omitted when the PRNG library is not installed. Then however some of the examples might not work.

The library uses three objects: `UNUR_DISTR`, `UNUR_PAR` and `UNUR_GEN`. It is not important to understand the details of these objects but it is important not to change the order of their creation. The distribution object can be destroyed *after* the generator object has been made. (The parameter object is freed automatically by the `unur_init` call.) It is also important to check the result of the `unur_init` call. If it has failed the `NULL` pointer is returned and causes a segmentation fault when used for sampling.

We give all examples with the UNURAN standard API and the more convenient string API.

2.1 As short as possible

Select a distribution and let UNURAN do all necessary steps.

```
/* ----- */
/* File: example0.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

int main()
{
    int    i;        /* loop variable */
    double x;        /* will hold the random number */

    /* Declare the three UNURAN objects. */
    UNUR_DISTR *distr; /* distribution object */
    UNUR_PAR   *par;   /* parameter object */
    UNUR_GEN   *gen;   /* generator object */

    /* Use a predefined standard distribution: */
    /* Gaussian with mean zero and standard deviation 1. */
    /* Since this is the standard form of the distribution, */
    /* we need not give these parameters. */
    distr = unur_distr_normal(NULL, 0);

    /* Use method AUTO: */
    /* Let UNURAN select a suitable method for you. */
    par = unur_auto_new(distr);

    /* Now you can change some of the default settings for the */
    /* parameters of the chosen method. We don't do it here. */

    /* Create the generator object. */
    gen = unur_init(par);

    /* Notice that this call has also destroyed the parameter */

```

```

/* object 'par' as a side effect. */

/* It is important to check if the creation of the generator
/* object was successful. Otherwise 'gen' is the NULL pointer
/* and would cause a segmentation fault if used for sampling.
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create
/* another generator object. If you do not need it any more,
/* it should be destroyed to free memory.
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from
/* the standard Gaussian distribution.
/* Eg.:
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you
/* can destroy it.
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.2 As short as possible (String API)

Select a distribution and let UNURAN do all necessary steps.

```

/* ----- */
/* File: example0_str.c */
/* ----- */
/* String API. */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

int main()
{
    int    i;        /* loop variable */
    double x;       /* will hold the random number */

    /* Declare UNURAN generator object. */
    UNUR_GEN *gen;   /* generator object */

```

```

/* Create the generator object. */
/* Use a predefined standard distribution: */
/* Standard Gaussian distribution. */
/* Use method AUTO: */
/* Let UNURAN select a suitable method for you. */
gen = unur_str2gen("normal()");

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* Now you can use the generator object 'gen' to sample from */
/* the standard Gaussian distribution. */
/* Eg.: */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.3 Select a method

Select method AROU and use it with default parameters.

```

/* ----- */
/* File: example1.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

int main()
{
    int    i;        /* loop variable */
    double x;       /* will hold the random number */

    /* Declare the three UNURAN objects. */
    UNUR_DISTR *distr; /* distribution object */
    UNUR_PAR    *par;  /* parameter object */
    UNUR_GEN    *gen;  /* generator object */
}

```

```

/* Use a predefined standard distribution: */
/* Gaussian with mean zero and standard deviation 1. */
/* Since this is the standard form of the distribution, */
/* we need not give these parameters. */
distr = unur_distr_normal(NULL, 0);

/* Choose a method: AROU. */
/* For other (suitable) methods replace "arou" with the */
/* respective name (in lower case letters). */
par = unur_arou_new(distr);

/* Now you can change some of the default settings for the */
/* parameters of the chosen method. We don't do it here. */

/* Create the generator object. */
gen = unur_init(par);

/* Notice that this call has also destroyed the parameter */
/* object 'par' as a side effect. */

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create */
/* another generator object. If you do not need it any more, */
/* it should be destroyed to free memory. */
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from */
/* the standard Gaussian distribution. */
/* Eg.: */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.4 Select a method (String API)

Select method AROU and use it with default parameters.

```

/* ----- */

```

```

/* File: example1_str.c */
/* ----- */
/* String API. */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

int main()
{
    int    i;        /* loop variable */
    double x;       /* will hold the random number */

    /* Declare UNURAN generator object. */
    UNUR_GEN *gen;   /* generator object */

    /* Create the generator object. */
    /* Use a predefined standard distribution: */
    /* Standard Gaussian distribution. */
    /* Choose a method: AROU. */
    /* For other (suitable) methods replace "arou" with the */
    /* respective name. */
    gen = unur_str2gen("normal() & method=arou");

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from */
    /* the standard Gaussian distribution. */
    /* Eg.: */
    for (i=0; i<10; i++){
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* When you do not need the generator object any more, you */
    /* can destroy it. */
    unur_free(gen);

    exit (EXIT_SUCCESS);
} /* end of main() */

/* ----- */

```

2.5 Arbitrary distributions

If you want to sample from a non-standard distribution, UNURAN might be exactly what you need. Depending on the information is available, a method must be chosen for sampling, see [Section 1.4 \[Concepts\]](#), page 5 for an overview and [Chapter 5 \[Methods\]](#), page 57 for details.

```

/* ----- */
/* File: example2.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* In this example we build a distribution object from scratch */
/* and sample from this distribution. */
/* */
/* We use method TDR (Transformed Density Rejection) which */
/* required a PDF and the derivative of the PDF. */

/* ----- */

/* Define the PDF and dPDF of our distribution. */
/* */
/* Our distribution has the PDF */
/* */
/*      / 1 - x*x if |x| <= 1 */
/* f(x) = < */
/*      \ 0      otherwise */
/* */

/* The PDF of our distribution: */
double mypdf( double x, const UNUR_DISTR *distr )
    /* The second argument ('distr') can be used for parameters */
    /* for the PDF. (We do not use parameters in our example.) */
{
    if (fabs(x) >= 1.)
        return 0.;
    else
        return (1.-x*x);
} /* end of mypdf() */

/* The derivative of the PDF of our distribution: */
double mydpdf( double x, const UNUR_DISTR *distr )
{
    if (fabs(x) >= 1.)
        return 0.;
    else
        return (-2.*x);
} /* end of mydpdf() */

/* ----- */

int main()
{
    int    i;      /* loop variable */
    double x;     /* will hold the random number */

```

```

/* Declare the three UNURAN objects. */
UNUR_DISTR *distr; /* distribution object */
UNUR_PAR *par; /* parameter object */
UNUR_GEN *gen; /* generator object */

/* Create a new distribution object from scratch. */
/* It is a continuous distribution, and we need a PDF and the */
/* derivative of the PDF. Moreover we set the domain. */

/* Get empty distribution object for a continuous distribution */
distr = unur_distr_cont_new();

/* Assign the PDF and dPDF (defined above). */
unur_distr_cont_set_pdf( distr, mypdf );
unur_distr_cont_set_dpdf( distr, mydpdf );

/* Set the domain of the distribution (optional for TDR). */
unur_distr_cont_set_domain( distr, -1., 1. );

/* Choose a method: TDR. */
par = unur_tdr_new(distr);

/* Now you can change some of the default settings for the */
/* parameters of the chosen method. We don't do it here. */

/* Create the generator object. */
gen = unur_init(par);

/* Notice that this call has also destroyed the parameter */
/* object 'par' as a side effect. */

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create */
/* another generator object. If you do not need it any more, */
/* it should be destroyed to free memory. */
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from */
/* the distribution. Eg.: */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

```

```

    exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.6 Arbitrary distributions (String API)

If you want to sample from a non-standard distribution, UNURAN might be exactly what you need. Depending on the information is available, a method must be chosen for sampling, see [Section 1.4 \[Concepts\], page 5](#) for an overview and [Chapter 5 \[Methods\], page 57](#) for details.

```

/* ----- */
/* File: example2_str.c */
/* ----- */
/* String API. */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* In this example we use a generic distribution object */
/* and sample from this distribution. */
/* */
/* The PDF of our distribution is given by */
/* */
/*      / 1 - x*x  if |x| <= 1 */
/* f(x) = < */
/*      \ 0        otherwise */
/* */
/* We use method TDR (Transformed Density Rejection) which */
/* required a PDF and the derivative of the PDF. */

/* ----- */

int main()
{
    int    i;      /* loop variable */
    double x;     /* will hold the random number

    /* Declare UNURAN generator object. */
    UNUR_GEN *gen; /* generator object

    /* Create the generator object. */
    /* Use a generic continuous distribution. */
    /* Choose a method: TDR. */
    gen = unur_str2gen("distr = cont; pdf=\"1-x*x\"; domain=(-1,1) & method=tdr");

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
    }
}

```

```

        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from
    /* the distribution. Eg.:
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* When you do not need the generator object any more, you
    /* can destroy it.
    unur_free(gen);

    exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.7 Change parameters of the method

Each method for generating random numbers allows several parameters to be modified. If you do not want to use default values, it is possible to change them. The following example illustrates how to change parameters. For details see [Chapter 5 \[Methods\], page 57](#).

```

/* ----- */
/* File: example3.c
/* ----- */

/* Include UNURAN header file.
#include <unuran.h>

/* ----- */

int main()
{
    int    i;          /* loop variable
    double x;         /* will hold the random number

    double fparams[2]; /* array for parameters for distribution

    /* Declare the three UNURAN objects.
    UNUR_DISTR *distr; /* distribution object
    UNUR_PAR   *par;   /* parameter object
    UNUR_GEN   *gen;   /* generator object

    /* Use a predefined standard distribution:
    /* Gaussian with mean 2. and standard deviation 0.5.
    fparams[0] = 2.;
    fparams[1] = 0.5;
    distr = unur_distr_normal( fparams, 2 );

    /* Choose a method: TDR.
    par = unur_tdr_new(distr);

```

```

/* Change some of the default parameters. */

/* We want to use  $T(x)=\log(x)$  for the transformation. */
unur_tdr_set_c( par, 0. );

/* We want to have the variant with immediate acceptance. */
unur_tdr_set_variant_ia( par );

/* We want to use 10 construction points for the setup */
unur_tdr_set_cpoints ( par, 10, NULL );

/* Create the generator object. */
gen = unur_init(par);

/* Notice that this call has also destroyed the parameter */
/* object 'par' as a side effect. */

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create */
/* another generator object. If you do not need it any more, */
/* it should be destroyed to free memory. */
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from */
/* the distribution. Eg.: */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* It is possible with method TDR to truncate the distribution */
/* for an existing generator object ... */
unur_tdr_chg_truncated( gen, -1., 0. );

/* ... and sample again. */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

```

```
/* ----- */
```

2.8 Change parameters of the method (String API)

Each method for generating random numbers allows several parameters to be modified. If you do not want to use default values, it is possible to change them. The following example illustrates how to change parameters. For details see [Chapter 5 \[Methods\]](#), page 57.

```
/* ----- */
/* File: example3_str.c                                     */
/* ----- */
/* String API.                                           */
/* ----- */

/* Include UNURAN header file.                            */
#include <unuran.h>

/* ----- */

int main()
{
    int    i;          /* loop variable          */
    double x;         /* will hold the random number */

    /* Declare UNURAN generator object.                    */
    UNUR_GEN *gen;    /* generator object      */

    /* Create the generator object.                        */
    /* Use a predefined standard distribution:              */
    /* Gaussian with mean 2. and standard deviation 0.5.  */
    /* Choose a method: TDR with parameters                */
    /* c = 0: use T(x)=log(x) for the transformation;      */
    /* variant "immediate acceptance";                    */
    /* number of construction points = 10.                 */
    gen = unur_str2gen("normal(2,0.5) & method=tdr; c=0.; variant_ia; cpoints=10");

    /* It is important to check if the creation of the generator
    /* object was successful. Otherwise 'gen' is the NULL pointer
    /* and would cause a segmentation fault if used for sampling.
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from
    /* the distribution. Eg.:
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* It is possible with method TDR to truncate the distribution
    /* for an existing generator object ...
    unur_tdr_chg_truncated( gen, -1., 0. );

    /* ... and sample again.
    for (i=0; i<10; i++) {
```

```

    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you      */
/* can destroy it.                                             */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

2.9 Change uniform random generator

All generator object use the same default uniform random number generator by default. This can be changed to any generator of your choice such that each generator object has its own random number generator or can share it with some other objects. It is also possible to change the default generator at any time. See [Chapter 6 \[Using uniform random number generators\]](#), page 103, for details.

The following example shows how the uniform random number generator can be set or changed for a generator object. It requires the PRNG library to be installed and used. Otherwise the example must be modified accordingly.

```

/* ----- */
/* File: example4.c                                           */
/* ----- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ----- */

/* This example makes use of the PRNG library (see           */
/* http://statistik.wu-wien.ac.at/prng/) for generating      */
/* uniform random numbers.                                   */
/* To compile this example you must have set                 */
/* #define UNUR_URNG_TYPE UNUR_URNG_PRNG                    */
/* in 'src/unuran_config.h'.                                 */

/* It also works with necessary modifications with other uniform */
/* random number generators.                                  */

/* ----- */

int main()
{
#if UNUR_URNG_TYPE == UNUR_URNG_PRNG

    int    i;          /* loop variable          */
    double x;          /* will hold the random number */
    double fparams[2]; /* array for parameters for distribution */

    /* Declare the three UNURAN objects.                      */
    UNUR_DISTR *distr; /* distribution object    */
    UNUR_PAR   *par;   /* parameter object      */

```

```

UNUR_GEN  *gen;          /* generator object          */

/* Declare objects for uniform random number generators. */
UNUR_URNG *urng1, *urng2; /* uniform generator objects */

/* PRNG only: */
/* Make a object for uniform random number generator. */
/* For details see http://statistik.wu-wien.ac.at/prng/ */
/* We use the Mersenne Twister. */
urng1 = prng_new("mt19937(1237)");
if (urng1 == NULL) exit (EXIT_FAILURE);

/* Use a predefined standard distribution: */
/* Beta with parameters 2 and 3. */
fparams[0] = 2.;
fparams[1] = 3.;
distr = unur_distr_beta( fparams, 2 );

/* Choose a method: TDR. */
par = unur_tdr_new(distr);

/* Set uniform generator in parameter object */
unur_set_urng( par, urng1 );

/* Create the generator object. */
gen = unur_init(par);

/* Notice that this call has also destroyed the parameter
/* object 'par' as a side effect.

/* It is important to check if the creation of the generator
/* object was successful. Otherwise 'gen' is the NULL pointer
/* and would cause a segmentation fault if used for sampling.
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create
/* another generator object. If you do not need it any more,
/* it should be destroyed to free memory.
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from
/* the distribution. Eg.:
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* Now we want to switch to a different uniform random number
/* generator.
/* Now we use an ICG (Inversive Congruential Generator).
urng2 = prng_new("icg(2147483647,1,1,0)");
if (urng2 == NULL) exit (EXIT_FAILURE);
unur_chg_urng( gen, urng2 );

```

```

/* ... and sample again. */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

/* We also should destroy the uniform random number generators.*/
prng_free(urng1);
prng_free(urng2);

exit (EXIT_SUCCESS);

#else
printf("You must use the PRNG library to run this example!\n\n");
exit (77); /* exit code for automake check routines */
#endif

} /* end of main() */

/* ----- */

```

2.10 Change uniform random generator (String API)

All generator object use the same default uniform random number generator by default. This can be changed to any generator of your choice such that each generator object has its own random number generator or can share it with some other objects. It is also possible to change the default generator at any time. See [Chapter 6 \[Using uniform random number generators\]](#), page 103, for details.

The following example shows how the uniform random number generator can be set or changed for a generator object. It requires the PRNG library to be installed and used. Otherwise the example must be modified accordingly.

```

/* ----- */
/* File: example4_str.c */
/* ----- */
/* String API. */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* This example makes use of the PRNG library (see */
/* http://statistik.wu-wien.ac.at/prng/) for generating */
/* uniform random numbers. */
/* To compile this example you must have set */
/* #define UNUR_URNG_TYPE UNUR_URNG_PRNG */
/* in 'src/unuran_config.h'. */

/* It also works with necessary modifications with other uniform */

```

```

/* random number generators. */

/* ----- */

int main()
{
#if UNUR_URNG_TYPE == UNUR_URNG_PRNG

    int    i;          /* loop variable */
    double x;          /* will hold the random number */

    /* Declare UNURAN generator object. */
    UNUR_GEN *gen;     /* generator object */

    /* Declare objects for uniform random number generators. */
    UNUR_URNG *urng1, *urng2; /* uniform generator objects */

    /* Create the generator object. */
    /* Use a predefined standard distribution: */
    /* Beta with parameters 2 and 3. */
    /* Choose a method: TDR. */
    /* Use the Mersenne Twister for uniform random number */
    /* generator (requires PRNG library). */
    gen = unur_str2gen("beta(2,3) & method=tdr & urng = mt19937(1237)");

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from */
    /* the distribution. Eg.: */
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* Now we want to switch to a different uniform random number */
    /* generator. */
    /* Now we use an ICG (Inversive Congruental Generator). */
    urng2 = prng_new("icg(2147483647,1,1,0)");
    if (urng2 == NULL) exit (EXIT_FAILURE);

    /* Change uniform random number generator. */
    /* Notice however that we should save the pointer to uniform */
    /* random number generator in the generator object. */
    urng1 = unur_chg_urng( gen, urng2 );

    /* ... and sample again. */
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }
}

```

```

/* When you do not need the generator object any more, you      */
/* can destroy it.                                             */
unur_free(gen);

/* We also should destroy the uniform random number generators.*/
prng_free(urng1);
prng_free(urng2);

exit (EXIT_SUCCESS);

#else
printf("You must use the PRNG library to run this example!\n\n");
exit (77); /* exit code for automake check routines */
#endif

} /* end of main() */

/* ----- */

```

2.11 Sample pairs of antithetic random variates

Using Method TDR it is easy to sample pairs of antithetic random variates.

```

/* ----- */
/* File: example_anti.c                                       */
/* ----- */

/* Include UNURAN header file.                                */
#include <unuran.h>

/* ----- */

/* Example how to sample from two streams of antithetic random */
/* variates from Gaussian N(2,5) and Gamma(4) distribution, resp.*/
/* ----- */

/* This example makes use of the PRNG library (see            */
/* http://statistik.wu-wien.ac.at/prng/) for generating */
/* uniform random numbers.                                    */
/* To compile this example you must have set                  */
/* #define UNUR_URNG_TYPE UNUR_URNG_PRNG                     */
/* in 'src/unuran_config.h'.                                  */

/* It also works with necessary modifications with other uniform */
/* random number generators.                                    */

/* ----- */

int main()
{
#if UNUR_URNG_TYPE == UNUR_URNG_PRNG

int i; /* loop variable */
double xn, xg; /* will hold the random number */
double fparams[2]; /* array for parameters for distribution */

```

```

/* Declare the three UNURAN objects.                               */
UNUR_DISTR *distr;        /* distribution object                    */
UNUR_PAR   *par;          /* parameter object          */
UNUR_GEN   *gen_normal, *gen_gamma;
                        /* generator objects        */

/* Declare objects for uniform random number generators.          */
UNUR_URNG *urng1, *urng2; /* uniform generator objects */

/* PRNG only:                                                     */
/* Make a object for uniform random number generator.            */
/* For details see http://statistik.wu-wien.ac.at/prng/. */

/* The first generator: Gaussian N(2,5) */

/* uniform generator: We use the Mersenne Twister.              */
urng1 = prng_new("mt19937(1237)");
if (urng1 == NULL) exit (EXIT_FAILURE);

/* UNURAN generator object for N(2,5) */
fparams[0] = 2.;
fparams[1] = 5.;
distr = unur_distr_normal( fparams, 2 );

/* Choose method TDR with variant PS.                             */
par = unur_tdr_new( distr );
unur_tdr_set_variant_ps( par );

/* Set uniform generator in parameter object.                     */
unur_set_urng( par, urng1 );

/* Set auxilliary uniform random number generator.              */
/* We use the default generator.                                  */
unur_use_urng_aux_default( par );

/* Alternatively you can create and use your own auxilliary     */
/* uniform random number generator:                               */
/* UNUR_URNG *urng_aux;                                          */
/* urng_aux = prng_new("tt800");                                 */
/* if (urng_aux == NULL) exit (EXIT_FAILURE);                   */
/* unur_set_urng_aux( par, urng_aux );                           */

/* Create the generator object.                                   */
gen_normal = unur_init(par);
if (gen_normal == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* Destroy distribution object (gen_normal has its own copy).    */
unur_distr_free(distr);

/* The second generator: Gamma(4) with antithetic variates.     */

```

```

/* uniform generator: We use the Mersenne Twister.          */
urng2 = prng_new("anti(mt19937(1237))");
if (urng2 == NULL) exit (EXIT_FAILURE);

/* UNURAN generator object for gamma(4) */
fparams[0] = 4.;
distr = unur_distr_gamma( fparams, 1 );

/* Choose method TDR with variant PS.                       */
par = unur_tdr_new( distr );
unur_tdr_set_variant_ps( par );

/* Set uniform generator in parameter object.               */
unur_set_urng( par, urng2 );

/* Set auxilliary uniform random number generator.         */
/* We use the default generator.                           */
unur_use_urng_aux_default( par );

/* Alternatively you can create and use your own auxilliary */
/* uniform random number generator (see above).            */
/* Notice that both generator objects gen_normal and      */
/* gen_gamma can share the same auxilliary URNG.          */

/* Create the generator object.                             */
gen_gamma = unur_init(par);
if (gen_gamma == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* Destroy distribution object (gen_normal has its own copy). */
unur_distr_free(distr);

/* Now we can sample pairs of negatively correlated random */
/* variates. E.g.:                                         */
for (i=0; i<10; i++) {
    xn = unur_sample_cont(gen_normal);
    xg = unur_sample_cont(gen_gamma);
    printf("%g, %g\n",xn,xg);
}

/* When you do not need the generator objects any more, you */
/* can destroy it.                                          */
unur_free(gen_normal);
unur_free(gen_gamma);

/* We also should destroy the uniform random number generators.*/
prng_free(urng1);
prng_free(urng2);

exit (EXIT_SUCCESS);

```

```

#else
    printf("You must use the PRNG library to run this example!\n\n");
    exit (77);    /* exit code for automake check routines */
#endif

} /* end of main() */

/* ----- */

```

2.12 Sample pairs of antithetic random variates (String API)

Using Method TDR it is easy to sample pairs of antithetic random variates.

```

/* ----- */
/* File: example_anti_str.c                               */
/* ----- */
/* String API.                                           */
/* ----- */

/* Include UNURAN header file.                            */
#include <unuran.h>

/* ----- */

/* Example how to sample from two streams of antithetic random */
/* variates from Gaussian N(2,5) and Gamma(4) distribution, resp.*/

/* ----- */

/* This example makes use of the PRNG library (see          */
/* http://statistik.wu-wien.ac.at/prng/) for generating */
/* uniform random numbers.                                  */
/* To compile this example you must have set                */
/* #define UNUR_URNG_TYPE UNUR_URNG_PRNG                    */
/* in 'src/unuran_config.h'.                                */

/* It also works with necessary modifications with other uniform */
/* random number generators.                                  */

/* ----- */

int main()
{
    #if UNUR_URNG_TYPE == UNUR_URNG_PRNG

        int    i;          /* loop variable          */
        double xn, xg;     /* will hold the random number */

        /* Declare UNURAN generator objects.                */
        UNUR_GEN    *gen_normal, *gen_gamma;

        /* PRNG only:                                       */
        /* Make a object for uniform random number generator. */
        /* For details see http://statistik.wu-wien.ac.at/prng/. */

        /* Create the first generator: Gaussian N(2,5)      */
        gen_normal = unur_str2gen("normal(2,5) & method=tdr; variant_ps & urng=mt19937(123)");
    #endif
}

```

```

if (gen_normal == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}
/* Set auxilliary uniform random number generator.          */
/* We use the default generator.                             */
unur_chgto_urng_aux_default(gen_normal);

/* The second generator: Gamma(4) with antithetic variates. */
gen_gamma = unur_str2gen("gamma(4) & method=tdr; variant_ps & urng=anti(mt19937(12)");
if (gen_gamma == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}
unur_chgto_urng_aux_default(gen_gamma);

/* Now we can sample pairs of negatively correlated random */
/* variates. E.g.:                                         */
for (i=0; i<10; i++) {
    xn = unur_sample_cont(gen_normal);
    xg = unur_sample_cont(gen_gamma);
    printf("%g, %g\n",xn,xg);
}

/* When you do not need the generator objects any more, you */
/* can destroy it.                                          */

/* But first we have to destroy the uniform random number */
/* generators.                                             */
prng_free(unur_get_urng(gen_normal));
prng_free(unur_get_urng(gen_gamma));

unur_free(gen_normal);
unur_free(gen_gamma);

exit (EXIT_SUCCESS);

#else
    printf("You must use the PRNG library to run this example!\n\n");
    exit (77); /* exit code for automake check routines */
#endif

} /* end of main() */

/* ----- */

```

2.13 More examples

See [Section 5.3 \[Methods for continuous univariate distributions\]](#), page 58.

See [Section 5.4 \[Methods for continuous empirical univariate distributions\]](#), page 85.

See [Section 5.6 \[Methods for continuous empirical multivariate distributions\]](#), page 91.

See [Section 5.7 \[Methods for discrete univariate distributions\]](#), page 93.

3 String Interface

The string interface (string API) provided by the `unur_str2gen` call is the easiest way to use UNURAN. This function takes a character string as its argument. The string is parsed and the information obtained is used to create a generator object. It returns `NULL` if this fails, either due to a syntax error, or due to invalid data. In both cases `unur_error` is set to the corresponding error codes (see [Section 8.1 \[Error reporting\]](#), page 117). Additionally there exists the call `unur_str2distr` that only produces a distribution object.

Notice that the string interface does not implement all features of the UNURAN library. For trickier tasks it might be necessary to use the UNURAN calls.

In [Chapter 2 \[Examples\]](#), page 9, all examples are given using both the UNURAN standard API and this convenient string API. The corresponding program codes are equivalent.

Function reference

`UNUR_GEN*` **unur_str2gen** (*const char* string*)

Get a generator object for the distribution, method and uniform random number generator as described in the given *string*. See [Section 3.1 \[Syntax of String Interface\]](#), page 29, for details.

`UNUR_DISTR*` **unur_str2distr** (*const char* string*)

Get a distribution object for the distribution described in *string*. See [Section 3.1 \[Syntax of String Interface\]](#), page 29, and [Section 3.2 \[Distribution String\]](#), page 30, for details. However, only the block for the distribution object is allowed.

3.1 Syntax of String Interface

The given string holds information about the requested distribution and (optional) about the sampling method and the uniform random number generator invoked. The interpretation of the string is not case-sensitive, all white spaces are ignored.

The string consists of up to three blocks, separated by ampersands `&`.

Each block consists of `<key>=<value>` pairs, separated by semicolons `;`.

The first key in each block is used to indicate each block. We have three different blocks with the following (first) keys:

`distr` definition of the distribution (see [Section 3.2 \[Distribution String\]](#), page 30).
`method` description of the transformation method (see [Section 3.4 \[Method String\]](#), page 35).
`urng` uniform random number generation (see [Section 3.5 \[Uniform RNG String\]](#), page 39).

The `distr` block must be the very first block and is obligatory. All the other blocks are optional and can be arranged in arbitrary order.

For details see the following description of each block.

In the following example

```
distr = normal(3.,0.75); domain = (0,inf) & method = tdr; c = 0
```

we have a distribution block for the truncated normal distribution with mean 3 and standard deviation 0.75 on domain (0,infinity); and block for choosing method TDR with parameter `c` set to 0.

The `<key>=<value>` pairs that follow the first (initial) pair in each block are used to set parameters. The name of the parameter is given by the `<key>` string. It is deduced from the UNURAN set calls by taking the part after `..._set_`. The `<value>` string holds the parameters to be set, separated by commata `,`. There are three types of parameters:

string "..."

i.e. any sequence of characters enclosed by double quotes "...",

list (...,...)

i.e. list of *numbers*, separated by commata ,, enclosed in parenthesis (...), and

number a sequence of characters that is not enclosed by quotes "... " or parenthesis (...). It is interpreted as float or integer depending on the type of the corresponding parameter.

The <value> string (including the character =) can be omitted when no argument is required.

At the moment not all **set** calls are supported. The syntax for the <value> can be directly derived from the corresponding **set** calls. To simplify the syntax additional shortcuts are possible. The following table lists the parameters for the **set** calls that are supported by the string interface; the entry in parenthesis gives the type of the argument as <value> string:

int (*number*):

The *number* is interpreted as an integer. **true** and **on** are transformed to 1, **false** and **off** are transformed to 0. A missing argument is interpreted as 1.

int, int (*number, number* or *list*):

The two numbers or the first two entries in the list are interpreted as a integers. **inf** and **-inf** are transformed to INT_MAX and INT_MIN respectively, i.e. the largest and smallest integers that can be represented by the computer.

unsigned (*number*):

The *number* is interpreted as an unsigned hexadecimal integer.

double (*number*):

The number is interpreted as a floating point number. **inf** is transformed to UNUR_INFINITY.

double, double (*number, number* or *list*):

The two numbers or the first two entries in the list are interpreted as a floating point numbers. **inf** is transformed to UNUR_INFINITY. However using **inf** in the list might not work for all versions of C. Then it is recommended to use two single numbers instead of a list.

int, double* (*[number,] list* or *number*):

- The list is interpreted as a double array. The (first) number as its length. If it is less than the actual size of the array only the first entries of the array are used.
- If only the list is given (i.e., if the first number is omitted), the first number is set to the actual size of the array.
- If only the number is given (i.e., if the list is omitted), the NULL pointer is used instead an array as argument.

double*, int (*list [,number]*):

The list is interpreted as a double array. The (second) number as its length. If the length is omitted, it is replaced by the actual size of the array. (Only in the **distribution** block!)

char* (*string*):

The character string is passed as is to the corresponding set call.

Notice that missing entries in a list of numbers are interpreted as 0. E.g, a the list (1,,3) is read as (1,0,3), the list (1,2,) as (1,2,0).

The the list of key strings in [Section 3.2.1 \[Keys for Distribution String\]](#), page 31, and [Section 3.4.1 \[Keys for Method String\]](#), page 35, for further details.

3.2 Distribution String

The **distr** block must be the very first block and is obligatory. For that reason the keyword **distr** is optional and can be omitted (together with the = character). Moreover it is ignored while parsing the string. However, to avoid some possible confusion it has to start with the letter d (if it is given at all).

The value of the `distr` key is used to get the distribution object, either via a `unur_distr_<value>` call for a standard distribution via a `unur_distr_<value>_new` call to get an object of a generic distribution. However not all generic distributions are supported yet.

The parameters for the standard distribution are given as a list. There must not be any character (other than white space) between the name of the standard distribution and the opening parenthesis (of this list. E.g., to get a beta distribution, use

```
distr = beta(2,4)
```

To get an object for a discrete distribution with probability vector (0.5,0.2,0.3), use

```
distr = discr; pv = (0.5,0.2,0.3)
```

It is also possible to set a PDF, PMF, or CDF using a string. E.g., to create a continuous distribution with PDF proportional to $\exp(-\sqrt{2+(x-1)^2} + (x-1))$ and domain (0,inf) use

```
distr = cont; pdf = "exp(-sqrt(2+(x-1)^2) + (x-1))"
```

(Notice: If this string is used in an `unur_str2distr` or `unur_str2gen` call the double quotes " must be protected by `\`.)

For the details of function strings see [Section 3.3 \[Function String\]](#), page 33.

3.2.1 Keys for Distribution String

List of standard distributions see [Chapter 7 \[Standard distributions\]](#), page 107

- [`distr =`] `beta(...)` ⇒ see [Section 7.1.1 \[beta\]](#), page 108
- [`distr =`] `binomial(...)` ⇒ see [Section 7.3.1 \[binomial\]](#), page 113
- [`distr =`] `cauchy(...)` ⇒ see [Section 7.1.2 \[cauchy\]](#), page 108
- [`distr =`] `chi(...)` ⇒ see [Section 7.1.3 \[chi\]](#), page 108
- [`distr =`] `chisquare(...)` ⇒ see [Section 7.1.4 \[chisquare\]](#), page 109
- [`distr =`] `exponential(...)` ⇒ see [Section 7.1.5 \[exponential\]](#), page 109
- [`distr =`] `extremeI(...)` ⇒ see [Section 7.1.6 \[extremeI\]](#), page 109
- [`distr =`] `extremeII(...)` ⇒ see [Section 7.1.7 \[extremeII\]](#), page 109
- [`distr =`] `gamma(...)` ⇒ see [Section 7.1.8 \[gamma\]](#), page 110
- [`distr =`] `geometric(...)` ⇒ see [Section 7.3.2 \[geometric\]](#), page 113
- [`distr =`] `hypergeometric(...)` ⇒ see [Section 7.3.3 \[hypergeometric\]](#), page 114
- [`distr =`] `laplace(...)` ⇒ see [Section 7.1.9 \[laplace\]](#), page 110
- [`distr =`] `logarithmic(...)` ⇒ see [Section 7.3.4 \[logarithmic\]](#), page 114
- [`distr =`] `logistic(...)` ⇒ see [Section 7.1.10 \[logistic\]](#), page 110
- [`distr =`] `lomax(...)` ⇒ see [Section 7.1.11 \[lomax\]](#), page 110
- [`distr =`] `negativebinomial(...)` ⇒ see [Section 7.3.5 \[negativebinomial\]](#), page 114
- [`distr =`] `normal(...)` ⇒ see [Section 7.1.12 \[normal\]](#), page 111
- [`distr =`] `pareto(...)` ⇒ see [Section 7.1.13 \[pareto\]](#), page 111
- [`distr =`] `poisson(...)` ⇒ see [Section 7.3.6 \[poisson\]](#), page 115
- [`distr =`] `powerexponential(...)` ⇒ see [Section 7.1.14 \[powerexponential\]](#), page 111
- [`distr =`] `rayleigh(...)` ⇒ see [Section 7.1.15 \[rayleigh\]](#), page 112
- [`distr =`] `student(...)` ⇒ see [Section 7.1.16 \[student\]](#), page 112
- [`distr =`] `triangular(...)` ⇒ see [Section 7.1.17 \[triangular\]](#), page 112
- [`distr =`] `uniform(...)` ⇒ see [Section 7.1.18 \[uniform\]](#), page 112
- [`distr =`] `weibull(...)` ⇒ see [Section 7.1.19 \[weibull\]](#), page 113

List of generic distributions see [Chapter 4 \[Handling Distribution Objects\]](#), page 41

- [`distr =`] `cemp` ⇒ see [Section 4.4 \[CEMP\]](#), page 48
- [`distr =`] `cont` ⇒ see [Section 4.2 \[CONT\]](#), page 42
- [`distr =`] `discr` ⇒ see [Section 4.7 \[DISCR\]](#), page 52

Notice: Order statistics for continuous distributions (see Section 4.3 [CORDER], page 46) are supported by using the key `orderstatistics` for distributions of type `CONT`.

List of keys that are available via the String API. For description see the corresponding UNURAN set calls.

- All distribution types

`name = "<string>"`

⇒ see [unur_distr_set_name], page 41

- `cemp` (*Distribution Type*) (see Section 4.4 [CEMP], page 48)

`data = (<list>) [, <int>]`

⇒ see [unur_distr_cemp_set_data], page 48

- `cont` (*Distribution Type*) (see Section 4.2 [CONT], page 42)

`cdf = "<string>"`

⇒ see [unur_distr_cont_set_cdfstr], page 43

`domain = <double>, <double> | (<list>)`

⇒ see [unur_distr_cont_set_domain], page 44

`hr = "<string>"`

⇒ see [unur_distr_cont_set_hrstr], page 45

`mode = <double>`

⇒ see [unur_distr_cont_set_mode], page 45

`pdf = "<string>"`

⇒ see [unur_distr_cont_set_pdfstr], page 43

`pdfarea = <double>`

⇒ see [unur_distr_cont_set_pdfarea], page 46

`pdfparams = (<list>) [, <int>]`

⇒ see [unur_distr_cont_set_pdfparams], page 44

`orderstatistics = <int>, <int> | (<list>)`

Make order statistics for given distribution. The first parameter gives the sample size, the second parameter its rank. It must be the last keyword for a distribution string. (see see [unur_distr_corder_new], page 46)

- `discr` (*Distribution Type*) (see Section 4.7 [DISCR], page 52)

`cdf = "<string>"`

⇒ see [unur_distr_discr_set_cdfstr], page 54

`domain = <int>, <int> | (<list>)`

⇒ see [unur_distr_discr_set_domain], page 54

`mode [= <int>]`

⇒ see [unur_distr_discr_set_mode], page 55

`pmf = "<string>"`

⇒ see [unur_distr_discr_set_pmfstr], page 54

`pmfparams = (<list>) [, <int>]`

⇒ see [unur_distr_discr_set_pmfparams], page 54

`pmfsum = <double>`

⇒ see [unur_distr_discr_set_pmfsum], page 55

`pv = (<list>) [, <int>]`

⇒ see [unur_distr_discr_set_pv], page 52

3.3 Function String

In unuran it is also possible to define functions (e.g. CDF or PDF) as strings. As you can see in Example 2 (Section 2.6 [Example_2_str], page 16) it is very easy to define the PDF of a distribution object by means of a string. The possibilities using this string interface are more restricted than using a pointer to a routine coded in C (Section 2.5 [Example_2], page 14). But the differences in evaluation time is small. When a distribution object is defined using this string interface then of course the same conditions on the given density or CDF must be satisfied for a chosen method as for the standard API. This string interface can be used for both within the UNURAN string API using the `unur_str2gen` call, and for calls that define the density or CDF for a particular distribution object as done with (e.g.) the call `unur_distr_cont_set_pdfstr`. Here is an example for the latter case:

```
unur_distr_cont_set_pdfstr(distr,"1-x*x");
```

Syntax

The syntax for the function string is case insensitive, white spaces are ignored. The expressions are similar to most programming languages and mathematical programs (see also the examples below). It is especially influenced by C. The usual precedence rules are used (from highest to lowest precedence: functions, power, multiplication, addition, relation operators). Use parentheses in case of doubt or when these precedences should be changed.

Relation operators can be used as indicator functions, i.e. the term $(x>1)$ is evaluated as 1 if this relation is satisfied, and as 0 otherwise.

The first unknown symbol (letter or word) is interpreted as the variable of the function. It is recommended to use `x`. Only one variable can be used.

Important: The symbol `e` is used twice, for Euler's constant ($= 2.7182\dots$) and as exponent. The multiplication operator `*` must not be omitted, i.e. `2 x` is interpreted as the string `2x` (which will result in a syntax error).

List of symbols

Numbers

Numbers are composed using digits and, optionally, a sign, a decimal point, and an exponent indicated by `e`.

Symbol	Explanation	Examples
0...9	<i>digits</i>	2343
.	<i>decimal point</i>	165.567
-	<i>negative sign</i>	-465.223
e	<i>exponent</i>	13.2e-4 (=0.00132)

Constants

<code>pi</code>	$pi = 3.1415\dots$	<code>3*pi+2</code>
<code>e</code>	<i>Euler's constant</i>	<code>3*e+2</code> ($= 10.15\dots$; do not confuse with <code>3e2 = 300</code>)
<code>inf</code>	<i>infinity</i>	(used for domains)

Special symbols

(<i>opening parenthesis</i>	$2*(3+x)$
)	<i>closing parenthesis</i>	$2*(3+x)$
,	<i>(argument) list separator</i>	$\text{mod}(13,2)$

Relation operators (Indicator functions)

<	<i>less than</i>	$(x<1)$
=	<i>equal</i>	$(2=x)$
==	<i>same as =</i>	$(x==3)$
>	<i>greater than</i>	$(x>0)$
<=	<i>less than or equal</i>	$(x<=1)$
!=	<i>not equal</i>	$(x!0)$
<>	<i>same as !=</i>	$(x<>\text{pi})$
>=	<i>greater or equal</i>	$(x>=1)$

Arithmetic operators

+	<i>addition</i>	$2+x$
-	<i>subtraction</i>	$2-x$
*	<i>multiplication</i>	$2*x$
/	<i>division</i>	$x/2$
^	<i>power</i>	x^2

Functions

mod	<i>mod(m,n) remainder of division m over n</i>	$\text{mod}(x,2)$
exp	<i>exponential function (same as e^x)</i>	$\text{exp}(-x^2)$ (same as e^{-x^2})
log	<i>natural logarithm</i>	$\log(x)$
sin	<i>sine</i>	$\sin(x)$
cos	<i>cosine</i>	$\cos(x)$
tan	<i>tangent</i>	$\tan(x)$
sec	<i>secant</i>	$\sec(x*2)$
sqrt	<i>square root</i>	$\text{sqrt}(2*x)$
abs	<i>absolute value</i>	$\text{abs}(x)$
sgn	<i>sign function</i>	$\text{sign}(x)*3$

Variable

x	<i>variable</i>	$3*x^2$
---	-----------------	---------

Examples

$$1.231+7.9876*x-1.234e-3*x^2+3.335e-5*x^3$$

```
sin(2*pi*x)+x^2
```

```
exp(-((x-3)/2.1)^2)
```

It is also possible to define functions using different terms on separate domains. However, instead of constructs using `if ... then ... else ...` indicator functions are available.

For example to define the density of triangular distribution with domain $(-1,1)$ and mode 0 use

```
(x>-1)*(x<0)*(1+x) + (x>=0)*(x<1)*(1-x)
```

3.4 Method String

The key `method` is obligatory, it must be the first key and its value is the name of a method suitable for the chosen standard distribution. E.g., if method AROU is chosen, use

```
method = arou
```

Of course the all following keys depend on the method chosen at first. All corresponding `set` calls of UNURAN are available and the key is the string after the `unur_<methodname>_set_` part of the command. E.g., UNURAN provides the command `unur_arou_set_max_sqratio` to set a parameter of method AROU. To call this function via the string-interface, the key `max_sqratio` can be used:

```
max_sqratio = 0.9
```

Additionally the keyword `debug` can be used to set debugging flags (see [Chapter 9 \[Debugging\]](#), [page 121](#), for details).

If this block is omitted, a suitable default method is used. Notice however that the default method may change in future versions of UNURAN.

3.4.1 Keys for Method String

List of methods and keys that are available via the String API. For description see the corresponding UNURAN set calls.

- `method = arou` ⇒ `unur_arou_new` (see [Section 5.3.1 \[AROU\]](#), [page 61](#))

```
center = <double>
```

```
⇒ see [unur\_arou\_set\_center], page 62
```

```
cpoints = <int> [, (<list>)] | (<list>)
```

```
⇒ see [unur\_arou\_set\_cpoints], page 62
```

```
guidefactor = <double>
```

```
⇒ see [unur\_arou\_set\_guidefactor], page 63
```

```
max_segments [= <int>]
```

```
⇒ see [unur\_arou\_set\_max\_segments], page 62
```

```
max_sqratio = <double>
```

```
⇒ see [unur\_arou\_set\_max\_sqratio], page 62
```

```
pedantic [= <int>]
```

```
⇒ see [unur\_arou\_set\_pedantic], page 63
```

```
usecenter [= <int>]
```

```
⇒ see [unur\_arou\_set\_usecenter], page 63
```

```
verify [= <int>]
```

```
⇒ see [unur\_arou\_set\_verify], page 63
```

- `method = auto` ⇒ `unur_auto_new` (see [Section 5.2 \[AUTO\]](#), [page 57](#))

```
logss [= <int>]
```

```
⇒ see [unur\_auto\_set\_logss], page 58
```

- `method = cstd` ⇒ `unur_cstd_new` (see [Section 5.3.2 \[CSTD\]](#), [page 63](#))

- variant = *<unsigned>*
 ⇒ see [unur_cstd_set_variant], page 64
- method = dari ⇒ unur_dari_new (see Section 5.7.1 [DARI], page 96)
 - cpfactor = *<double>*
 ⇒ see [unur_dari_set_cpfactor], page 97

squeeze [= *<int>*]
 ⇒ see [unur_dari_set_squeeze], page 97

tablesize [= *<int>*]
 ⇒ see [unur_dari_set_tablesize], page 97

verify [= *<int>*]
 ⇒ see [unur_dari_set_verify], page 97
- method = dau ⇒ unur_dau_new (see Section 5.7.2 [DAU], page 98)
 - urnfactor = *<double>*
 ⇒ see [unur_dau_set_urnfactor], page 98
- method = dgt ⇒ unur_dgt_new (see Section 5.7.3 [DGT], page 98)
 - guidefactor = *<double>*
 ⇒ see [unur_dgt_set_guidefactor], page 99

variant = *<unsigned>*
 ⇒ see [unur_dgt_set_variant], page 99
- method = dsrou ⇒ unur_dsrou_new (see Section 5.7.4 [DSROU], page 99)
 - cdfatmode = *<double>*
 ⇒ see [unur_dsrou_set_cdfatmode], page 100

verify [= *<int>*]
 ⇒ see [unur_dsrou_set_verify], page 100
- method = dstd ⇒ unur_dstd_new (see Section 5.7.5 [DSTD], page 101)
 - variant = *<unsigned>*
 ⇒ see [unur_dstd_set_variant], page 102
- method = empk ⇒ unur_empk_new (see Section 5.4.1 [EMPK], page 87)
 - beta = *<double>*
 ⇒ see [unur_empk_set_beta], page 89

kernel = *<unsigned>*
 ⇒ see [unur_empk_set_kernel], page 88

positive [= *<int>*]
 ⇒ see [unur_empk_set_positive], page 89

smoothing = *<double>*
 ⇒ see [unur_empk_set_smoothing], page 89

varcor [= *<int>*]
 ⇒ see [unur_empk_set_varcor], page 89
- method = hinv ⇒ unur_hinv_new (see Section 5.3.3 [HINV], page 65)
 - boundary = *<double>*, *<double>* | (*<list>*)
 ⇒ see [unur_hinv_set_boundary], page 66

cpoints = (*<list>*), *<int>*
 ⇒ see [unur_hinv_set_cpoints], page 65

- guidefactor = *<double>*
 ⇒ see [unur_hinv_set_guidefactor], page 66
- max_intervals [= *<int>*]
 ⇒ see [unur_hinv_set_max_intervals], page 66
- order [= *<int>*]
 ⇒ see [unur_hinv_set_order], page 65
- u_resolution = *<double>*
 ⇒ see [unur_hinv_set_u_resolution], page 65
- method = hrb ⇒ unur_hrb_new (see Section 5.3.4 [HRB], page 67)

upperbound = *<double>*
 ⇒ see [unur_hrb_set_upperbound], page 67

verify [= *<int>*]
 ⇒ see [unur_hrb_set_verify], page 67
 - method = hrd ⇒ unur_hrd_new (see Section 5.3.5 [HRD], page 68)

verify [= *<int>*]
 ⇒ see [unur_hrd_set_verify], page 68
 - method = hri ⇒ unur_hri_new (see Section 5.3.6 [HRI], page 68)

p0 = *<double>*
 ⇒ see [unur_hri_set_p0], page 69

verify [= *<int>*]
 ⇒ see [unur_hri_set_verify], page 69
 - method = ninv ⇒ unur_ninv_new (see Section 5.3.7 [NINV], page 69)

max_iter [= *<int>*]
 ⇒ see [unur_ninv_set_max_iter], page 70

start = *<double>*, *<double>* | (*<list>*)
 ⇒ see [unur_ninv_set_start], page 70

table [= *<int>*]
 ⇒ see [unur_ninv_set_table], page 70

usnewton
 ⇒ see [unur_ninv_set_usnewton], page 70

useregula
 ⇒ see [unur_ninv_set_useregula], page 70

x_resolution = *<double>*
 ⇒ see [unur_ninv_set_x_resolution], page 70
 - method = srou ⇒ unur_srou_new (see Section 5.3.8 [SROU], page 71)

cdfatmode = *<double>*
 ⇒ see [unur_srou_set_cdfatmode], page 72

pdfatmode = *<double>*
 ⇒ see [unur_srou_set_pdfatmode], page 72

r = *<double>*
 ⇒ see [unur_srou_set_r], page 72

usemirror [= *<int>*]
 ⇒ see [unur_srou_set_usemirror], page 73

- usesqueeze [= *<int>*]
 ⇒ see [unur_srou_set_usesqueeze], page 73
- verify [= *<int>*]
 ⇒ see [unur_srou_set_verify], page 73
- method = `ssr` ⇒ `unur_ssr_new` (see Section 5.3.9 [SSR], page 74)
 - `cdfatmode` = *<double>*
 ⇒ see [unur_ssr_set_cdfatmode], page 75
 - `pdfatmode` = *<double>*
 ⇒ see [unur_ssr_set_pdfatmode], page 75
 - `usesqueeze` [= *<int>*]
 ⇒ see [unur_ssr_set_usesqueeze], page 75
 - `verify` [= *<int>*]
 ⇒ see [unur_ssr_set_verify], page 75
 - method = `tabl` ⇒ `unur_tabl_new` (see Section 5.3.10 [TABL], page 76)
 - `areafraction` = *<double>*
 ⇒ see [unur_tabl_set_areafraction], page 78
 - `boundary` = *<double>*, *<double>* | (*<list>*)
 ⇒ see [unur_tabl_set_boundary], page 78
 - `guidefactor` = *<double>*
 ⇒ see [unur_tabl_set_guidefactor], page 78
 - `max_intervals` [= *<int>*]
 ⇒ see [unur_tabl_set_max_intervals], page 78
 - `max_sqratio` = *<double>*
 ⇒ see [unur_tabl_set_max_sqratio], page 77
 - `nstp` [= *<int>*]
 ⇒ see [unur_tabl_set_nstp], page 78
 - `slopes` = (*<list>*), *<int>*
 ⇒ see [unur_tabl_set_slopes], page 78
 - `variant_setup` = *<unsigned>*
 ⇒ see [unur_tabl_set_variant_setup], page 77
 - `variant_splitmode` = *<unsigned>*
 ⇒ see [unur_tabl_set_variant_splitmode], page 77
 - `verify` [= *<int>*]
 ⇒ see [unur_tabl_set_verify], page 79
 - method = `tdr` ⇒ `unur_tdr_new` (see Section 5.3.11 [TDR], page 79)
 - `c` = *<double>*
 ⇒ see [unur_tdr_set_c], page 80
 - `center` = *<double>*
 ⇒ see [unur_tdr_set_center], page 82
 - `cpoints` = *<int>* [, (*<list>*)] | (*<list>*)
 ⇒ see [unur_tdr_set_cpoints], page 81
 - `darsfactor` = *<double>*
 ⇒ see [unur_tdr_set_darsfactor], page 80
 - `guidefactor` = *<double>*
 ⇒ see [unur_tdr_set_guidefactor], page 82

- max_intervals [= <int>]
⇒ see [unur_tdr_set_max_intervals], page 81
- max_sqratio = <double>
⇒ see [unur_tdr_set_max_sqratio], page 81
- pedantic [= <int>]
⇒ see [unur_tdr_set_pedantic], page 82
- usecenter [= <int>]
⇒ see [unur_tdr_set_usecenter], page 82
- usedars [= <int>]
⇒ see [unur_tdr_set_usedars], page 80
- usemode [= <int>]
⇒ see [unur_tdr_set_usemode], page 82
- variant_gw
⇒ see [unur_tdr_set_variant_gw], page 80
- variant_ia
⇒ see [unur_tdr_set_variant_ia], page 80
- variant_ps
⇒ see [unur_tdr_set_variant_ps], page 80
- verify [= <int>]
⇒ see [unur_tdr_set_verify], page 82
- method = utdr ⇒ unur_utdr_new (see Section 5.3.12 [UTDR], page 83)
 - cpfactor = <double>
⇒ see [unur_utdr_set_cpfactor], page 84
 - deltafactor = <double>
⇒ see [unur_utdr_set_deltafactor], page 84
 - pdfatmode = <double>
⇒ see [unur_utdr_set_pdfatmode], page 83
 - verify [= <int>]
⇒ see [unur_utdr_set_verify], page 84
- method = vempk ⇒ unur_vempk_new (see Section 5.6.1 [VEMPK], page 92)
 - smoothing = <double>
⇒ see [unur_vempk_set_smoothing], page 93
 - varcor [= <int>]
⇒ see [unur_vempk_set_varcor], page 93

3.5 Uniform RNG String

The value of the `urng` key is passed to the PRNG interface (see [PRNG manual](#) for details). However it only works when using the PRNG library is enabled, see [Section 1.2 \[Installation\]](#), [page 3](#) for details. There are no other keys.

IMPORTANT: UNURAN creates a new uniform random number generator for the generator object. The pointer to this uniform generator has to be read and saved via a `unur_get_urng` call in order to clear the memory *before* the UNURAN generator object is destroyed.

If this block is omitted the UNURAN default generator is used (which *must not* be destroyed).

4 Handling distribution objects

Objects of type `UNUR_DISTR` are used for handling distributions. All data about a distribution are stored in this object. UNURAN provides functions that return such objects for standard distributions (see [Chapter 7 \[Standard distributions\], page 107](#)). It is then possible to change this distribution object by various set calls. Moreover it is possible to build a distribution object entirely from scratch. For this purpose there exists an `unur_distr_<type>_new` call that returns an empty object of this type for each object type (eg. univariate continuous) which can be filled with the appropriate set calls.

Notice that there are essential data about a distribution, eg. the PDF, a list of (shape, scale, location) parameters for the distribution, and the domain of (the possibly truncated) distribution. And there exist parameters that are/can be derived from these, eg. the mode of the distribution or the area below the given PDF (which need not be normalized for many methods). UNURAN keeps track of parameters which are known. Thus if one of the essential parameters is changed all derived parameters are marked as unknown and must be set again if these are required for the chosen generation method.

The library can handle truncated distributions, that is, distribution that are derived from (standard) distribution by simply restricting its domain to a subset. However there is a subtle difference between changing the domain of a distribution object by a `unur_distr_cont_set_domain` call and changing the (truncated) domain for an existing generator object. The domain of the distribution object is used to create the generator object with hats, squeezes, tables, etc. Whereas truncating the domain of an existing generator object need not necessarily require a recomputation of these data. Thus by a `unur_<method>_chg_truncated` call (if available) the sampling region is restricted to the subset of the domain of the given distribution object. However generation methods that require a recreation of the generator object when the domain is changed have a `unur_<method>_chg_domain` call instead. For this call there are of course no restrictions on the given domain (i.e., it is possible to increase the domain of the distribution) (see [Chapter 5 \[Methods\], page 57](#), for details).

For the objects provided by the UNURAN library of standard distributions, calls for updating these parameters exist (one for each parameter to avoid computational overhead since not all parameters are required for all generator methods).

The calls listed below only handle distribution object. Since every generator object has its own copy of a distribution object, there are calls for a chosen method that change this copy of distribution object. NEVER extract the distribution object out of the generator object and run one of the below set calls on it. (How should the poor generator object know what has happen?)

4.1 Functions for all kinds of distribution objects

Function reference

```
void unur_distr_free (UNUR_DISTR* distribution)           —
    Destroy a distribution object.

int unur_distr_set_name (UNUR_DISTR* distribution, const char* name) —
const char* unur_distr_get_name (const UNUR_DISTR* distribution) —
    Set and get name of distribution.

int unur_distr_get_dim (const UNUR_DISTR* distribution) —
    Get number of components of random vector (its dimension). For univariate distributions
    it returns dimension 1.
```

- unsigned int unur_distr_get_type** (const *UNUR_DISTR** *distribution*) –
 Get type of *distribution*. Possible types are
- UNUR_DISTR_CONT*
univariate continuous distributions
 - UNUR_DISTR_CEMP*
empirical continuous univariate distributions (ie. samples)
 - UNUR_DISTR_CVEC*
continuous multivariate distributions
 - UNUR_DISTR_CVEMP*
empirical continuous multivariate distributions (ie. samples)
 - UNUR_DISTR_DISCR*
discrete univariate distributions
- Alternatively the `unur_distr_is_<TYPE>` calls can be used.
- int unur_distr_is_cont** (const *UNUR_DISTR** *distribution*) –
 TRUE if *distribution* is a continuous univariate distribution.
- int unur_distr_is_cvec** (const *UNUR_DISTR** *distribution*) –
 TRUE if *distribution* is a continuous multivariate distribution.
- int unur_distr_is_cemp** (const *UNUR_DISTR** *distribution*) –
 TRUE if *distribution* is an empirical continuous univariate distribution, i.e. a sample.
- int unur_distr_is_cvemp** (const *UNUR_DISTR** *distribution*) –
 TRUE if *distribution* is an empirical continuous multivariate distribution.
- int unur_distr_is_discr** (const *UNUR_DISTR** *distribution*) –
 TRUE if *distribution* is a discrete univariate distribution.

4.2 Continuous univariate distributions

Function reference

- UNUR_DISTR** **unur_distr_cont_new** (void) –
 Create a new (empty) object for univariate continuous distribution.

Essential parameters

- int unur_distr_cont_set_pdf** (*UNUR_DISTR** *distribution*, *UNUR_FUNCT_CONT** *pdf*) –
- int unur_distr_cont_set_dpdf** (*UNUR_DISTR** *distribution*, *UNUR_FUNCT_CONT** *dpdf*) –
- int unur_distr_cont_set_cdf** (*UNUR_DISTR** *distribution*, *UNUR_FUNCT_CONT** *cdf*) –

Set respective pointer to the probability density function (PDF), the derivative of the probability density function (dPDF) and the cumulative distribution function (CDF) of the *distribution*. The type of each of these functions must be of type `double funct(double x, const UNUR_DISTR *distr)`.

Due to the fact that some of the methods do not require a normalized PDF the following is important:

- The given CDF must be the cumulative distribution function of the (non-truncated) distribution. If a distribution from the UNURAN library of standard distributions (see [Chapter 7 \[Standard distributions\], page 107](#)) is truncated, there is no need to change the CDF.
- If both the CDF and the PDF are used (for a method or for order statistics), the PDF must be the derivative of the CDF. If a truncated distribution for one of the standard distributions from the UNURAN library of standard distributions is used, there is no need to change the PDF.
- If the area below the PDF is required for a given distribution it must be given by the `unur_distr_cont_set_pdfarea` call. For a truncated distribution this must be of course the integral of the PDF in the given truncated domain. For distributions from the UNURAN library of standard distributions this is done automatically by the `unur_distr_cont_upd_pdfarea` call.

It is important to note that all these functions must return a result for all floats x . Eg., if the domain of a given PDF is the interval $[-1,1]$, then the given function must return 0.0 for all points outside this interval. In case of an overflow the PDF should return `UNUR_INFINITY`.

It is not possible to change such a function. Once the PDF or CDF is set it cannot be overwritten. This also holds when the PDF is given by the `unur_distr_cont_set_pdfstr` call. A new distribution object has to be used instead.

`UNUR_FUNCT_CONT*` **unur_distr_cont_get_pdf** (const *UNUR_DISTR** *distribution*) –

`UNUR_FUNCT_CONT*` **unur_distr_cont_get_dpdf** (const *UNUR_DISTR** *distribution*) –

`UNUR_FUNCT_CONT*` **unur_distr_cont_get_cdf** (const *UNUR_DISTR** *distribution*) –

Get the respective pointer to the PDF, the derivative of the PDF and the CDF of the *distribution*. The pointer is of type `double funct(double x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the distribution, the NULL pointer is returned.

`double` **unur_distr_cont_eval_pdf** (double x , const *UNUR_DISTR** *distribution*) –

`double` **unur_distr_cont_eval_dpdf** (double x , const *UNUR_DISTR** *distribution*) –

`double` **unur_distr_cont_eval_cdf** (double x , const *UNUR_DISTR** *distribution*) –

Evaluate the PDF, derivative of the PDF and the CDF, respectively, at x . Notice that *distribution* must not be the NULL pointer. If the corresponding function is not available for the distribution, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

IMPORTANT: In the case of a truncated standard distribution these calls always return the respective values of the *untruncated* distribution!

`int` **unur_distr_cont_set_pdfstr** (*UNUR_DISTR** *distribution*, const *char** *pdfstr*) –

This function provides an alternative way to set a PDF and its derivative of the *distribution*. *pdfstr* is a character string that contains the formula for the PDF, see [Section 3.3 \[Function String\], page 33](#), for details. See also the remarks for the `unur_distr_cont_set_pdf` call.

It is not possible to call this function twice or to call this function after a `unur_distr_cont_set_pdf` call.

`int` **unur_distr_cont_set_cdfstr** (*UNUR_DISTR** *distribution*, const *char** *cdfstr*) –

This function provides an alternative way to set a CDF; analogously to the `unur_distr_cont_set_pdfstr` call.

- char* unur_distr_cont_get_pdfstr** (const *UNUR_DISTR** *distribution*) –
char* unur_distr_cont_get_dpdfstr (const *UNUR_DISTR** *distribution*) –
char* unur_distr_cont_get_cdfstr (const *UNUR_DISTR** *distribution*) –
 Get pointer to respective string for PDF, derivate of PDF, and CDF of *distribution* that is given via the string interface. This call allocates memory to produce this string. It should be freed when it is not used any more.
- int unur_distr_cont_set_pdfparams** (*UNUR_DISTR** *distribution*, const double* *params*, int *n_params*) –
 Set array of parameters for *distribution*. There is an upper limit for the number of parameters *n_params*. It is given by the macro `UNUR_DISTR_MAXPARAMS` in ‘`unuran_config.h`’. (It is set to 5 by default but can be changed to any appropriate nonnegative number.) If *n_params* is negative or exceeds this limit no parameters are copied into the distribution object and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.
- For standard distributions from the UNURAN library the parameters are checked. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_cont_set_domain` call. If these parameters are invalid, then no parameters are set and 0 is returned. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.
- int unur_distr_cont_get_pdfparams** (const *UNUR_DISTR** *distribution*, const double** *params*) –
 Get number of parameters of the PDF and set pointer *params* to array of parameters. If no parameters are stored in the object, 0 is returned and *params* is set to NULL.
Important: Do **not** change the entries in *params*!
- int unur_distr_cont_set_domain** (*UNUR_DISTR** *distribution*, double *left*, double *right*) –
 Set the left and right borders of the domain of the distribution. This can also be used to truncate an existing distribution. For setting the boundary to +/- infinity use +/- `UNUR_INFINITY`. If *right* is not strictly greater than *left* no domain is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`.
- Important:* For some technical reasons it is assumed that the density is unimodal and thus monotone on either side of the mode! This is used in the case when the given mode is outside of the original domain. Then the mode is set to the corresponding boundary of the new domain.
- int unur_distr_cont_get_domain** (const *UNUR_DISTR** *distribution*, double* *left*, double* *right*) –
 Get the left and right borders of the domain of the distribution. If the domain is not set explicitly +/- `UNUR_INFINITY` is assumed and returned. No error is reported in this case.
- int unur_distr_cont_get_truncated** (const *UNUR_DISTR** *distribution*, double* *left*, double* *right*) –
 Get the left and right borders of the (truncated) domain of the distribution. For non-truncated distribution this call is equivalent to the `unur_distr_cont_get_domain` call. If the (truncated) domain is not set explicitly +/- `UNUR_INFINITY` is assumed and returned. No error is reported in this case.
- This call is only useful in connection with a `unur_get_distr` call to get the boundaries of the sampling region of a generator object.

```
int unur_distr_cont_set_hr (UNUR_DISTR* distribution, UNUR_FUNCT_CONT*
    hazard)
```

Set pointer to the hazard rate (HR) of the *distribution*.

The *hazard rate* (or failure rate) is a mathematical way of describing aging. If the lifetime X is a random variable with density $f(x)$ and CDF $F(x)$ the hazard rate $h(x)$ is defined as $h(x) = f(x) / (1-F(x))$. In other words, $h(x)$ represents the (conditional) rate of failure of a unit that has survived up to time x with probability $1-F(x)$. The key distribution is the exponential distribution as it has constant hazard rate of value 1. Hazard rates tending to infinity describe distributions with sub-exponential tails whereas distributions with hazard rates tending to zero have heavier tails than the exponential distribution.

It is important to note that all these functions must return a result for all floats x . In case of an overflow the PDF should return UNUR_INFINITY.

Important: Do not simply use $f(x) / (1-F(x))$, since this is numerically very unstable and results in numerical noise if $F(x)$ is (very) close to 1. Moreover, if the density $f(x)$ is known a generation method that uses the density is more appropriate.

It is not possible to change such a function. Once the HR is set it cannot be overwritten. This also holds when the HR is given by the `unur_distr_cont_set_hrstr` call. A new distribution object has to be used instead.

```
UNUR_FUNCT_CONT* unur_distr_cont_get_hr (const UNUR_DISTR*
    distribution)
```

Get the pointer to the hazard rate of the *distribution*. The pointer is of type `double funct(double x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the distribution, the NULL pointer is returned.

```
double unur_distr_cont_eval_hr (double x, const UNUR_DISTR* distribution)
```

Evaluate the hazard rate at x . Notice that *distribution* must not be the NULL pointer. If the corresponding function is not available for the distribution, UNUR_INFINITY is returned and `unur_errno` is set to UNUR_ERR_DISTR_DATA.

```
int unur_distr_cont_set_hrstr (UNUR_DISTR* distribution, const char* hrstr)
```

This function provides an alternative way to set a hazard rate and its derivative of the *distribution*. *hrstr* is a character string that contains the formula for the HR, see [Section 3.3 \[Function String\]](#), page 33, for details. See also the remarks for the `unur_distr_cont_set_hr` call.

It is not possible to call this function twice or to call this function after a `unur_distr_cont_set_hr` call.

```
char* unur_distr_cont_get_hrstr (const UNUR_DISTR* distribution)
```

Get pointer to string for HR of *distribution* that is given via the string interface. This call allocates memory to produce this string. It should be freed when it is not used any more.

Derived parameters

The following parameters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

```
int unur_distr_cont_set_mode (UNUR_DISTR* distribution, double mode)
```

Set mode of *distribution*.

```
int unur_distr_cont_upd_mode (UNUR_DISTR* distribution)
```

Recompute the mode of the *distribution*. This call works properly for distribution objects from the UNURAN library of standard distributions when the corresponding function is available. Otherwise a (slow) numerical mode finder is used. If it fails `unur_errno` is set to UNUR_ERR_DISTR_DATA.

double unur_distr_cont_get_mode (UNUR_DISTR* *distribution*) —
 Get mode of *distribution*. If the mode is not marked as known, `unur_distr_cont_upd_mode` is called to compute the mode. If this is not successful UNUR_INFINITY is returned and `unur_errno` is set to UNUR_ERR_DISTR_GET. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the distribution at all.)

int unur_distr_cont_set_pdfarea (UNUR_DISTR* *distribution*, double *area*) —
 Set the area below the PDF. If *area* is non-positive, no area is set and `unur_errno` is set to UNUR_ERR_DISTR_SET.
 For a distribution object created by the UNURAN library of standard distributions you always should use the `unur_distr_cont_upd_pdfarea`. Otherwise there might be ambiguous side-effects.

int unur_distr_cont_upd_pdfarea (UNUR_DISTR* *distribution*) —
 Recompute the area below the PDF of the distribution. It only works for distribution objects from the UNURAN library of standard distributions when the corresponding function is available. Otherwise `unur_errno` is set to UNUR_ERR_DISTR_DATA.
 This call sets the normalization constant such that the given PDF is the derivative of a given CDF, i.e. the area is 1. However for truncated distributions the area is smaller than 1.
 The call does not work for distributions from the UNURAN library of standard distributions with truncated domain when the CDF is not available.

double unur_distr_cont_get_pdfarea (UNUR_DISTR* *distribution*) —
 Get the area below the PDF of the distribution. If this area is not known, `unur_distr_cont_upd_pdfarea` is called to compute it. If this is not successful UNUR_INFINITY is returned and `unur_errno` is set to UNUR_ERR_DISTR_GET.

4.3 Continuous univariate order statistics

Function reference

UNUR_DISTR* **unur_distr_corder_new** (const UNUR_DISTR* *distribution*, int *n*, int *k*) —
 Create an object for order statistics of sample size *n* and rank *k*. *distribution* must be a pointer to a univariate continuous distribution. The resulting generator object is of the same type as of a `unur_distr_cont_new` call. (However it cannot be used to make an order statistics out of an order statistics.)

To have a PDF for the order statistics, the given distribution object must contain a CDF and a PDF. Moreover it is assumed that the given PDF is the derivative of the given CDF. Otherwise the area below the PDF of the order statistics is not computed correctly.

Important: There is no warning when the computed area below the PDF of the order statistics is wrong.

const UNUR_DISTR* **unur_distr_corder_get_distribution** (const UNUR_DISTR* *distribution*) —
 Get pointer to distribution object for underlying distribution.

Essential parameters

int unur_distr_corder_set_rank (UNUR_DISTR* *distribution*, int *n*, int *k*) –
 Change sample size *n* and rank *k* of order statistics. In case of invalid data, no parameters are changed and 0 is returned. The area below the PDF can be set to that of the underlying distribution by a `unur_distr_corder_upd_pdfarea` call.

int unur_distr_corder_get_rank (const UNUR_DISTR* *distribution*, int* *n*, int* *k*) –
 Get sample size *n* and rank *k* of order statistics. In case of error 0 is returned.

Additionally most of the set and get calls for continuous univariate distributions work. The most important exceptions are that the PDF and CDF cannot be changed and `unur_distr_cont_upd_mode` uses in any way a (slow) numerical method that might fail.

UNUR_FUNCT_CONT* **unur_distr_corder_get_pdf** (UNUR_DISTR* *distribution*) –
 UNUR_FUNCT_CONT* **unur_distr_corder_get_dpdf** (UNUR_DISTR* *distribution*) –
 UNUR_FUNCT_CONT* **unur_distr_corder_get_cdf** (UNUR_DISTR* *distribution*) –
 Get the respective pointer to the PDF, the derivative of the PDF and the CDF of the distribution, respectively. The pointer is of type `double funct(double x, UNUR_DISTR* distr)`. If the corresponding function is not available for the distribution, the NULL pointer is returned. See also `unur_distr_cont_get_pdf`. (Macro)

double unur_distr_corder_eval_pdf (double *x*, UNUR_DISTR* *distribution*) –
double unur_distr_corder_eval_dpdf (double *x*, UNUR_DISTR* *distribution*) –
double unur_distr_corder_eval_cdf (double *x*, UNUR_DISTR* *distribution*) –
 Evaluate the PDF, derivative of the PDF, and the CDF, respectively, at *x*. Notice that *distribution* must not be the NULL pointer. If the corresponding function is not available for the distribution, UNUR_INFINITY is returned and `unur_errno` is set to UNUR_ERR_DISTR_DATA. See also `unur_distr_cont_eval_pdf`. (Macro)
IMPORTANT: In the case of a truncated standard distribution these calls always return the respective values of the *untruncated* distribution!

int unur_distr_corder_set_pdfparams (UNUR_DISTR* *distribution*, double* *params*, int *n-params*) –
 Set array of parameters for underlying distribution. See `unur_distr_cont_set_pdfparams` for details. (Macro)

int unur_distr_corder_get_pdfparams (UNUR_DISTR* *distribution*, double** *params*) –
 Get number of parameters of the PDF of the underlying distribution and set pointer *params* to array of parameters. See `unur_distr_cont_get_pdfparams` for details. (Macro)

int unur_distr_corder_set_domain (UNUR_DISTR* *distribution*, double *left*, double *right*) –
 Set the left and right borders of the domain of the distribution. See `unur_distr_cont_set_domain` for details. (Macro)

int unur_distr_corder_get_domain (UNUR_DISTR* *distribution*, double* *left*, double* *right*) –
 Get the left and right borders of the domain of the distribution. See `unur_distr_cont_get_domain` for details. (Macro)

int unur_distr_corder_get_truncated (UNUR_DISTR* *distribution*, double* *left*, double* *right*) –
 Get the left and right borders of the (truncated) domain of the distribution. See `unur_distr_cont_get_truncated` for details. (Macro)

Derived parameters

The following parameters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

int unur_distr_corder_set_mode (UNUR_DISTR* *distribution*, double *mode*) –
 Set mode of distribution. See also `unur_distr_corder_set_mode`. (Macro)

double unur_distr_corder_upd_mode (UNUR_DISTR* *distribution*) –
 Recompute the mode of the distribution numerically. Notice that this routine is slow and might not work properly in every case. See also `unur_distr_cont_upd_mode` for further details. (Macro)

double unur_distr_corder_get_mode (UNUR_DISTR* *distribution*) –
 Get mode of distribution. See `unur_distr_cont_get_mode` for details. (Macro)

int unur_distr_corder_set_pdfarea (UNUR_DISTR* *distribution*, double *area*) –
 Set the area below the PDF. See `unur_distr_cont_set_pdfarea` for details. (Macro)

double unur_distr_corder_upd_pdfarea (UNUR_DISTR* *distribution*) –
 Recompute the area below the PDF of the distribution. It only works for order statistics for distribution objects from the UNURAN library of standard distributions when the corresponding function is available. `unur_distr_cont_upd_pdfarea` assumes that the PDF of the underlying distribution is normalized, i.e. it is the derivative of its CDF. Otherwise the computed area is wrong and there is **no** warning about this failure. See `unur_distr_cont_upd_pdfarea` for further details. (Macro)

double unur_distr_corder_get_pdfarea (UNUR_DISTR* *distribution*) –
 Get the area below the PDF of the distribution. See `unur_distr_cont_get_pdfarea` for details. (Macro)

4.4 Continuous empirical univariate distributions

Function reference

UNUR_DISTR* **unur_distr_cemp_new** (void) –
 Create a new (empty) object for empirical univariate continuous distribution.

Essential parameters

int unur_distr_cemp_set_data (UNUR_DISTR* *distribution*, const double* *sample*, int *n_sample*) –
 Set observed sample for empirical distribution.

int unur_distr_cemp_read_data (UNUR_DISTR* *distribution*, const char* *filename*) —

Read data from file ‘*filename*’. It reads the first double number from each line. Lines that do not start with +, -, ., or a digit are ignored. (Beware of lines starting with a blank!)

In case of an error (file cannot be opened, invalid string for double in line) no data are copied into the distribution object and 0 is returned.

int unur_distr_cemp_get_data (const UNUR_DISTR* *distribution*, const double** *sample*) —

Get number of samples and set pointer *sample* to array of observations. If no sample has been given, 0 is returned and *sample* is set to NULL.

Important: Do **not** change the entries in *params*!

4.5 Continuous multivariate distributions

Function reference

UNUR_DISTR* **unur_distr_cvec_new** (int *dim*) —

Create a new (empty) object for multivariate continuous distribution. *dim* is the number of components of the random vector (i.e. its dimension). It must be at least 2; otherwise `unur_distr_cont_new` should be used to create an object for a univariate distribution.

Essential parameters

int unur_distr_cvec_set_pdf (UNUR_DISTR* *distribution*, UNUR_FUNCT_CVEC* *pdf*) —

Set respective pointer to the PDF of the *distribution*. This function must be of type `double funct(const double *x, const UNUR_DISTR *distr)`, where *x* must be a pointer to a double array of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call).

It is not necessary that the given PDF is normalized, i.e. the integral need not be 1. Nevertheless the volume below the PDF can be provided by a `unur_distr_cvec_set_pdfvol` call.

int unur_distr_cvec_set_dpdf (UNUR_DISTR* *distribution*, UNUR_VFUNCT_CVEC* *dpdf*) —

Set pointer to the gradient of the PDF. The type of this function must be `int funct(double *result, const double *x, const UNUR_DISTR *distr)`, where *result* and *x* must be pointers to double arrays of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call). The gradient of the PDF is stored in the array *result*. The function should return 0 in case of an error and must return a non-zero value otherwise.

The given function must be proved the gradient of the function given by a `unur_distr_cvec_set_pdf` call.

UNUR_FUNCT_CVEC* **unur_distr_cvec_get_pdf** (const UNUR_DISTR* *distribution*) —

Get the pointer to the PDF of the *distribution*. The pointer is of type `double funct(const double *x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the *distribution*, the NULL pointer is returned.

`UNUR_VFUNCT_CVEC* unur_distr_cvec_get_dpdf (const UNUR_DISTR* distribution)` —

Get the pointer to the gradient of the PDF of the *distribution*. The pointer is of type `int double funct(double *result, const double *x, const UNUR_DISTR *distr)`. If the corresponding function is not available for the *distribution*, the NULL pointer is returned.

`double unur_distr_cvec_eval_pdf (const double* x, const UNUR_DISTR* distribution)` —

Evaluate the PDF of the *distribution* at *x*. *x* must be a pointers to a double arrays of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call) that contains the vector for which the function has to be evaluated.

Notice that *distribution* must not be the NULL pointer. If the corresponding function is not available for the *distribution*, `UNUR_INFINITY` is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`.

`int unur_distr_cvec_eval_dpdf (double* result, const double* x, const UNUR_DISTR* distribution)` —

Evaluate the gradient of the PDF of the *distribution* at *x*. The result is stored in the double array *result*. Both *result* and *x* must be pointer to double arrays of appropriate size (i.e. of the same size as given to the `unur_distr_cvec_new` call).

Notice that *distribution* must not be the NULL pointer. If the corresponding function is not available for the *distribution*, 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA` (*result* is left unmodified).

`int unur_distr_cvec_set_mean (UNUR_DISTR* distribution, const double* mean)` —

Set mean vector for multivariate *distribution*. *mean* must be a pointer to an array of size `dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. A NULL pointer for *mean* is interpreted as the zero vector (0, . . . ,0).

`const double* unur_distr_cvec_get_mean (const UNUR_DISTR* distribution)` —

Get the mean vector of the *distribution*. The function returns a pointer to an array of size `dim`. If the mean vector is not marked as known the NULL pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`. (However note that the NULL pointer also indicates the zero vector to avoid unnecessary computations. But then `unur_errno` is not set.)

Important: Do **not** modify the array that holds the mean vector!

`int unur_distr_cvec_set_covar (UNUR_DISTR* distribution, const double* covar)` —

Set covariance matrix for multivariate *distribution*. *covar* must be a pointer to an array of size `dim x dim`, where `dim` is the dimension returned by `unur_distr_get_dim`. The rows of the matrix have to be stored consecutively in this array.

covar must be a variance-covariance matrix of the *distribution*, i.e. it must be symmetric and positive definite and its diagonal entries (i.e. the variance of the components of the random vector) must be positive. There is no check for the positive definitnes yet.

A NULL pointer for *covar* is interpreted as the identity matrix.

`const double* unur_distr_cvec_get_covar (const UNUR_DISTR* distribution)` —

Get covariance matrix of *distribution*. The function returns a pointer to an array of size `dim x dim`. The rows of the matrix have to be stored consecutively in this array. If the covariance matrix is not marked as known the NULL pointer is returned and `unur_errno` is set to `UNUR_ERR_DISTR_GET`. (However note that the NULL pointer also indicates the identity matrix to avoid unnecessary computations. But then `unur_errno` is not set.)

Important: Do **not** modify the array that holds the covariance matrix!

```
int unur_distr_cvec_set_pdfparams (UNUR_DISTR* distribution, int par,
    const double* params, int n_params)
```

This function provides an interface for additional parameters for a multivariate *distribution* besides mean vector and covariance matrix which have their own calls.

It sets the parameter with number *par*. *par* indicates directly which of the parameters is set and must be a number between 0 and UNUR_DISTR_MAXPARAMS-1 (the upper limit of possible parameters defined in 'unuran_config.h'; it is set to 5 but can be changed to any appropriate nonnegative number.)

The entries of a this parameter are given by the array *params* of size *n_params*. Notice that using this interface an $(n \times m)$ -matrix has to be stored in an array of length $n_params = n$ times m ; where the rows of the matrix are stored consecutively in this array.

Due to great variety of possible parameters for a multivariate *distribution* there is no simpler interface.

If an error occurs no parameters are copied into the parameter object `unur_errno` is set to UNUR_ERR_DISTR_DATA.

```
int unur_distr_cvec_get_pdfparams (const UNUR_DISTR* distribution, int
    par, const double** params)
```

Get parameter of the PDF with number *par*. The pointer to the parameter array is stored in *params*, its size is returned by the function. If the requested parameter is not set, then 0 is returned and *params* is set to NULL.

Important: Do **not** change the entries in *params*!

Derived parameters

The following paramters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

```
int unur_distr_cvec_set_mode (UNUR_DISTR* distribution, const double*
    mode)
```

Set mode of *distribution*. *mode* must be a pointer to an array of the size returned by `unur_distr_get_dim`.

```
const double* unur_distr_cvec_get_mode (const UNUR_DISTR*
    distribution)
```

Get mode of *distribution*. The function returns a pointer to an array of the size returned by `unur_distr_get_dim`. If the mode is not marked as known the NULL pointer is returned and `unur_errno` is set to UNUR_ERR_DISTR_GET. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the *distribution* at all.)

Important: Do **not** modify the array that holds the mode!

```
int unur_distr_cvec_set_pdfvol (UNUR_DISTR* distribution, double volume)
```

Set the volume below the PDF. If *vol* is non-positive, no volume is set and `unur_errno` is set to UNUR_ERR_DISTR_SET.

```
double unur_distr_cvec_get_pdfvol (const UNUR_DISTR* distribution)
```

Get the volume below the PDF of the *distribution*. If this volume is not known, `unur_distr_cont_upd_pdfarea` is called to compute it. If this is not successful UNUR_INFINITY is returned and `unur_errno` is set to UNUR_ERR_DISTR_GET.

4.6 Continuous empirical multivariate distributions

Function reference

UNUR_DISTR* **unur_distr_cvemp_new** (int *dim*)

Create a new (empty) object for an empirical multivariate continuous distribution. *dim* is the number of components of the random vector (i.e. its dimension). It must be at least 2; otherwise `unur_distr_cemp_new` should be used to create an object for an empirical univariate distribution.

Essential parameters

int **unur_distr_cvemp_set_data** (UNUR_DISTR* *distribution*, const double* *sample*, int *n_sample*)

Set observed sample for empirical *distribution*. *sample* is an array of doubles of size `dim x n_sample`, where `dim` is the dimension of the *distribution* returned by `unur_distr_get_dim`. The data points must be stored consecutively in *sample*.

int **unur_distr_cvemp_read_data** (UNUR_DISTR* *distribution*, const char* *filename*)

Read data from file '*filename*'. It reads the first `dim` double numbers from each line, where `dim` is the dimension of the *distribution* returned by `unur_distr_get_dim`. Lines that do not start with +, -, ., or a digit are ignored. (Beware of lines starting with a blank!)

In case of an error (file cannot be opened, too few entries in a line, invalid string for double in line) no data are copied into the distribution object and 0 is returned.

In case of an error no data are copied into the distribuion object and 0 is returned.

int **unur_distr_cvemp_get_data** (const UNUR_DISTR* *distribution*, const double** *sample*)

Get number of samples and set pointer *sample* to array of observations. If no sample has been given, 0 is returned and *sample* is set to NULL. If successful *sample* points to an array of length `dim x n_sample`, where `dim` is the dimension of the distribution returned by `unur_distr_get_dim` and `n_sample` the return value of the function.

Important: Do **not** modify the array *sample*.

4.7 Discrete univariate distributions

Function reference

UNUR_DISTR* **unur_distr_discr_new** (void)

Create a new (empty) object for a univariate discrete distribution.

Essential parameters

There are two interfaces for discrete univariate distributions: Either provide a (finite) probability vector (PV). Or provide a probability mass function (PMF). For the latter case there are also a couple of derived parameters that are not required when a PV is given.

It is not possible to set both a PMF and a PV directly. However a PV can be computed from PMF by means of a `unur_distr_discr_make_pv` call. If both a PV and a PMF are given in the distribution object it depends on the generation method which of these is used.

```
int unur_distr_discr_set_pv (UNUR_DISTR* distribution, const double* pv, int
    n_pv)
    Set finite probability vector (PV) for a distribution. It is not necessary that the entries in
    the given PV sum to 1. n_pv must be positive. However there is no testing whether all
    entries in pv are non-negative.
    If no domain has been set, then the left boundary is set to 0, by default. If n_pv is too
    large, e.g. because left boundary + n_pv exceeds the range of integers, then the call fails.
    Notice it not possible to set both a PV and a PMF. (E.g., it is not possible to set a PV
    for a distribution from UNURAN library of standard distributions.)
```

```
int unur_distr_discr_make_pv (UNUR_DISTR* distribution)
    Compute a PV when a PMF is given. However when the domain is not given or is too
    large and the sum over the PMF is given then the (right) tail of the distribution is chopped
    off such that the probability for the tail region is less than  $10^{-8}$ . If the sum over the PMF
    is not given a PV of maximal length is computed.
    The maximal size of the created PV is bounded by the macro UNUR_MAX_AUTO_PV that is
    defined in 'unuran_config.h'.
    If successful, the length of the generated PV is returned. If the sum over the PMF on the
    chopped tail is not negligible small (i.e. greater than  $10^{-8}$  or unknown) than the negative
    of the length of the PV is returned and unur_errno is set to UNUR_ERR_DISTR_SET.
    Notice that when a discrete distribution object is created from scratch, then the left
    boundary of the PV is set to 0 by default.
    If computing a PV fails for some reasons, 0 is returned and unur_errno is set to UNUR_
    ERR_DISTR_SET.
```

```
int unur_distr_discr_get_pv (const UNUR_DISTR* distribution, const
    double** pv)
    Get length of PV of the distribution and set pointer pv to array of probabilities. If no PV
    is given, 0 is returned and pv is set to NULL.
    (It does not call unur_distr_discr_make_pv !)
```

```
int unur_distr_discr_set_pmf (UNUR_DISTR* distribution, UNUR_FUNCT_DISCR*
    pmf)
int unur_distr_discr_set_cdf (UNUR_DISTR* distribution, UNUR_FUNCT_DISCR*
    cdf)
    Set respective pointer to the PMF and the CDF of the distribution. These functions must
    be of type double funct(int k, const UNUR_DISTR *distr).
    It is important to note that all these functions must return a result for all integers k. Eg.,
    if the domain of a given PMF is the interval {1,2,3,...,100}, than the given function must
    return 0.0 for all points outside this interval.
    It is not possible to change such a function. Once the PMF or CDF is set it cannot be
    overwritten. A new distribution object has to be used instead.
    Notice it not possible to set both a PV and a PMF, i.e. it is not possible to use this call
    after a unur_distr_discr_set_pv call.
```

```
double unur_distr_discr_eval_pv (int k, const UNUR_DISTR* distribution)
double unur_distr_discr_eval_pmf (int k, const UNUR_DISTR* distribution)
double unur_distr_discr_eval_cdf (int k, const UNUR_DISTR* distribution)
    Evaluate the PV, PMF, and the CDF, respectively, at k. Notice that distribution must
    not be the NULL pointer. If no PV is set for the distribution, then unur_distr_discr_
    eval_pv behaves like unur_distr_discr_eval_pmf. If the corresponding function is not
    available for the distribution, UNUR_INFINITY is returned and unur_errno is set to UNUR_
    ERR_DISTR_DATA.
    IMPORTANT: In the case of a truncated standard distribution these calls always return
    the respective values of the untruncated distribution!
```

- int unur_distr_discr_set_pmfstr** (UNUR_DISTR* *distribution*, const char* *pmfstr*) –
- This function provides an alternative way to set a PMF of the *distribution*. *pmfstr* is a character string that contains the formula for the PMF, see [Section 3.3 \[Function String\]](#), [page 33](#), for details. See also the remarks for the `unur_distr_discr_set_pmf` call.
- It is not possible to call this function twice or to call this function after a `unur_distr_discr_set_pmf` call.
- int unur_distr_discr_set_cdfstr** (UNUR_DISTR* *distribution*, const char* *cdfstr*) –
- This function provides an alternative way to set a CDF; analogously to the `unur_distr_discr_set_pmfstr` call.
- char* unur_distr_discr_get_pmfstr** (const UNUR_DISTR* *distribution*) –
- char* unur_distr_discr_get_cdfstr** (const UNUR_DISTR* *distribution*) –
- Get pointer to respective string for PMF and CDF of *distribution* that is given via the string interface. This call allocates memory to produce this string. It should be freed when it is not used any more.
- int unur_distr_discr_set_pmfparams** (UNUR_DISTR* *distribution*, const double* *params*, int *n_params*) –
- Set array of parameters for *distribution*. There is an upper limit for the number of parameters *n_params*. It is given by the macro `UNUR_DISTR_MAXPARAMS` in ‘`unuran_config.h`’. (It is set to 5 but can be changed to any appropriate nonnegative number.) If *n_params* is negative or exceeds this limit no parameters are copied into the *distribution* object and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.
- For standard distributions from the UNURAN library the parameters are checked. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_cont_set_domain` call. If these parameters are invalid, then no parameters are set and 0 is returned. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.
- Important:* Integer parameter must be given as doubles.
- int unur_distr_discr_get_pmfparams** (const UNUR_DISTR* *distribution*, const double** *params*) –
- Get number of parameters of the PMF and set pointer *params* to array of parameters. If no parameters are stored in the object, 0 is returned and *params* is set to NULL.
- int unur_distr_discr_set_domain** (UNUR_DISTR* *distribution*, int *left*, int *right*) –
- Set the left and right borders of the domain of the *distribution*. This can also be used to truncate an existing distribution. For setting the boundary to +/- infinity use `INT_MAX` and `INT_MIN`, respectively. If *right* is not strictly greater than *left* no domain is set and `unur_errno` is set to `UNUR_ERR_DISTR_SET`. It is allowed to use this call to increase the domain. If the PV of the discrete distribution is used, then the right boundary is ignored (and internally set to *left* + size of PV - 1). Notice that `INT_MAX` and `INT_MIN` are interpreted as (minus) infinity. Default is [`INT_MIN`, `INT_MAX`] when a PMF is used for generation, and [0, size of PV - 1] when a PV is used.
- int unur_distr_discr_get_domain** (const UNUR_DISTR* *distribution*, int* *left*, int* *right*) –
- Get the left and right borders of the domain of the *distribution*. If the domain is not set explicitly the interval [`INT_MIN`, `INT_MAX`] is assumed and returned. When a PV is given then the domain is set automatically to [0, size of PV - 1].

Derived parameters

The following parameters **must** be set whenever one of the essential parameters has been set or changed (and the parameter is required for the chosen method).

int unur_distr_discr_set_mode (UNUR_DISTR* *distribution*, int *mode*) —
Set mode of *distribution*.

int unur_distr_discr_upd_mode (UNUR_DISTR* *distribution*) —
Recompute the mode of the *distribution*. This call works properly for distribution objects from the UNURAN library of standard distributions when the corresponding function is available. Otherwise a (slow) numerical mode finder is used. If it fails **unur_errno** is set to UNUR_ERR_DISTR_DATA.

int unur_distr_discr_get_mode (UNUR_DISTR* *distribution*) —
Get mode of *distribution*. If the mode is not marked as known, **unur_distr_discr_upd_mode** is called to compute the mode. If this is not successful INT_MAX is returned and **unur_errno** is set to UNUR_ERR_DISTR_GET. (There is no difference between the case where no routine for computing the mode is available and the case where no mode exists for the distribution at all.)

int unur_distr_discr_set_pmfsum (UNUR_DISTR* *distribution*, double *sum*) —
Set the sum over the PMF. If *sum* is non-positive, no sum is set and **unur_errno** is set to UNUR_ERR_DISTR_SET.
For a distribution object created by the UNURAN library of standard distributions you always should use the **unur_distr_discr_upd_pmfsum**. Otherwise there might be ambiguous side-effects.

int unur_distr_discr_upd_pmfsum (UNUR_DISTR* *distribution*) —
Recompute the sum over the PMF of the *distribution*. In most cases the normalization constant is recomputed and thus the sum is 1. This call only works for distribution objects from the UNURAN library of standard distributions when the corresponding function is available. Otherwise **unur_errno** is set to UNUR_ERR_DISTR_DATA.
The call does not work for distributions from the UNURAN library of standard distributions with truncated domain when the CDF is not available.

double unur_distr_discr_get_pmfsum (UNUR_DISTR* *distribution*) —
Get the sum over the PMF of the *distribution*. If this sum is not known, **unur_distr_discr_upd_pmfsum** is called to compute it. If this is not successful UNUR_INFINITY is returned and **unur_errno** is set to UNUR_ERR_DISTR_GET.

5 Methods for generating non-uniform random variates

5.1 Routines for all generator objects

Routines for all generator objects.

Function reference

- `UNUR_GEN*` **unur_init** (`UNUR_PAR*` *parameters*) —
 Initialize a generator object. All necessary information must be stored in the parameter object.
- Important:** If an error has occurred a NULL pointer is return. This must not be used for the sampling routines (this causes a segmentation fault).
- Always** check whether the call was successful or not!
- Important:* This call destroys the *parameter* object automatically. Thus it is not necessary/allowed to free it.
- `int` **unur_sample_discr** (`UNUR_GEN*` *generator*) —
`double` **unur_sample_cont** (`UNUR_GEN*` *generator*) —
`void` **unur_sample_vec** (`UNUR_GEN*` *generator*, `double*` *vector*) —
 Sample from generator object. The three routines depend on the type of the generator object (discrete or continuous univariate distribution, or multivariate distribution).
- Important:** These routines do **not** check if generator is an invalid NULL pointer.
- `void` **unur_free** (`UNUR_GEN*` *generator*) —
 Destroy (free) the given generator object.
- `int` **unur_get_dimension** (`const UNUR_GEN*` *generator*) —
 Get the number of dimension of a (multivariate) distribution. For a univariate distribution 1 is return.
- `const char*` **unur_get_genid** (`const UNUR_GEN*` *generator*) —
 Get identifier string for generator. If `UNUR_ENABLE_GENID` is not defined in `'unuran_config.h'` then only the method used for the generator is returned.
- `const UNUR_DISTR*` **unur_get_distr** (`const UNUR_GEN*` *generator*) —
 Get pointer to distribution object from generator object. This function should be used with extreme care. **Never** manipulate the distribution object returned by this call. (How should the poor generator object know what you have done?)

5.2 AUTO – Select method automatically

AUTO selects a an appropriate method for the given distribution object automatically. There are no parameters for this method, yet. But it is planned to give some parameter to describe the task for which the random variate generator is used for and thus make the choice of the generating method more appropriate. Notice that the required sampling routine for the generator object depends on the type of the given distribution object.

The chosen method also depends on the sample size for which the generator object will be used. If only a few random variates the order of magnitude of the sample size should be set via a `unur_auto_set_logss` call.

IMPORTANT: This is an experimental version and the method chosen may change in future releases of UNURAN.

For an example see [Section 2.1 \[Example: As short as possible\]](#), page 9.

Function reference

UNUR_PAR* `unur_auto_new` (`const UNUR_DISTR*` *distribution*)

Get default parameters for generator.

int `unur_auto_set_logss` (`UNUR_PAR*` *parameters*, `int` *logss*)

Set the order of magnitude for the size of the sample that will be generated by the generator, i.e., the the common logarithm of the sample size.

Default is 10.

Notice: This feature will be used in future releases of UNURAN only.

5.3 Methods for continuous univariate distributions

Overview of methods

Methods for **continuous univariate distributions**

sample with `unur_sample_cont`

method	PDF	dPDF	mode	area	other
AROU	x	x	[x]		T-concave
CSTD					build-in standard distribution
HINV	[x]	[x]			CDF
NINV	[x]				CDF
SROU	x		x	x	T-concave
SSR	x		x	x	T-concave
TABLE	x		x	[~]	all local extrema
TDR	x	x			T-concave
UTDR	x		x	~	T-concave

Example

```

/* ----- */
/* File: example_cont.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* Example how to sample from a continuous univariate */
/* distribution. */
/* */
/* We build a distribution object from scratch and sample. */

/* ----- */

/* Define the PDF and dPDF of our distribution. */
/* */
/* Our distribution has the PDF */
/* */
/* / 1 - x*x if |x| <= 1 */
/* f(x) = < */

```

```

/*          \ 0          otherwise          */
/*          */

/* The PDF of our distribution:          */
double mypdf( double x, const UNUR_DISTR *distr )
    /* The second argument ('distr') can be used for parameters */
    /* for the PDF. (We do not use parameters in our example.) */
{
    if (fabs(x) >= 1.)
        return 0.;
    else
        return (1.-x*x);
} /* end of mypdf() */

/* The derivative of the PDF of our distribution:          */
double mydpdf( double x, const UNUR_DISTR *distr )
{
    if (fabs(x) >= 1.)
        return 0.;
    else
        return (-2.*x);
} /* end of mydpdf() */

/* ----- */

int main()
{
    int    i;        /* loop variable          */
    double x;        /* will hold the random number          */

    /* Declare the three UNURAN objects.          */
    UNUR_DISTR *distr; /* distribution object          */
    UNUR_PAR    *par;  /* parameter object          */
    UNUR_GEN    *gen;  /* generator object          */

    /* Create a new distribution object from scratch.          */

    /* Get empty distribution object for a continuous distribution */
    distr = unur_distr_cont_new();

    /* Fill the distribution object -- the provided information */
    /* must fulfill the requirements of the method chosen below. */
    unur_distr_cont_set_pdf(distr, mypdf); /* PDF          */
    unur_distr_cont_set_dpdf(distr, mydpdf); /* its derivative */
    unur_distr_cont_set_mode(distr, 0.); /* mode          */
    unur_distr_cont_set_domain(distr, -1., 1.); /* domain          */

    /* Choose a method: TDR.          */
    par = unur_tdr_new(distr);

    /* Set some parameters of the method TDR.          */
    unur_tdr_set_variant_gw(par);
    unur_tdr_set_max_sqratio(par, 0.90);
    unur_tdr_set_c(par, -0.5);
    unur_tdr_set_max_intervals(par, 100);
    unur_tdr_set_cpoints(par, 10, NULL);

```

```

/* Create the generator object. */
gen = unur_init(par);

/* Notice that this call has also destroyed the parameter */
/* object 'par' as a side effect. */

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create */
/* another generator object. If you do not need it any more, */
/* it should be destroyed to free memory. */
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from */
/* the distribution. Eg.: */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you */
/* can destroy it. */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

Example (String API)

```

/* ----- */
/* File: example_cont_str.c */
/* ----- */
/* String API. */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* Example how to sample from a continuous univariate */
/* distribution. */

/* We use a generic distribution object and sample. */
/* */
/* The PDF of our distribution is given by */
/* */

```

```

/*          / 1 - x*x  if |x| <= 1          */
/* f(x) = <          */
/*          \ 0          otherwise          */
/*          */
/* ----- */

int main()
{
    int    i;      /* loop variable          */
    double x;     /* will hold the random number */

    /* Declare UNURAN generator object.          */
    UNUR_GEN *gen; /* generator object          */

    /* Create the generator object.          */
    /* Use a generic continuous distribution.  */
    /* Choose a method: TDR.                  */
    gen = unur_str2gen("distr = cont; pdf=\"1-x*x\"; domain=(-1,1); mode=0. & \
method=tdr; variant_gw; max_sqhratio=0.90; c=-0.5; \
max_intervals=100; cpoints=10");

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from */
    /* the distribution. Eg.:          */
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* When you do not need the generator object any more, you */
    /* can destroy it.          */
    unur_free(gen);

    exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

5.3.1 AROU – Automatic Ratio-Of-Uniforms method

Required: T-concave PDF, dPDF

Optional: mode

Speed: Set-up: slow, Sampling: fast

reference: [LJa00]

AROU is a variant of the ratio-of-uniforms method that uses the fact that the transformed region is convex for many distributions. It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

It is possible to use this method for correlation induction by setting an auxiliary uniform random number generator via the `unur_set_urng_aux` call. (Notice that this must be done after a possible `unur_set_urng` call.) When an auxiliary generator is used then the number of used uniform random numbers that is used up for one generated random variate is constant and equal to 1.

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on by calling `unur_arou_set_verify` and `unur_arou_chg_verify`, respectively. Notice however that sampling is (much) slower then.

For densities with modes not close to 0 it is suggested either to set the mode of the distribution or to use the `unur_arou_set_center` call to provide some information about the main part of the PDF to avoid numerical problems.

Function reference

- UNUR_PAR*** `unur_arou_new` (*const UNUR_DISTR** *distribution*) —
Get default parameters for generator.
- int** `unur_arou_set_max_sqratio` (*UNUR_PAR** *parameters*, *double max_ratio*) —
Set upper bound for the ratio (area inside squeeze) / (area inside envelope). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via adaptive rejection sampling. Use 0 if no construction points should be added after the setup. Use 1 if adding new construction points should not be stopped until the maximum number of construction points is reached.
Default is 0.99.
- double** `unur_arou_get_sqratio` (*const UNUR_GEN** *generator*) —
Get the current ratio (area inside squeeze) / (area inside envelope) for the generator. (In case of error 0 is returned.)
- double** `unur_arou_get_hatarea` (*const UNUR_GEN** *generator*) —
Get the area below the hat for the generator. (In case of an error 0 is returned.)
- double** `unur_arou_get_squeezearea` (*const UNUR_GEN** *generator*) —
Get the area below the squeeze for the generator. (In case of an error 0 is returned.)
- int** `unur_arou_set_max_segments` (*UNUR_PAR** *parameters*, *int max_segs*) —
Set maximum number of segments. No construction points are added *after* the setup when the number of segments succeeds *max_segs*.
Default is 100.
- int** `unur_arou_set_cpoints` (*UNUR_PAR** *parameters*, *int n_stp*, *const double** *stp*) —
Set construction points for enveloping polygon. If *stp* is NULL, then a heuristical rule of thumb is used to get *n_stp* construction points. This is the default behavior when this routine is not called. The (default) number of construction points is 30, then.

- int unur_arou_set_center** (UNUR_PAR* *parameters*, double *center*) –
 Set the center (approximate mode) of the PDF. It is used to find construction points by means of a heuristical rule of thumb. If the mode is given the center is set equal to the mode.
 It is suggested to use this call to provide some information about the main part of the PDF to avoid numerical problems, when the most important part of the PDF is not close to 0.
 By default the mode is used as center if available. Otherwise 0 is used.
- int unur_arou_set_usecenter** (UNUR_PAR* *parameters*, int *usecenter*) –
 Use the center as construction point. Default is TRUE.
- int unur_arou_set_guidefactor** (UNUR_PAR* *parameters*, double *factor*) –
 Set factor for relative size of the guide table for indexed search (see also method DGT [Section 5.7.3 \[DGT\], page 98](#)). It must be greater than or equal to 0. When set to 0, then sequential search is used.
 Default is 2.
- int unur_arou_set_verify** (UNUR_PAR* *parameters*, int *verify*) –
int unur_arou_chg_verify (UNUR_GEN* *generator*, int *verify*) –
 Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PDF}(x) \leq \text{hat}(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
 Default is FALSE.
- int unur_arou_set_pedantic** (UNUR_PAR* *parameters*, int *pedantic*) –
 Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not T-concave.
 With *pedantic* being TRUE, the sampling routine is then exchanged by a routine that simply returns `UNUR_INFINITY`. Otherwise the new point is not added to the list of construction points. At least the hat function remains T-concave.
 Setting *pedantic* to FALSE allows sampling from a distribution which is “almost” T-concave and small errors are tolerated. However it might happen that the hat function cannot be improved significantly. When the hat function that has been constructed by the `unur_init` call is extremely large then it might happen that the generation times are extremely high (even hours are possible in extremely rare cases).
 Default is FALSE.

5.3.2 CSTD – Continuous STandard distributions

Required: standard distribution from UNURAN library (see [Chapter 7 \[Standard distributions\], page 107](#)).

Speed: Set-up: fast, Sampling: depends on distribution and generator

CSTD is a wrapper for special generators for continuous univariate standard distributions. It only works for distributions in the UNURAN library of standard distributions (see [Chapter 7 \[Standard distributions\], page 107](#)). If a distribution object is provided that is build from scratch, or if no special generator for the given standard distribution is provided, the NULL pointer is returned.

For some distributions more than one special generator (*variants*) is possible. These can be chosen by a `unur_cstd_set_variant` call. For possible variants see [Chapter 7 \[Standard distributions\], page 107](#). However the following are common to all distributions:

UNUR_STDGEN_DEFAULT
the default generator.

UNUR_STDGEN_FAST
the fastest available special generator.

UNUR_STDGEN_INVERSION
the inversion method (if available).

Notice that the variant **UNUR_STDGEN_FAST** for a special generator may be slower than one of the universal algorithms! Additional variants may exist for particular distributions.

Sampling from truncated distributions (which can be constructed by changing the default domain of a distribution by means of `unur_distr_cont_set_domain` or `unur_cstd_chg_truncated` calls) is possible but requires the inversion method.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object.

Function reference

UNUR_PAR* unur_cstd_new (const *UNUR_DISTR** *distribution*) —

Get default parameters for new generator. It requires a distribution object for a continuous univariant distribution from the UNURAN library of standard distributions (see [Chapter 7 \[Standard distributions\]](#), page 107).

Using a truncated distribution is allowed only if the inversion method is available and selected by the `unur_cstd_set_variant` call immediately after creating the parameter object. Use a `unur_distr_cont_set_domain` call to get a truncated distribution. To change the domain of a (truncated) distribution of a generator use the `unur_cstd_chg_truncated` call.

int unur_cstd_set_variant (UNUR_PAR* *parameters*, unsigned *variant*) —

Set variant (special generator) for sampling from a given distribution. For possible variants see [Chapter 7 \[Standard distributions\]](#), page 107.

Common variants are **UNUR_STDGEN_DEFAULT** for the default generator, **UNUR_STDGEN_FAST** for (one of the) fastest implemented special generators, and **UNUR_STDGEN_INVERSION** for the inversion method (if available). If the selected variant number is not implemented, then 0 is returned and the variant is not changed.

int unur_cstd_chg_pdfparams (UNUR_GEN* *generator*, double* *params*, int *n_params*) —

Change array of parameters of the distribution in a given generator object. If the given parameters are invalid for the distribution, no parameters are set. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.

int unur_cstd_chg_truncated (UNUR_GEN* *generator*, double *left*, double *right*) —

Change left and right border of the domain of the (truncated) distribution. This is only possible if the inversion method is used. Otherwise this call has no effect and 0 is returned. Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call.

Important: If the CDF is (almost) the same for *left* and *right* and (almost) equal to 0 or 1, then the truncated domain is not changed and the call returns 0.

Notice: If the parameters of the distribution has been changed by a `unur_cstd_chg_pdfparams` call it is recommended to set the truncated domain again, since the former call might change the domain of the distribution but not update the values for the boundaries of the truncated distribution.

5.3.3 HINV – Hermite interpolation based INVersion of CDF

Required: CDF

Optional: PDF, dPDF

Speed: Set-up: (very) slow, Sampling: (very) fast

HINV is a variant of numerical inversion, where the inverse CDF is approximated using Hermite interpolation. These splines have to be computed in a setup step. However, it only works for distributions with bounded domain; for distributions with unbounded domain the tails are chopped off such that the probability for the tail regions is small compared to the given u-resolution. For finding these cut points the algorithm starts with the region $[-1.e20, 1.e20]$. For the exceptional case where this might be too small (or one knows this region and wants to avoid this search heuristics) it can be changed using the `unur_hinv_set_boundary` call.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an `unur_hinv_chg_truncated` call.

This method is not exact, as it only produces random variates of the approximated distribution. Nevertheless, the numerical error in "u-direction" (i.e. for $X = \text{"approximate inverse CDF"}(U) \mid |U - \text{CDF}(X)|$) can be controlled by means of the `unur_hinv_set_u_resolution`. Notice that very small values of the u-resolution are possible but may increase the cost for the setup step.

As the possible maximal error is only estimated in the setup it may be necessary to set some special design points for computing the Hermite interpolation to guarantee that the maximal u-error can not be bigger than desired. Such points are points where the density is not differentiable or has a local extremum. Notice that there is no necessity to do so. However, if you do not provide these points to the algorithm there might be a small chance that the approximation error is larger than the given u-resolution, or that the required number of intervals is larger than necessary. Setting such design points can be done using the `unur_hinv_set_cpoin` call. If the mode for a unimodal distribution is set in the distribution object this mode is automatically used as design-point if the `unur_hinv_set_cpoin` call is not used.

Function reference

`UNUR_PAR*` `unur_hinv_new` (`const UNUR_DISTR*` *distribution*)

Get default parameters for generator.

`int` `unur_hinv_set_order` (`UNUR_PAR*` *parameters*, `int` *order*)

Set order of Hermite interpolation. Valid orders are 1, 3, and 5. Notice that *order* greater than 1 requires the density of the distribution, and *order* greater than 3 even requires the derivative of the density. Using *order* 1 results for most distributions in a huge number of intervals and is therefore not recommended. If the maximal error in u-direction is very small (say smaller than $1.e-10$), *order* 5 is recommended as it leads to considerably fewer design points.

Default is 3 if the density is given and 1 otherwise.

`int` `unur_hinv_set_u_resolution` (`UNUR_PAR*` *parameters*, `double` *u_resolution*)

Set maximal error in u-direction. However, the given u-error must not be smaller than machine epsilon (`DBL_EPSILON`) and should not be too close to this value. As the resolution of most uniform random number sources is $2^{(-32)} = 2.3e-10$, a value of $1.e-10$ leads to an inversion algorithm that could be called exact. For most simulations slightly bigger values for the maximal error are enough as well.

Default is 10^{-8} .

- int unur_hinv_set_cpoints** (UNUR_PAR* *parameters*, const double* *stp*, int *n_stp*) –
- Set starting construction points (nodes) for Hermite interpolation.
- As the possible maximal error is only estimated in the setup it may be necessary to set some special design points for computing the Hermite interpolation to guarantee that the maximal u-error can not be bigger than desired. We suggest to include as special design points all local extrema of the density, all points where the density is not differentiable, and isolated points inside of the domain with density 0. If there is an interval with density constant equal to 0 inside of the given domain of the density, both endpoints of this interval should be included as special design points. Notice that there is no necessity to do so. However, if these points are not provided to the algorithm the approximation error might be larger than the given u-resolution, or the required number of intervals could be larger than necessary.
- Important:* Notice that the given points must be in increasing order and they must be disjoint.
- Important:* The boundary point of the computational region must not be given in this list! Points outside the boundary of the computational region are ignored.
- Default is for unimodal densities - if known - the mode of the density, if it is not equal to the border of the domain.
- int unur_hinv_set_boundary** (UNUR_PAR* *parameters*, double *left*, double *right*) –
- Set the left and right boundary of the computation interval. The approximate CDF is only constructed inside this interval. The probability outside of this region must not be of computational relevance. Of course +/- UNUR_INFINITY is not allowed.
- Important:* This call does not change the domain of the given distribution itself. But it restricts the domain for the resulting random variates.
- Default is 1.e20.
- int unur_hinv_set_guidefactor** (UNUR_PAR* *parameters*, double *factor*) –
- Set factor for relative size of the guide table for indexed search (see also method DGT [Section 5.7.3 \[DGT\], page 98](#)). It must be greater than or equal to 0. When set to 0, then sequential search is used.
- Default is 1.
- int unur_hinv_set_max_intervals** (UNUR_PAR* *parameters*, int *max_ivs*) –
- Set maximum number of intervals. No generator object is created if the necessary number of intervals for the Hermite interpolation exceeds *max_ivs*. It is used to prevent the algorithm to eat up all memory for very badly shaped CDFs.
- Default is 1000000 (1.e6).
- int unur_hinv_get_n_intervals** (const UNUR_GEN* *generator*) –
- Get number of nodes (design points) used for Hermite interpolation in the generator object. The number of intervals is the number of nodes - 1. It returns 0 in case of an error.
- double unur_hinv_eval_approxinvcdf** (const UNUR_GEN* *generator*, double *u*) –
- Evaluate Hermite interpolation of inverse CDF at *u*. If *u* is out of the domain [0,1] then *unur_errno* is set to UNUR_ERR_DOMAIN and the respective bound of the domain of the distribution are returned (which is -UNUR_INFINITY or UNUR_INFINITY in the case of unbounded domains).
- Notice:* This function always evaluates the inverse CDF of the given distribution. A call to *unur_hinv_chg_truncated* call has no effect.

int unur_hinv_chg_truncated (UNUR_GEN* *generator*, double *left*, double *right*) –

Changes the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. The tables of splines are not recomputed. Thus it might happen that the relative error for the generated variates from the truncated distribution is greater than the bound for the non-truncated distribution. This call also fails when the CDF values of the boundary points are too close, i.e. when only a few different floating point numbers would be computed due to round-off errors with floating point arithmetic.

It returns 0 in case of an error.

Important: Always check the return code since the domain is not changed in case of an error.

int unur_hinv_estimate_error (const UNUR_GEN* *generator*, int *samplesize*,
double* *max_error*, double* *MAE*) –

Estimate maximal u-error and mean absolute error (MAE) for *generator* by means of Monte-Carlo simulation with sample size *samplesize*. The results are stored in *max_error* and *MAE*, respectively.

It returns 1 if successful.

5.3.4 HRB – Hazard Rate Bounded

Required: bounded hazard rate

Optional: upper bound for hazard rate

Speed: Set-up: fast, Sampling: slow

Generates random variate with given hazard rate. It requires that the distribution object contains a hazard rate and it requires an upper bound for the hazard rate which must be set using `unur_hrb_set_upperbound` call. If no such upper bound is given it is assumed that the upper bound can be achieved by evaluating the hazard rate at the left hand boundary of the domain of the distribution.

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, the left border must not be negative. Otherwise it is set to 0. This is also the default if no domain is given at all. For computational reasons the right border is always set to UNUR_INFINITY independently of the given domain. Thus for domains bounded from right the function for computing the hazard rate should return UNUR_INFINITY right of this domain.

For distributions with decreasing hazard rates use method HRD, which is faster. For distributions with increasing hazard rate method HRI is required.

Function reference

UNUR_PAR* **unur_hrb_new** (const UNUR_DISTR* *distribution*) –

Get default parameters for generator.

int unur_hrb_set_upperbound (UNUR_PAR* *parameters*, double *upperbound*) –

Set upper bound for hazard rate. If this call is not used it is assumed that the maximum of the hazard rate is achieved at the left hand boundary of the domain of the distribution.

```
int unur_hrb_set_verify (UNUR_PAR* parameters, int verify)      -
int unur_hrb_chg_verify (UNUR_GEN* generator, int verify)      -
    Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by
    the given bound, then unur_errno is set to UNUR_ERR_GEN_CONDITION.

    Default is FALSE.
```

5.3.5 HRD – Hazard Rate Decreasing

Required: decreasing (non-increasing) hazard rate

Speed: Set-up: fast, Sampling: slow

Generates random variate with given non-increasing hazard rate. It is necessary that the distribution object contains this hazard rate. Decreasing hazard rate implies that the corresponding PDF of the distribution has heavier tails than the exponential distribution (which has constant hazard rate).

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, only the left hand boundary is used. For computational reasons the right hand boundary is always reset to `UNUR_INFINITY`. If no domain is given by the user then the left hand boundary is set to 0.

For distributions which do not have decreasing hazard rates but are bounded from above use method HRB. For distributions with increasing hazard rate method HRI is required.

Function reference

```
UNUR_PAR* unur_hrd_new (const UNUR_DISTR* distribution)      -
    Get default parameters for generator.
```

```
int unur_hrd_set_verify (UNUR_PAR* parameters, int verify)  -
int unur_hrd_chg_verify (UNUR_GEN* generator, int verify)  -
    Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by
    the given bound, then unur_errno is set to UNUR_ERR_GEN_CONDITION.

    Default is FALSE.
```

5.3.6 HRI – Hazard Rate Increasing

Required: increasing (non-decreasing) hazard rate

Speed: Set-up: fast, Sampling: slow

Generates random variate with given non-increasing hazard rate. It is necessary that the distribution object contains this hazard rate. Increasing hazard rate implies that the corresponding PDF of the distribution has heavier tails than the exponential distribution (which has constant hazard rate).

It is important to note that the domain of the distribution can be set via a `unur_distr_cont_set_domain` call. However, only the left hand boundary is used. For computational reasons the right hand boundary is always reset to `UNUR_INFINITY`. If no domain is given by the user then the left hand boundary is set to 0.

For distributions with decreasing hazard rate method HRD is required. For distributions which do not have increasing or decreasing hazard rates but are bounded from above use method HRB.

Function reference

- UNUR_PAR* unur_hri_new** (const *UNUR_DISTR** *distribution*) –
 Get default parameters for generator.
- int unur_hri_set_p0** (UNUR_PAR* *parameters*, double *p0*) –
 Set design point for algorithm. It is used to split the domain of the distribution. Values for *p0* close to the expectation of the distribution results in a relatively good performance of the algorithm. It is important that the hazard rate at this point must be greater than 0 and less than UNUR_INFINITY.
 Default: left boundary of domain + 1.
- int unur_hri_set_verify** (UNUR_PAR* *parameters*, int *verify*) –
int unur_hri_chg_verify (UNUR_GEN* *generator*, int *verify*) –
 Turn verifying of algorithm while sampling on/off. If the hazard rate is not bounded by the given bound, then *unur_errno* is set to UNUR_ERR_GEN_CONDITION.
 Default is FALSE.

5.3.7 NINV – Numerical INVersion

Required: CDF

Optional: PDF

Speed: Set-up: optional, Sampling: (very) slow

NINV is the implementation of numerical inversion. For finding the root it is possible to choose between Newton's method and the regula falsi (combined with interval bisectioning). The regula falsi requires only the CDF while Newton's method also requires the PDF.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an *unur_ninv_chg_truncated* call.

To speed up the marginal generation time a table with suitable starting points can be computed in the setup. Using such a table can be switched on by means of a *unur_ninv_set_table* call where the table size is given as a parameter. The table is still useful when the (truncated) domain is changed often, since it is computed for the domain of the given distribution. (It is not possible to enlarge this domain.) If it is necessary to recalculate the table during sampling, the command *unur_ninv_chg_table* can be used.

As a rule of thumb using such a table is appropriate when the number of generated points exceeds the table size by a factor of 100.

The standard number of iterations of NINV should be enough for all reasonable cases. Nevertheless it is possible to adjust the maximal number of iterations with the command *unur_ninv_[set|chg]_max_iter*.

To speed up this method (at the expense of the accuracy) it is possible to change the maximum error allowed in *x* with *unur_ninv_[set|chg]_x_resolution*.

NINV tries to use proper starting values for both the regula falsi and Newton's method. Of course the user might have more knowledge about the properties of the underlying distribution and is able to share his wisdom with NINV using the command *unur_ninv_[set|chg]_start*.

It is also possible to change the parameters of the given distribution by a *unur_ninv_chg_pdfparams* call. If a table exists, it will be recomputed immediately.

Default algorithm is regula falsi. It is slightly slower but numerically much more stable than Newton's algorithm.

It might happen that NINV aborts *unur_sample_cont* without computing the correct value (because the maximal number iterations has been exceeded). Then the last approximate value for *x* is returned (with might be fairly false) and *unur_error* is set to UNUR_ERR_GEN_SAMPLING.

Function reference

- UNUR_PAR* unur_ninv_new** (const *UNUR_DISTR** *distribution*) –
 Get default parameters for generator.
- int unur_ninv_set_useregula** (*UNUR_PAR** *parameters*) –
 Switch to regula falsi combined with interval bisectioning. (This the default.)
- int unur_ninv_set_usenewton** (*UNUR_PAR** *parameters*) –
 Switch to Newton's method. Notice that it is numerically less stable than regula falsi. It is not possible to invert the CDF for a particular random number U when calling `unur_sample_cont`, `unur_error` is set to `UNUR_ERR_` and `UNUR_INFINITY` is returned. Thus it is recommended to check `unur_error` before using the result of the sampling routine.
- int unur_ninv_set_max_iter** (*UNUR_PAR** *parameters*, int *max_iter*) –
 Set number of maximal iterations. Default is 40.
- int unur_ninv_set_x_resolution** (*UNUR_PAR** *parameters*, double *x_resolution*) –
 Set maximal relative error. Default is 10^{-8} .
- int unur_ninv_set_start** (*UNUR_PAR** *parameters*, double *left*, double *right*) –
 Set starting points. If not set, suitable values are chosen automatically.
- | | | |
|---------------|------------------------------|-------------------------------|
| Newton: | <i>left</i> : | starting point |
| Regula falsi: | <i>left</i> , <i>right</i> : | boundary of starting interval |
- If the starting points are not set then the following points are used by default:
- | | | |
|---------------|----------------|------------------------------------|
| Newton: | <i>left</i> : | $\text{CDF}(\textit{left}) = 0.5$ |
| Regula falsi: | <i>left</i> : | $\text{CDF}(\textit{left}) = 0.1$ |
| | <i>right</i> : | $\text{CDF}(\textit{right}) = 0.9$ |
- If *left* == *right*, then UNURAN always uses the default starting points!
- int unur_ninv_set_table** (*UNUR_PAR** *parameters*, int *no_of_points*) –
 Generates a table with *no_of_points* points containing suitable starting values for the iteration. The value of *no_of_points* must be at least 10 (otherwise it will be set to 10 automatically).
- The table points are chosen such that the CDF at these points form an equidistance sequence in the interval (0,1).
- If a table is used, then the starting points given by `unur_ninv_set_start` are ignored.
- No table is used by default.
- int unur_ninv_chg_max_iter** (*UNUR_GEN** *generator*, int *max_iter*) –
 Change the maximum number of iterations.
- int unur_ninv_chg_x_resolution** (*UNUR_GEN** *generator*, double *x_resolution*) –
 Change the maximal relative error in x.
- int unur_ninv_chg_start** (*UNUR_GEN** *gen*, double *left*, double *right*) –
 Change the starting points for numerical inversion. If *left*==*right*, then UNURAN uses the default starting points (see `unur_ninv_set_start`).

int unur_ninv_chg_table (UNUR_GEN* *gen*, int *no_of_points*)
 Recomputes a table as described in `unur_ninv_set_table`.

int unur_ninv_chg_truncated (UNUR_GEN* *gen*, double *left*, double *right*)
 Changes the borders of the domain of the (truncated) distribution.

Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. Moreover the starting point(s) will not be changed.

Important: If the CDF is (almost) the same for *left* and *right* and (almost) equal to 0 or 1, then the truncated domain is *not* changed and the call returns 0.

Notice: If the parameters of the distribution has been changed by a `unur_ninv_chg_pdfparams` call it is recommended to set the truncated domain again, since the former call might change the domain of the distribution but not update the values for the boundaries of the truncated distribution.

int unur_ninv_chg_pdfparams (UNUR_GEN* *generator*, double* *params*, int *n_params*)
 Change array of parameters of the distribution in a given generator object.

For standard distributions from the UNURAN library the parameters are checked. If these are invalid, then 0 is returned. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.

For other distributions *params* is simply copied into to distribution object. It is only checked that *n_params* does not exceed the maximum number of parameters allowed. Then 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.

5.3.8 SROU – Simple Ratio-Of-Uniforms method

Required: T-concave PDF, mode, area

Speed: Set-up: fast, Sampling: slow

reference: [LJa01] [LJa02]

SROU is based on the ratio-of-uniforms method but uses universal inequalities for constructing a (universal) bounding rectangle. It works for all T-concave distributions (including log-concave and T-concave distributions with $T(x) = -1/\sqrt{x}$).

It requires the PDF, the (exact) location of the mode and the area below the given PDF. Moreover an (optional) parameter *r* can be given, to adjust the generator to the given distribution. This parameter is strongly related parameter *c* for transformed density rejection via the formula $c = -r/(r+1)$. *r* should be set as small as possible but the given density must be T-c-concave for the corresponding *c*. The default setting for *r* is 1.

The parameter *r* can be any value larger than or equal to 1. The rejection constant depends on the chosen parameter *r* but not on the particular distribution. It is 4 for *r* equal to 1 and higher for higher values of *r*. It is important to note that different algorithms for different values of *r*: If *r* equal to 1 this is much faster than the algorithm for *r* greater than 1.

Optionally the CDF at the mode can be given to increase the performance of the algorithm by means of the `unur_srou_set_cdfatmode` call. Then the rejection constant is reduced by 1/2 and (if *r*=1) even a universal squeeze can (but need not be) used. A way to increase the performance of the algorithm when the CDF at the mode is not provided is the usage of the mirror principle (only if *r*=1). However using squeezes and using the mirror principle is not recommended in general (see below).

If the exact location of the mode is not known, then use the approximate location and provide the (exact) value of the PDF at the mode by means of the `unur_srou_set_pdfatmode` call. But

then `unur_srou_set_cdfatmode` must not be used. Notice if no mode is given at all, a (slow) numerical mode finder will be used.

If the (exact) area below the PDF is not known, then an upper bound can be used instead (which of course increases the rejection constant). But then the squeeze flag must not be set and `unur_srou_set_cdfatmode` must not be used.

It is even possible to give an upper bound for the area below the PDF only. However then the (upper bound for the) area below the PDF has to be multiplied by the ratio between the upper bound and the lower bound of the PDF at the mode. Again setting the squeeze flag and using `unur_srou_set_cdfatmode` is not allowed.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object using the `unur_srou_chg_pdfparams` and `unur_srou_chg_domain` call, respectively. But then `unur_srou_chg_pdfarea`, `unur_srou_chg_mode` and `unur_srou_chg_cdfatmode` have to be used to reset the corresponding figures whenever they have changed. If the PDF at the mode has been provided by a `unur_srou_set_pdfatmode` call, additionally `unur_srou_chg_pdfatmode` must be used (otherwise this call is not necessary since then this figure is computed directly from the PDF). If any of mode, PDF or CDF at the mode, or the area below the mode has been changed, then `unur_srou_reinit` must be executed. (Otherwise the generator produces garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on by calling `unur_srou_set_verify` and `unur_srou_chg_verify`, respectively. Notice however that sampling is (a little bit) slower then.

Function reference

`UNUR_PAR*` **unur_srou_new** (`const UNUR_DISTR*` *distribution*) —

Get default parameters for generator.

`int` **unur_srou_reinit** (`UNUR_GEN*` *generator*) —

Update an existing generator object after the distribution has been modified. It must be executed whenever the parameters or the domain of the distributions have been changed (see below). It is faster than destroying the existing object and building a new one from scratch. If reinitialization has been successful 1 is returned, in case of a failure 0 is returned.

`int` **unur_srou_set_r** (`UNUR_PAR*` *parameters*, `double` *r*) —

Set parameter *r* for transformation. Only values greater than or equal to 1 are allowed. The performance of the generator decreases when *r* is increased. On the other hand *r* must not be set to small, since the given density must be T_c-concave for $c = -r/(r+1)$.

Notice: If *r* is set to 1 a simpler and much faster algorithm is used then for *r* greater than one.

For computational reasons values of *r* that are greater than 1 but less than 1.01 are always set to 1.01.

Default is 1.

`int` **unur_srou_set_cdfatmode** (`UNUR_PAR*` *parameters*, `double` *Fmode*) —

Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_srou_chg_cdfatmode` has to be used to update this value.

Default: not set.

- int unur_srou_set_pdfatmode** (UNUR_PAR* *parameters*, double *fmode*) —
 Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters of the distribution.
IMPORTANT: This call has to be executed after a possible call of `unur_srou_set_r`.
 Default: not set.
- int unur_srou_set_usesqueeze** (UNUR_PAR* *parameters*, int *usesqueeze*) —
 Set flag for using universal squeeze (default: off). Using squeezes is only useful when the evaluation of the PDF is (extremely) expensive. Using squeezes is automatically disabled when the CDF at the mode is not given (then no universal squeezes exist).
 Default is FALSE.
- int unur_srou_set_usemirror** (UNUR_PAR* *parameters*, int *usemirror*) —
 Set flag for using mirror principle (default: off). Using the mirror principle is only useful when the CDF at the mode is not known and the evaluation of the PDF is rather cheap compared to the marginal generation time of the underlying uniform random number generator. It is automatically disabled when the CDF at the mode is given. (Then there is no necessity to use the mirror principle. However disabling is only done during the initialization step but not at a re-initialization step.)
 Default is FALSE.
- int unur_srou_set_verify** (UNUR_PAR* *parameters*, int *verify*) —
int unur_srou_chg_verify (UNUR_GEN* *generator*, int *verify*) —
 Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PDF}(x) \leq \hat{\text{PDF}}(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
 Default is FALSE.
- int unur_srou_chg_pdfparams** (UNUR_GEN* *generator*, double* *params*, int *n_params*) —
 Change array of parameters of the distribution in a given generator object.
 For standard distributions from the UNURAN library the parameters are checked. If these are invalid, then 0 is returned. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.
 For other distributions *params* is simply copied into to distribution object. It is only checked that *n_params* does not exceed the maximum number of parameters allowed. Then 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.
- int unur_srou_chg_domain** (UNUR_GEN* *generator*, double *left*, double *right*) —
 Change left and right border of the domain of the (truncated) distribution. If the mode changes when the domain of the (truncated) distribution is changed, then a corresponding `unur_srou_chg_mode` is required. (There is no checking whether the domain is set or not as in the `unur_init` call.)
- int unur_srou_chg_mode** (UNUR_GEN* *generator*, double *mode*) —
 Change mode of distribution. `unur_srou_reinit` must be executed before sampling from the generator again.

- int unur_srou_upd_mode** (UNUR_GEN* *generator*) –
 Recompute the mode of the distribution. See `unur_distr_cont_upd_mode` for more details. `unur_srou_reinit` must be executed before sampling from the generator again.
- int unur_srou_chg_cdfatmode** (UNUR_GEN* *generator*, double *Fmode*) –
 Change CDF at mode of distribution. `unur_srou_reinit` must be executed before sampling from the generator again.
- int unur_srou_chg_pdfatmode** (UNUR_GEN* *generator*, double *fmode*) –
 Change PDF at mode of distribution. `unur_srou_reinit` must be executed before sampling from the generator again.
- int unur_srou_chg_pdfarea** (UNUR_GEN* *generator*, double *area*) –
 Change area below PDF of distribution. `unur_srou_reinit` must be executed before sampling from the generator again.
- int unur_srou_upd_pdfarea** (UNUR_GEN* *generator*) –
 Recompute the area below the PDF of the distribution. It only works when a distribution objects from the UNURAN library of standard distributions is used (see [Chapter 7 \[Standard distributions\]](#), page 107). Otherwise `unur_errno` is set to `UNUR_ERR_DISTR_DATA`. `unur_srou_reinit` must be executed before sampling from the generator again.

5.3.9 SSR – Simple Setup Rejection

Required: T-concave PDF, mode, area

Speed: Set-up: fast, Sampling: slow

reference: [LJa01]

SSR is an acceptance/rejection method that uses universal inequalities for constructing (universal) hats and squeezes. It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

It requires the PDF, the (exact) location of the mode and the area below the given PDF. The rejection constant is 4 for all T-concave distributions with unbounded domain and is less than 4 when the domain is bounded. Optionally the CDF at the mode can be given to increase the performance of the algorithm by means of the `unur_ssr_set_cdfatmode` call. Then the rejection constant is reduced by one half and even a universal squeeze can (but need not be) used. However using squeezes is not recommended unless the evaluation of the PDF is rather expensive. (The mirror principle is not implemented.)

If the exact location of the mode is not known, then use the approximate location and provide the (exact) value of the PDF at the mode by means of the `unur_ssr_set_pdfatmode` call. But then `unur_ssr_set_cdfatmode` must not be used. Notice if no mode is given at all, a (slow) numerical mode finder will be used.

If the (exact) area below the PDF is not known, then an upper bound can be used instead (which of course increases the rejection constant). But then the squeeze flag must not be set and `unur_ssr_set_cdfatmode` must not be used.

It is even possible to give an upper bound for the PDF only. However then the (upper bound for the) area below the PDF has to be multiplied by the ratio between the upper bound and the lower bound of the PDF at the mode. Again setting the squeeze flag and using `unur_ssr_set_cdfatmode` is not allowed.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object using the `unur_ssr_chg_pdfparams` and `unur_ssr_chg_domain` call, respectively. But then `unur_ssr_chg_pdfarea`, `unur_ssr_chg_mode` and `unur_ssr_chg_cdfatmode` have to be used to reset the corresponding figures whenever they have changed. If the PDF at the mode has been provided by a `unur_ssr_set_pdfatmode` call, additionally `unur_ssr_chg_pdfatmode` must be used (otherwise this call is not necessary since then this

figure is computed directly from the PDF). If any of mode, PDF or CDF at the mode, or the area below the mode has been changed, then `unur_ssr_reinit` must be executed. (Otherwise the generator produces garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on by calling `unur_ssr_set_verify` and `unur_ssr_chg_verify`, respectively. Notice however that sampling is (a little bit) slower then.

Function reference

- UNUR_PAR*** `unur_ssr_new` (*const UNUR_DISTR** *distribution*) —
Get default parameters for generator.
- int** `unur_ssr_reinit` (*UNUR_GEN** *generator*) —
Update an existing generator object after the distribution has been modified. It must be executed whenever the parameters or the domain of the distributions has been changed (see below). It is faster than destroying the existing object and build a new one from scratch. If reinitialization has been successful 1 is returned, in case of a failure 0 is returned.
- int** `unur_ssr_set_cdfatmode` (*UNUR_PAR** *parameters*, *double Fmode*) —
Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_ssr_chg_cdfatmode` has to be used to update this value.
Default: not set.
- int** `unur_ssr_set_pdfatmode` (*UNUR_PAR** *parameters*, *double fmode*) —
Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters for the distribution.
Default: not set.
- int** `unur_ssr_set_usesqueeze` (*UNUR_PAR** *parameters*, *int usesqueeze*) —
Set flag for using universal squeeze (default: off). Using squeezes is only useful when the evaluation of the PDF is (extremely) expensive. Using squeezes is automatically disabled when the CDF at the mode is not given (then no universal squeezes exist).
Default is FALSE.
- int** `unur_ssr_set_verify` (*UNUR_PAR** *parameters*, *int verify*) —
int `unur_ssr_chg_verify` (*UNUR_GEN** *generator*, *int verify*) —
Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PDF}(x) \leq \hat{\text{PDF}}(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
Default is FALSE.
- int** `unur_ssr_chg_pdfparams` (*UNUR_GEN** *generator*, *double** *params*, *int n_params*) —
Change array of parameters of the distribution in a given generator object.
For standard distributions from the UNURAN library the parameters are checked. If these are invalid, then 0 is returned. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. Notice that

optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.

For other distributions *params* is simply copied into to distribution object. It is only checked that *n_params* does not exceed the maximum number of parameters allowed. Then 0 is returned and *unur_errno* is set to *UNUR_ERR_DISTR_NPARAMS*.

- int *unur_ssr_chg_domain*** (*UNUR_GEN* generator*, *double left*, *double right*) —
 Change left and right border of the domain of the distribution. If the mode changes when the domain of the distribution is changed, then a correspondig *unur_ssr_chg_mode* is required. (There is no domain checking as in the *unur_init* call.)
- int *unur_ssr_chg_mode*** (*UNUR_GEN* generator*, *double mode*) —
 Change mode of distribution. *unur_ssr_reinit* must be executed before sampling from the generator again.
- int *unur_ssr_upd_mode*** (*UNUR_GEN* generator*) —
 Recompute the mode of the distribution. See *unur_distr_cont_upd_mode* for more details. *unur_srou_reinit* must be executed before sampling from the generator again.
- int *unur_ssr_chg_cdfatmode*** (*UNUR_GEN* generator*, *double Fmode*) —
 Change CDF at mode of distribution. *unur_ssr_reinit* must be executed before sampling from the generator again.
- int *unur_ssr_chg_pdfatmode*** (*UNUR_GEN* generator*, *double fmode*) —
 Change PDF at mode of distribution. *unur_ssr_reinit* must be executed before sampling from the generator again.
- int *unur_ssr_chg_pdfarea*** (*UNUR_GEN* generator*, *double area*) —
 Change area below PDF of distribution. *unur_ssr_reinit* must be executed before sampling from the generator again.
- int *unur_ssr_upd_pdfarea*** (*UNUR_GEN* generator*) —
 Recompute the area below the PDF of the distribution. It only works when a distribution objects from the UNURAN library of standard distributions is used (see [Chapter 7 \[Standard distributions\]](#), [page 107](#)). Otherwise *unur_errno* is set to *UNUR_ERR_DISTR_DATA*. *unur_srou_reinit* must be executed before sampling from the generator again.

5.3.10 TABL – a TABLE method with piecewise constant hats

- Required:* PDF, all local extrema
Optional: approximate area
Speed: Set-up: slow, Sampling: fast
reference: [AJa93] [AJa95]

TABL is an acceptance/rejection method that uses piecewise constant hat and squeezes. Immediate acceptance of points below the squeeze reduces the expected number of uniform random numbers to less than two and makes this method extremely fast.

The method only works for distributions with bounded domain. Thus for unbounded domains the left and right tails are cut off (the cutting points can be set by a *unur_tabl_set_boundary* call). This is no problem when the probability of falling into these tail regions is beyond computational relevance.

The method works for all probability density functions where the regions of monotonicity (called slopes) are given. This can be done explicitly by a `unur_tabl_set_slopes` call. If (and only if) no slopes are given, the domain and the mode of the PDF are used to compute the slopes. If neither slopes nor the mode and the domain are given initializing of the generator fails.

In the setup first the equal area rule is used to construct a hat function, i.e., the interval boundaries are chosen such that the area below each interval is equal to a given fraction of the total area below the given PDF. This fraction can be set by a `unur_tabl_set_areafraction` call. Additionally these intervals are split until the maximum number of intervals is reached or the ratio between the area below squeeze and the area below the hat is exceeded.

It is possible to switch off this second setup step. Then adaptive rejection sampling is used to split these intervals. There are three variants for adaptive rejection sampling. These differ in the way how an interval is split:

1. use the generated point to split the interval;
2. use the mean point of the interval; or
3. use the arcmean point.

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_tabl_set_verify` and `unur_tabl_chg_verify`, respectively. Notice however that sampling is (much) slower then.

Function reference

UNUR_PAR* `unur_tabl_new` (*const UNUR_DISTR** *distribution*)

Get default parameters for generator.

int `unur_tabl_set_variant_setup` (*UNUR_PAR** *parameters*, *unsigned variant*)

Set variant for setup. Two modes are possible for *variant*:

- 1 only use the equal area rule to construct hat.
- 2 additionally split the intervals created by the equal area rule until the maximum number of intervals is reached or the ratio between the area below the squeeze and the area below the hat is exceeded.

Default is variant 2.

int `unur_tabl_set_variant_splitmode` (*UNUR_PAR** *parameters*, *unsigned splitmode*)

There are three variants for adaptive rejection sampling. These differ in the way how an interval is split:

- `splitmode 1`
use the generated point to split the interval.
- `splitmode 2`
use the mean point of the interval.
- `splitmode 3`
use the arcmean point; suggested for distributions with heavy tails.

Default is `splitmode 2`.

int `unur_tabl_set_max_sqhratio` (*UNUR_PAR** *parameters*, *double max_ratio*)

Set upper bound for the ratio (area below squeeze) / (area below hat). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via adaptive rejection sampling. Use 0 if no construction points should be added after the setup. Use 1 if added new construction points should not be stopped until the maximum number of construction points is reached. If *max_ratio* is close to one, many construction points are used.

Default is 0.9.

- double unur_tabl_get_sqhratio** (const UNUR_GEN* generator) –
 Get the current ratio (area below squeeze) / (area below hat) for the generator. (In case of an error 0 is returned.)
- double unur_tabl_get_hatarea** (const UNUR_GEN* generator) –
 Get the area below the hat for the generator. (In case of an error 0 is returned.)
- double unur_tabl_get_squeezearea** (const UNUR_GEN* generator) –
 Get the area below the squeeze for the generator. (In case of an error 0 is returned.)
- int unur_tabl_set_max_intervals** (UNUR_PAR* parameters, int max_ivs) –
 Set maximum number of intervals. No construction points are added after the setup when the number of intervals succeeds *max_ivs*.
 Default is 1000.
- int unur_tabl_set_areafraction** (UNUR_PAR* parameters, double fraction) –
 Set parameter for equal area rule. During the setup a piecewise constant hat is constructed, such that the area below each of these pieces (strips) is the same and equal to the (given) area below the distribution times *fraction* (which must be greater than zero).
Important: If the area below the PDF is not set, then 1 is assumed.
 Default is 0.1.
- int unur_tabl_set_nstp** (UNUR_PAR* parameters, int n_stp) –
 Set number of construction points for the hat function. *n_stp* must be greater than zero. After the setup there are about *n_stp* construction points. However it might be larger when a small fraction is given by the `unur_tabl_set_areafraction` call. It also might be smaller for some variants.
 Default is 30.
- int unur_tabl_set_slopes** (UNUR_PAR* parameters, const double* slopes, int n_slopes) –
 Set slopes for the PDF. A slope <a,b> is an interval [a,b] or [b,a] where the PDF is monotone and $\text{PDF}(a) \geq \text{PDF}(b)$. The list of slopes are given by an array *slopes* where each consecutive tuples (i.e. (`slopes[0]`, `slopes[1]`), (`slopes[2]`, `slopes[3]`), etc.) is one slopes. Slopes must be sorted (i.e. both `slopes[0]` and `slopes[1]` must not be greater than any entry of the slope (`slopes[2]`, `slopes[3]`), etc.) and must not overlapping. Otherwise no slopes are set and `unur_errno` is set to `UNUR_ERR_PAR_SET`.
Notice: *n_slopes* is the number of slopes (and not the length of the array *slopes*).
Notice that setting slopes resets the given domain for the distribution. However in case of a standard distribution the area below the PDF is not updated.
- int unur_tabl_set_guidefactor** (UNUR_PAR* parameters, double factor) –
 Set factor for relative size of the guide table for indexed search (see also method DGT [Section 5.7.3 \[DGT\], page 98](#)). It must be greater than or equal to 0. When set to 0, then sequential search is used.
 Default is 1.
- int unur_tabl_set_boundary** (UNUR_PAR* parameters, double left, double right) –
 Set the left and right boundary of the computation interval. The piecewise hat is only constructed inside this interval. The probability outside of this region must not be of computational relevance. Of course +/- `UNUR_INFINITY` is not allowed.
 Default is `1.e20`.

```
int unur_tabl_set_verify (UNUR_PAR* parameters, int verify)      -
int unur_tabl_chg_verify (UNUR_GEN* generator, int verify)      -
    Turn verifying of algorithm while sampling on/off. If the condition  $\text{squeeze}(x) \leq \text{PDF}(x) \leq \hat{\text{hat}}(x)$  is violated for some  $x$  then unur_errno is set to UNUR_ERR_GEN_CONDITION. However notice that this might happen due to round-off errors for a few values of  $x$  (less than 1%).
    Default is FALSE.
```

5.3.11 TDR – Transformed Density Rejection

Required: T-concave PDF, dPDF
Optional: mode
Speed: Set-up: slow, Sampling: fast
reference: [GWA92] [HWA95]

TDR is an acceptance/rejection method that uses the concavity of a transformed density to construct hat function and squeezes automatically. Such PDFs are called T-concave. Currently the following transformations are implemented and can be selected by setting their `c`-values by a `unur_tdr_set_c` call:

`c = 0` $T(x) = \log(x)$
`c = -0.5` $T(x) = -1/\sqrt{x}$ (Default)

In future releases the transformations $T(x) = -(x)^c$ will be available for any c with $0 > c > -1$. Notice that if a PDF is T-concave for a c then it also T-concave for every $c' < c$. However the performance decreases when c' is smaller than c . For computational reasons we suggest the usage of `c = -0.5` (this is the default). For $c \leq -1$ the hat is not bounded any more if the domain of the PDF is unbounded. But in the case of a bounded domain using method TABL is preferred to a TDR with $c < -1$ (except in a few special cases).

We offer three variants of the algorithm.

GW squeezes between construction points
PS squeezes proportional to hat function (Default)
IA same as variant PS but uses a composition method with “immediate acceptance” in the region below the squeeze.

GW has a slightly faster setup but higher marginal generation times. PS is faster than GW. IA uses less uniform random numbers and is therefore faster than PS.

It is also possible to evaluate the CDF of the hat distribution directly using the `unur_tdr_eval_invcdfhat` call.

There are lots of parameters for these methods, see below.

It is possible to use this method for correlation induction by setting an auxiliary uniform random number generator via the `unur_set_urng_aux` call. (Notice that this must be done after a possible `unur_set_urng` call.) When an auxiliary generator is used then the number of uniform random numbers from the first URNG that are used for one generated random variate is constant and given in the following table:

GW ... 2
 PS ... 2
 IA ... 1

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_tdr_set_verify` and `unur_tdr_chg_verify`, respectively. Notice however that sampling is (much) slower then.

For densities with modes not close to 0 it is suggested either to set the mode of the distribution or to use the `unur_tdr_set_center` call for provide some information about the main part of the PDF to avoid numerical problems.

It is possible to use this method for generating from truncated distributions. It even can be changed for an existing generator object by an `unur_tdr_chg_truncated` call.

Important: The ratio between the area below the hat and the area below the squeeze changes when the sampling region is restricted. Especially it becomes (very) small when sampling from the (far) tail of the distribution. Then it is better to create a new generator object for the tail of the distribution only.

Function reference

- UNUR_PAR*** `unur_tdr_new` (*const UNUR_DISTR** *distribution*) —
Get default parameters for generator.
- int** `unur_tdr_set_c` (*UNUR_PAR** *parameters*, *double c*) —
Set parameter *c* for transformation T. Currently only values between 0 and -0.5 are allowed. If *c* is between 0 and -0.5 it is set to -0.5.
Default is -0.5.
- int** `unur_tdr_set_variant_gw` (*UNUR_PAR** *parameters*) —
Use original version with squeezes between construction points as proposed by Gilks & Wild (1992).
- int** `unur_tdr_set_variant_ps` (*UNUR_PAR** *parameters*) —
Use squeezes proportional to the hat function. This is faster than the original version. This is the default.
- int** `unur_tdr_set_variant_ia` (*UNUR_PAR** *parameters*) —
Use squeezes proportional to the hat function together with a composition method that required less uniform random numbers.
- int** `unur_tdr_set_usedars` (*UNUR_PAR** *parameters*, *int usedars*) —
If *usedars* is set to `TRUE`, “derandomized adaptive rejection sampling” (DARS) is used in setup. Intervals, where the area between hat and squeeze is too large compared to the average area between hat and squeeze over all intervals, are splitted. This procedure is repeated until the ratio between squeeze and hat exceeds the bound given by `unur_tdr_set_max_sqhratio` call or the maximum number of intervals is reached. Moreover, it also aborts when no more intervals can be found for splitting.
For finding splitting points the following rules are used (in this order, i.e., is if the first rule cannot be applied, the next one is used):
1. Use the expected value of adaptive rejection sampling.
 2. Use the arc-mean rule (a mixture of arithmetic mean and harmonic mean).
 3. Use the arithmetic mean of the interval boundaries.
- Notice, however, that for unbounded intervals neither rule 1 nor rule 3 can be used.
As an additional feature, it is possible to choose among these rules. If *usedars* is set to 1 or `TRUE` the expected point (rule 1) is used (it switches to rule 2 for a particular interval if rule 1 cannot be applied). If it is set to 2 the arc-mean rule is used. If it is set to 3 the mean is used. Notice that rule 3 can only be used if the domain of the distribution is bounded. It is faster than the other two methods but for heavy-tailed distribution and large domain the hat converges extremely slowly.
The default depends on the given construction points. If the user has provided such points via a `unur_tdr_set_cpoints` call, then *usedars* is set to `FALSE` by default, i.e., there is no further splitting. If the user has only given the number of construction points (or only uses the default number), then *usedars* is set to `TRUE` (i.e., use rule 1).

- int unur_tdr_set_darsfactor** (UNUR_PAR* *parameters*, double *factor*) –
 Set factor for “derandomized adaptive rejection sampling”. This factor is used to determine the intervals that are “too large”, that is, all intervals where the area between squeeze and hat is larger than *factor* times the average area over all intervals between squeeze and hat. Notice that all intervals are splitted when *factor* is set to 0., and that there is no splitting at all when *factor* is set to UNUR_INFINITY.
 Default is 0.99. There is no need to change this parameter.
- int unur_tdr_chg_truncated** (UNUR_GEN* *gen*, double *left*, double *right*) –
 Change the borders of the domain of the (truncated) distribution.
 Notice that the given truncated domain must be a subset of the domain of the given distribution. The generator always uses the intersection of the domain of the distribution and the truncated domain given by this call. The hat function will not be changed.
Important: The ratio between the area below the hat and the area below the squeeze changes when the sampling region is restricted. Especially it becomes (very) small when sampling from the (far) tail of the distribution. Then it is better to create a generator object for the tail of distribution only.
Important: This call does not work for variant IA (immediate acceptance). In this case UNURAN switches *automatically* to variant PS.
Important: It is not a good idea to use adaptave rejection sampling while sampling from a domain that is a strict subset of the domain that has been used to construct the hat. For that reason adaptive adding of construction points is *automatically disabled* by this call.
Important: If the CDF of the hat is (almost) the same for *left* and *right* and (almost) equal to 0 or 1, then the truncated domain is not chanced and the call returns 0.
- int unur_tdr_set_max_sqhratio** (UNUR_PAR* *parameters*, double *max_ratio*) –
 Set upper bound for the ratio (area below squeeze) / (area below hat). It must be a number between 0 and 1. When the ratio exceeds the given number no further construction points are inserted via adaptive rejection sampling. Use 0 if no construction points should be added after the setup. Use 1 if added new construction points should not be stopped until the maximum number of construction points is reached.
 Default is 0.99.
- double unur_tdr_get_sqhratio** (const UNUR_GEN* *generator*) –
 Get the current ratio (area below squeeze) / (area below hat) for the generator. (In case of an error 0 is returned.)
- double unur_tdr_get_hatarea** (const UNUR_GEN* *generator*) –
 Get the area below the hat for the generator. (In case of an error 0 is returned.)
- double unur_tdr_get_squeezearea** (const UNUR_GEN* *generator*) –
 Get the area below the squeeze for the generator. (In case of an error 0 is returned.)
- int _unur_tdr_is_ARS_running** (const UNUR_GEN* *generator*) –
 Check whether more points will be added by adaptive rejection sampling. (Internal call)
- int unur_tdr_set_max_intervals** (UNUR_PAR* *parameters*, int *max_ivs*) –
 Set maximum number of intervals. No construction points are added after the setup when the number of intervals succeeds *max_ivs*. It is increased automatically to twice the number of construction points if this is larger.
 Default is 100.

- int unur_tdr_set_cpoints** (UNUR_PAR* *parameters*, int *n_stp*, const double* *stp*) –
 Set construction points for the hat function. If *stp* is NULL than a heuristic rule of thumb is used to get *n_stp* construction points. This is the default behavior.
 The default number of construction points is 30.
- int unur_tdr_set_center** (UNUR_PAR* *parameters*, double *center*) –
 Set the center (approximate mode) of the PDF. It is used to find construction points by means of a heuristical rule of thumb. If the mode is given the center is set equal to the mode.
 It is suggested to use this call to provide some information about the main part of the PDF to avoid numerical problems.
 By default the mode is used as center if available. Otherwise 0 is used.
- int unur_tdr_set_usecenter** (UNUR_PAR* *parameters*, int *usecenter*) –
 Use the center as construction point. Default is TRUE.
- int unur_tdr_set_usemode** (UNUR_PAR* *parameters*, int *usemode*) –
 Use the (exact!) mode as construction point. Notice that the behavior of the algorithm is different to simply adding the mode in the list of construction points via a `unur_tdr_set_cpoints` call. In the latter case the mode is treated just like any other point. However when *usemode* is TRUE, the tangent in the mode is always set to 0. Then the hat of the transformed density can never cut the x-axis which must never happen if $c < 0$, since otherwise the hat would not be bounded.
 Default is TRUE.
- int unur_tdr_set_guidefactor** (UNUR_PAR* *parameters*, double *factor*) –
 Set factor for relative size of the guide table for indexed search (see also method DGT [Section 5.7.3 \[DGT\], page 98](#)). It must be greater than or equal to 0. When set to 0, then sequential search is used.
 Default is 2.
- int unur_tdr_set_verify** (UNUR_PAR* *parameters*, int *verify*) –
int unur_tdr_chg_verify (UNUR_GEN* *generator*, int *verify*) –
 Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PDF}(x) \leq \text{hat}(x)$ is violated for some x then `unur_errno` is set to UNUR_ERR_GEN_CONDITION. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
 Default is FALSE.
- int unur_tdr_set_pedantic** (UNUR_PAR* *parameters*, int *pedantic*) –
 Sometimes it might happen that `unur_init` has been executed successfully. But when additional construction points are added by adaptive rejection sampling, the algorithm detects that the PDF is not T-concave.
 With *pedantic* being TRUE, the sampling routine is exchanged by a routine that simply returns UNUR_INFINITY. Otherwise the new point is not added to the list of construction points. At least the hat function remains T-concave.
 Setting *pedantic* to FALSE allows sampling from a distribution which is “almost” T-concave and small errors are tolerated. However it might happen that the hat function cannot be improved significantly. When the hat functions that has been constructed by the `unur_init` call is extremely large then it might happen that the generation times are extremely high (even hours are possible in extremely rare cases).
 Default is FALSE.

double unur_tdr_eval_invcdfhat (const UNUR_GEN* generator, double u,
double* hx, double* fx, double* sqx)

Evaluate the CDF of the hat distribution at u . As a side effect the values of the hat, the density, and the squeeze at the computed point x are stored in hx , fx , and sqx , respectively. However, these computations are suppressed if the corresponding variable is set to NULL.

If u is out of the domain $[0,1]$ then `unur_errno` is set to `UNUR_ERR_DOMAIN` and the respective bound of the domain of the distribution are returned (which is `-UNUR_INFINITY` or `UNUR_INFINITY` in the case of unbounded domains).

Important: This call does not work for variant IA (immediate acceptance). In this case the hat CDF is evaluated as if variant PS is used.

Notice: This function always evaluates the inverse CDF of the hat distribution. A call to `unur_tdr_chg_truncated` call has no effect.

5.3.12 UTDR – Universal Transformed Density Rejection

Required: T-concave PDF, mode, approximate area

Speed: Set-up: moderate, Sampling: Moderate

reference: [HWa95]

UTDR is based on the transformed density rejection and uses three almost optimal points for constructing hat and squeezes. It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

It requires the PDF and the (exact) location of the mode. Notice that if no mode is given at all, a (slow) numerical mode finder will be used. Moreover the approximate area below the given PDF is used. (If no area is given for the distribution the algorithm assumes that it is approximately 1.) The rejection constant is bounded from above by 4 for all T-concave distributions.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object by using the `unur_utdr_chg_pdfparams` and `unur_utdr_chg_domain` call, respectively. But then `unur_utdr_chg_mode` and `unur_utdr_chg_pdfarea` have to be used to reset the corresponding figures whenever these have changed. Before sampling from the distribution again, `unur_utdr_reinit` must be executed. (Otherwise the generator produces garbage).

When the PDF does not change at the mode for varying parameters, then this value can be set with `unur_utdr_set_pdfatmode` to avoid some computations. Since this value will not be updated any more when the parameters of the distribution are changed, the `unur_utdr_chg_pdfatmode` call is necessary to do this manually.

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_utdr_set_verify` and `unur_utdr_chg_verify`, respectively. Notice however that sampling is slower then.

Function reference

UNUR_PAR* **unur_utdr_new** (const UNUR_DISTR* distribution)
Get default parameters for generator.

int **unur_utdr_reinit** (UNUR_GEN* generator)

Update an existing generator object after the distribution has been modified. It must be executed whenever the parameters or the domain of the distributions has been changed (see below). It is faster than destroying the existing object and building a new one from scratch. If reinitialization has been successful 1 is returned, in case of a failure 0 is returned.

Important: Do not use the *generator* object for sampling after a failed reinit, since otherwise it may produce garbage.

- int unur_utdr_set_pdfatmode** (UNUR_PAR* *parameters*, double *fmode*) —
 Set pdf at mode. When set, the PDF at the mode is never changed. This is to avoid additional computations, when the PDF does not change when parameters of the distributions vary. It is only useful when the PDF at the mode does not change with changing parameters for the distribution.
 Default: not set.
- int unur_utdr_set_cpfactor** (UNUR_PAR* *parameters*, double *cp_factor*) —
 Set factor for position of left and right construction point. The *cp_factor* is used to find almost optimal construction points for the hat function. There is no need to change this factor in almost all situations.
 Default is 0.664.
- int unur_utdr_set_deltafactor** (UNUR_PAR* *parameters*, double *delta*) —
 Set factor for replacing tangents by secants. higher factors increase the rejection constant but reduces the risk of serious round-off errors. There is no need to change this factor it almost all situations.
 Default is 1.e-5.
- int unur_utdr_set_verify** (UNUR_PAR* *parameters*, int *verify*) —
int unur_utdr_chg_verify (UNUR_GEN* *generator*, int *verify*) —
 Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PDF}(x) \leq \text{hat}(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
 Default is FALSE.
- int unur_utdr_chg_pdfparams** (UNUR_GEN* *generator*, double* *params*, int *n_params*) —
 Change array of parameters of the distribution in a given generator object.
 For standard distributions from the UNURAN library the parameters are checked. If these are invalid, then 0 is returned. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.
 For other distributions *params* is simply copied into to distribution object. It is only checked that *n_params* does not exceed the maximum number of parameters allowed. Then 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.
- int unur_utdr_chg_domain** (UNUR_GEN* *generator*, double *left*, double *right*) —
 Change left and right border of the domain of the (truncated) distribution. If the mode changes when the domain of the (truncated) distribution is changed, then a corresponding `unur_utdr_chg_mode` is required. (There is no domain checking as in the `unur_init` call.)
- int unur_utdr_chg_mode** (UNUR_GEN* *generator*, double *mode*) —
 Change mode of distribution. `unur_utdr_reinit` must be executed before sampling from the generator again.
- int unur_utdr_upd_mode** (UNUR_GEN* *generator*) —
 Recompute the mode of the distribution. See `unur_distr_cont_upd_mode` for more details. `unur_srou_reinit` must be executed before sampling from the generator again.

```

int unur_utdr_chg_pdfatmode (UNUR_GEN* generator, double fmode)
    Change PDF at mode of distribution. unur_utdr_reinit must be executed before sam-
    pling from the generator again.

int unur_utdr_chg_pdfarea (UNUR_GEN* generator, double area)
    Change area below PDF of distribution. unur_utdr_reinit must be executed before
    sampling from the generator again.

int unur_utdr_upd_pdfarea (UNUR_GEN* generator)
    Recompute the area below the PDF of the distribution. It only works when a distribution
    objects from the UNURAN library of standard distributions is used (see Chapter 7 \[Stan-
    dard distributions\], page 107). Otherwise unur_errno is set to UNUR_ERR_DISTR_DATA.
    unur_srou_reinit must be executed before sampling from the generator again.

```

5.4 Methods for continuous empirical univariate distributions

Overview of methods

Methods for **continuous empirical univariate distributions**

sample with `unur_sample_cont`

EMPK: Requires an observed sample. EMPL: Requires an observed sample.

Example

```

/* ----- */
/* File: example_emp.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* Example how to sample from an empirical continuous univariate */
/* distribution. */

/* ----- */

int main()
{
    int    i;
    double x;

    /* data points */
    double data[15] = { -0.1,  0.05, -0.5,  0.08,  0.13,\
        -0.21,-0.44, -0.43, -0.33, -0.3, \
        0.18, 0.2,  -0.37, -0.29, -0.9 };

    /* Declare the three UNURAN objects. */
    UNUR_DISTR *distr; /* distribution object */
    UNUR_PAR   *par;   /* parameter object */
    UNUR_GEN   *gen;   /* generator object */

```

```

/* Create a distribution object and set empirical sample.      */
distr = unur_distr_cemp_new();
unur_distr_cemp_set_data(distr, data, 15);

/* Choose a method: EMPK.                                     */
par = unur_empk_new(distr);

/* Set smoothing factor.                                     */
unur_empk_set_smoothing(par, 0.8);

/* Create the generator object.                              */
gen = unur_init(par);

/* It is important to check if the creation of the generator */
/* object was successful. Otherwise 'gen' is the NULL pointer */
/* and would cause a segmentation fault if used for sampling. */
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create */
/* another generator object. If you do not need it any more, */
/* it should be destroyed to free memory.                   */
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from */
/* the distribution. Eg.:                                    */
for (i=0; i<10; i++) {
    x = unur_sample_cont(gen);
    printf("%f\n",x);
}

/* When you do not need the generator object any more, you  */
/* can destroy it.                                           */
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

Example (String API)

```

/* ----- */
/* File: example_emp_str.c                                  */
/* ----- */
/* String API.                                            */
/* ----- */

/* Include UNURAN header file.                            */
#include <unuran.h>

/* ----- */

/* Example how to sample from an empirical continuous univariate */

```

```

/* distribution. */
/* ----- */

int main()
{
    int    i;
    double x;

    /* Declare UNURAN generator object. */
    UNUR_GEN *gen; /* generator object */

    /* Create the generator object. */
    gen = unur_str2gen("distr = cemp; \
                      data=(-0.10, 0.05,-0.50, 0.08, 0.13, \
                          -0.21,-0.44,-0.43,-0.33,-0.30, \
                          0.18, 0.20,-0.37,-0.29,-0.90) & \
                      method=empk; smoothing=0.8");

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Now you can use the generator object 'gen' to sample from */
    /* the distribution. Eg.: */
    for (i=0; i<10; i++) {
        x = unur_sample_cont(gen);
        printf("%f\n",x);
    }

    /* When you do not need the generator object any more, you */
    /* can destroy it. */
    unur_free(gen);

    exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

5.4.1 EMPK – EMPirical distribution with Kernel smoothing

Required: observed sample

Speed: Set-up: slow (as sample is sorted), Sampling: fast (depends on kernel)

reference: [HLa00]

EMPK generates random variates from an empirical distribution that is given by an observed sample. The idea is that simply choosing a random point from the sample and to return it with some added noise results in a method that has very nice properties, as it can be seen as sampling from a kernel density estimate. If the underlying distribution is continuous, especially the fine structure of the resulting empirical distribution is much better than using only resampling without noise.

Clearly we have to decide about the density of the noise (called kernel) and about the standard deviation of the noise. The mathematical theory of kernel density estimation shows us that we are comparatively free in choosing the kernel. It also supplies us with a simple formula to compute the optimal standard deviation of the noise, called bandwidth (or window width) of the kernel.

For most applications it is perfectly ok to use the default values offered. Unless you have some knowledge on density estimation we do not recommend to change anything. There are two exceptions:

- A. In the case that the unknown underlying distribution is not continuous but discrete you should "turn off" the adding of the noise by setting:

```
unur_empk_set_smoothing(par, 0.)
```

- B. In the case that you are especially interested in a fast sampling algorithm use the call

```
unur_empk_set_kernel(par, UNUR_DISTR_BOXCAR);
```

to change the used noise distribution from the default Gaussian distribution to the uniform distribution. For other possible kernels see `unur_empk_set_kernel` and `unur_empk_set_kernelgen` below.

All other parameters are only useful for people knowing the theory of kernel density estimation. It is not necessary to change them if the true underlying distribution is somehow comparable with a bell-shaped curve, even skewed or with some not too sharp extra peaks. In all these cases the simple robust reference method implemented to find a good standard deviation of the noise (i.e. the bandwidth of kernel density estimation) should give sensible results. However, it might be necessary to overwrite this automatic method to find the bandwidth eg. when resampling from data with two or more sharp distinct peaks. Then the distribution has nearly discrete components as well and our automatic method may easily choose too large a bandwidth which results in an empirical distribution which is oversmoothed (i.e. it has lower peaks than the original distribution). Then it is recommended to decrease the bandwidth using the `unur_empk_set_smoothing` call. A smoothing factor of 1 is the default. A smoothing factor of 0 leads to naive resampling of the data. Thus an appropriate value between these extremes should be chosen. We recommend to consult a reference on kernel smoothing when doing so; but it is not a simple problem to determine an optimal bandwidth for distributions with sharp peaks.

Function reference

UNUR_PAR* `unur_empk_new` (*const UNUR_DISTR** *distribution*)

Get default parameters for generator.

int `unur_empk_set_kernel` (*UNUR_PAR** *parameters*, *unsigned kernel*)

Select one of the supported kernel distributions. Currently the following kernels are supported:

`UNUR_DISTR_GAUSSIAN`

Gaussian (normal) kernel

`UNUR_DISTR_EPANECHNIKOV`

Epanechnikov kernel

`UNUR_DISTR_BOXCAR`

Boxcar (uniform, rectangular) kernel

`UNUR_DISTR_STUDENT`

t3 kernel (Student's distribution with 3 degrees of freedom)

`UNUR_DISTR_LOGISTIC`

logistic kernel

For other kernels (including kernels with Student's distribution with other than 3 degrees of freedom) use the `unur_empk_set_kernelgen` call.

It is not possible to call `unur_empk_set_kernel` twice.

Default is the Gaussian kernel.

int unur_empk_set_kernelgen (UNUR_PAR* *parameters*, const UNUR_GEN* *kernelgen*, double *alpha*, double *kernelvar*) —

Set generator for the kernel used for density estimation.

alpha is used to compute the optimal bandwidth from the point of view of minimizing the mean integrated square error (MISE). It depends on the kernel K and is given by

$$\alpha(K) = \text{Var}(K)^{-2/5} \left\{ \int K(t)^2 dt \right\}^{1/5}$$

For standard kernels (see above) *alpha* is computed by the algorithm.

kernvar is the variance of the used kernel. It is only required for the variance corrected version of density estimation (which is used by default); otherwise it is ignored. If *kernelvar* is nonpositive, variance correction is disabled. For standard kernels (see above) *kernvar* is computed by the algorithm.

It is not possible to call `unur_empk_set_kernelgen` after a standard kernel has been selected by a `unur_empk_set_kernel` call.

Notice that the uniform random number generator of the kernel generator is overwritten during the `unur_init` call and at each `unur_chg_urng` call with the uniform generator used for the empirical distribution.

Default is the Gaussian kernel.

int unur_empk_set_beta (UNUR_PAR* *parameters*, double *beta*) —

beta is used to compute the optimal bandwidth from the point of view of minimizing the mean integrated square error (MISE). *beta* depends on the (unknown) distribution of the sampled data points. By default Gaussian distribution is assumed for the sample (*beta* = 1.3637439). There is no requirement to change *beta*.

Default: 1.3637439

int unur_empk_set_smoothing (UNUR_PAR* *parameters*, double *smoothing*) —

int unur_empk_chg_smoothing (UNUR_GEN* *generator*, double *smoothing*) —

Set and change the smoothing factor. The smoothing factor controls how “smooth” the resulting density estimation will be. A smoothing factor equal to 0 results in naive resampling. A very large smoothing factor (together with the variance correction) results in a density which is approximately equal to the kernel. Default is 1 which results in a smoothing parameter minimising the MISE (mean integrated squared error) if the data are not too far away from normal. If a large smoothing factor is used, then variance correction must be switched on.

Default: 1

int unur_empk_set_varcor (UNUR_PAR* *parameters*, int *varcor*) —

int unur_empk_chg_varcor (UNUR_GEN* *generator*, int *varcor*) —

Switch variance correction in generator on/off. If *varcor* is TRUE then the variance of the used density estimation is the same as the sample variance. However this increases the MISE of the estimation a little bit.

Default is FALSE.

int unur_empk_set_positive (UNUR_PAR* *parameters*, int *positive*) —

If *positive* is TRUE then only nonnegative random variates are generated. This is done by means of a mirroring technique.

Default is FALSE.

5.4.2 EMPL – EMPirical distribution with Linear interpolation

Required: observed sample

Speed: Set-up: slow (as sample is sorted), Sampling: very fast (inversion)

reference: [HLa00]

EMPL generates random variates from an empirical distribution that is given by an observed sample. This is done by linear interpolation of the empirical CDF. Although this method is suggested in the books of Law and Keltn (2000) and Bratly, Fox, and Schrage (1987) we do not recommend this method at all since it has many theoretical drawbacks: The variance of empirical distribution function does not coincide with the variance of the given sample. Moreover, when the sample increases the empirical density function does not converge to the density of the underlying random variate. Notice that the range of the generated point set is always given by the range of the given sample.

This method is provided in UNURAN for the sake of completeness. We always recommend to use method EMPK (see [Section 5.4.1 \[EMPirical distribution with Kernel smoothing\]](#), page 87).

If the data seem to be far away from having a bell shaped histogram, then we think that naive resampling is still better than linear interpolation.

Important: Using this method is not recommended!

Function reference

UNUR_PAR* `unur_empl_new` (`const UNUR_DISTR*` *distribution*)

Get default parameters for generator.

5.5 Methods for continuous multivariate distributions

Overview of methods

Methods for **continuous multivariate distributions**

sample with `unur_sample_vec`

VMT: Requires the mean vector and the covariance matrix.

5.5.1 VMT – Vector Matrix Transformation

Required: mean vector, covariance matrix

Optional: marginal distribution (for non-Gaussian distributions)

Speed: Set-up: slow, Sampling: depends on dimension

VMT generates random vectors for distributions with given mean vector μ and covariance matrix Σ . It produces random vectors of the form $X = L Y + \mu$, where L is the Cholesky factor of Σ , i.e. $L L^t = \Sigma$, and Y has independent components of the same distribution with mean 0 and standard deviation 1.

By default the standard normal distribution is used for the components of Y . Thus VMT produces multinormal random vectors when this distribution of Y is not set explicitly.

The method VMT has been implemented especially to sample from a multinormal distribution. Nevertheless it can also be used (or abused) for other distributions. However notice that the univariate distribution provided by a `unur_vmt_set_marginalgen` call should have mean 0 and standard deviation 1. Otherwise μ and Σ are not the mean vector and covariance matrix, respectively, of the resulting distribution. Moreover notice that except for the multinormal distribution the given univariate distribution is *not* the marginal distribution of the resulting random vector.

Function reference

UNUR_PAR* **unur_vmt_new** (const UNUR_DISTR* *distribution*)

Get default parameters for generator.

int **unur_vmt_set_marginalgen** (UNUR_PAR* *parameters*, const UNUR_GEN* *uvgen*)

Set generator for (univariate) marginal distribution.

Default: Generator for (univariate) standard normal distribution.

5.6 Methods for continuous empirical multivariate distributions

Overview of methods

Methods for **continuous empirical multivariate distributions**
sample with `unur_sample_vec`

VEMPK: Requires an observed sample.

Example

```

/* ----- */
/* File: example_vemp.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* Example how to sample from an empirical continuous */
/* multivariate distribution. */

/* ----- */

int main()
{
    int    i;

    /* 4 data points of dimension 2 */
    double data[] = { 1. ,1., /* 1st data point */
                    -1.,1., /* 2nd data point */
                    1.,-1., /* 3rd data point */
                    -1.,-1. }; /* 4th data point */

    double result[2];

    /* Declare the three UNURAN objects. */
    UNUR_DISTR *distr; /* distribution object */
    UNUR_PAR   *par;   /* parameter object */
    UNUR_GEN   *gen;   /* generator object */

    /* Create a distribution object with dimension 2.

```

```

distr = unur_distr_cvemp_new( 2 );

/* Set empirical sample. */
unur_distr_cvemp_set_data(distr, data, 4);

/* Choose a method: VEMPK. */
par = unur_vempk_new(distr);

/* Use variance correction. */
unur_vempk_set_varcor( par, 1 );

/* Create the generator object. */
gen = unur_init(par);

/* It is important to check if the creation of the generator
/* object was successful. Otherwise 'gen' is the NULL pointer
/* and would cause a segmentation fault if used for sampling.
if (gen == NULL) {
    fprintf(stderr, "ERROR: cannot create generator object\n");
    exit (EXIT_FAILURE);
}

/* It is possible to reuse the distribution object to create
/* another generator object. If you do not need it any more,
/* it should be destroyed to free memory.
unur_distr_free(distr);

/* Now you can use the generator object 'gen' to sample from
/* the distribution. Eg.:
for (i=0; i<10; i++) {
    unur_sample_vec(gen, result);
    printf("(%f,%f)\n", result[0], result[1]);
}

/* When you do not need the generator object any more, you
/* can destroy it.
unur_free(gen);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

Example (String API)

(not implemented)

5.6.1 VEMPK – (Vector) EMPIRICAL distribution with Kernel smoothing

Required: observed sample

Speed: Set-up: slow, Sampling: slow (depends on dimension)

reference: [HLa00]

VEMPK generates random variates from a multivariate empirical distribution that is given by an observed sample. The idea is that simply choosing a random point from the sample and to return it with some added noise results in a method that has very nice properties, as it can be seen as sampling from a kernel density estimate. Clearly we have to decide about the density of the noise (called kernel) and about the covariance matrix of the noise. The mathematical theory of kernel density estimation shows us that we are comparatively free in choosing the kernel. It also supplies us with a simple formula to compute the optimal standard deviation of the noise, called bandwidth (or window width) of the kernel.

Currently only a Gaussian kernel with the same covariance matrix as the given sample is implemented. However it is possible to choose between a variance corrected version or those with optimal MISE. Additionally a smoothing factor can be set.

Function reference

`UNUR_PAR*` **unur_vempk_new** (`const UNUR_DISTR*` *distribution*) –

Get default parameters for generator.

`int` **unur_vempk_set_smoothing** (`UNUR_PAR*` *parameters*, `double` *smoothing*) –

`int` **unur_vempk_chg_smoothing** (`UNUR_GEN*` *generator*, `double` *smoothing*) –

Set and change the smoothing factor. The smoothing factor controls how “smooth” the resulting density estimation will be. A smoothing factor equal to 0 results in naive resampling. A very large smoothing factor (together with the variance correction) results in a density which is approximately equal to the kernel. Default is 1 which results in a smoothing parameter minimising the MISE (mean integrated squared error) if the data are not too far away from normal. If a large smoothing factor is used, then variance correction must be switched on.

Default: 1

`int` **unur_vempk_set_varcor** (`UNUR_PAR*` *parameters*, `int` *varcor*) –

`int` **unur_vempk_chg_varcor** (`UNUR_GEN*` *generator*, `int` *varcor*) –

Switch variance correction in generator on/off. If *varcor* is TRUE then the variance of the used density estimation is the same as the sample variance. However this increases the MISE of the estimation a little bit.

Default is FALSE.

5.7 Methods for discrete univariate distributions

Overview of methods

Methods for **discrete univariate distributions**

sample with `unur_sample_discr`

method	PMF	PV	mode	sum	other
DARI	x		x	~	T-concave
DAU	[x]	x			
DGT	[x]	x			
DSTD					build-in standard distribution

Example

```

/* ----- */
/* File: example_discr.c */
/* ----- */

/* Include UNURAN header file. */
#include <unuran.h>

/* ----- */

/* Example how to sample from a discrete univariate distribution.*/

/* ----- */

int main()
{
    int    i;
    double param = 0.3;

    double probvec[10] = {1.0, 2.0, 3.0, 4.0, 5.0,\
                          6.0, 7.0, 8.0, 4.0, 3.0};

    /* Declare the three UNURAN objects. */
    UNUR_DISTR *distr1, *distr2; /* distribution objects */
    UNUR_PAR   *par1, *par2;    /* parameter objects */
    UNUR_GEN   *gen1, *gen2;    /* generator objects */

    /* First distribution: defined by PMF. */
    distr1 = unur_distr_geometric(&param, 1);
    unur_distr_discr_set_mode(distr1, 0);

    /* Choose a method: DARI. */
    par1 = unur_dari_new(distr1);
    gen1 = unur_init(par1);

    /* It is important to check if the creation of the generator */
    /* object was successful. Otherwise 'gen' is the NULL pointer */
    /* and would cause a segmentation fault if used for sampling. */
    if (gen1 == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Second distribution: defined by (finite) PV. */
    distr2 = unur_distr_discr_new();
    unur_distr_discr_set_pv(distr2, probvec, 10);

    /* Choose a method: DGT. */
    par2 = unur_dgt_new(distr2);
    gen2 = unur_init(par2);
    if (gen2 == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* print some random integers */

```

```

for (i=0; i<10; i++){
    printf("number %d: %d\n", i*2,    unur_sample_discr(gen1) );
    printf("number %d: %d\n", i*2+1, unur_sample_discr(gen2) );
}

/* Destroy all objects.                                     */
unur_distr_free(distr1);
unur_distr_free(distr2);
unur_free(gen1);
unur_free(gen2);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

Example (String API)

```

/* ----- */
/* File: example_discr_str.c                               */
/* ----- */
/* String API.                                           */
/* ----- */

/* Include UNURAN header file.                            */
#include <unuran.h>

/* ----- */

/* Example how to sample from a discrete univariate distribution.*/
/* ----- */

int main()
{
    int    i;      /* loop variable                                     */

    /* Declare UNURAN generator objects.                   */
    UNUR_GEN *gen1, *gen2;      /* generator objects                                     */

    /* First distribution: defined by PMF.                  */
    gen1 = unur_str2gen("geometric(0.3); mode=0 & method=dari");

    /* It is important to check if the creation of the generator
     * object was successful. Otherwise 'gen' is the NULL pointer
     * and would cause a segmentation fault if used for sampling.
     */
    if (gen1 == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
        exit (EXIT_FAILURE);
    }

    /* Second distribution: defined by (finite) PV.         */
    gen2 = unur_str2gen("distr=discr; pv=(1,2,3,4,5,6,7,8,4,3) & method=dgt");
    if (gen2 == NULL) {
        fprintf(stderr, "ERROR: cannot create generator object\n");
    }
}

```

```

    exit (EXIT_FAILURE);
}

/* print some random integers */
for (i=0; i<10; i++){
    printf("number %d: %d\n", i*2,    unur_sample_discr(gen1) );
    printf("number %d: %d\n", i*2+1, unur_sample_discr(gen2) );
}

/* Destroy all objects. */
unur_free(gen1);
unur_free(gen2);

exit (EXIT_SUCCESS);

} /* end of main() */

/* ----- */

```

5.7.1 DARI – discrete automatic rejection inversion

Required: T-concave PMF, mode, approximate area

Speed: Set-up: moderate, Sampling: fast

reference: [HDa96]

DARI is based on rejection inversion, which can be seen as an adaptation of transformed density rejection to discrete distributions. The used transformation is $-1/\sqrt{x}$.

DARI uses three almost optimal points for constructing the (continuous) hat. Rejection is then done in horizontal direction. Rejection inversion uses only one uniform random variate per trial.

DARI has moderate set-up times (the PMF is evaluated nine times), and good marginal speed, especially if an auxiliary array is used to store values during generation.

DARI works for all T- $(-1/2)$ -concave distributions. It requires the PMF and the location of the mode. Moreover the approximate sum over the PMF is used. (If no sum is given for the distribution the algorithm assumes that it is approximately 1.) The rejection constant is bounded from above by 4 for all T-concave distributions.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object by using the `unur_dari_chg_pmfparams` and `unur_dari_chg_domain` call, respectively. But then `unur_dari_chg_mode` and `unur_dari_chg_pmfsum` have to be used to reset the corresponding figures whenever they were changed. Before sampling from the distribution again, `unur_dari_reinit` must be executed. (Otherwise the generator might produce garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not. It can be switched on by calling `unur_dari_set_verify` and `unur_dari_chg_verify`, respectively. Notice however that sampling is (much) slower then.

Function reference

`UNUR_PAR*` **unur_dari_new** (`const UNUR_DISTR*` *distribution*)

Get default parameters for generator.

`int` **unur_dari_reinit** (`UNUR_GEN*` *generator*)

Update an existing generator object after the distribution has been modified. It must be executed whenever the parameters or the domain of the distributions has been changed

(see below). It is faster than destroying the existing object and building a new one from scratch. If reinitialization has been successful 1 is returned, in case of a failure 0 is returned.

- int unur_dari_set_squeeze** (UNUR_PAR* *parameters*, int *squeeze*) —
 Turn utilization of the squeeze of the algorithm on/off. This squeeze does not resample the squeeze of the continuous TDR method. It was especially designed for rejection inversion. The squeeze is not necessary if the size of the auxiliary table is big enough (for the given distribution). Using a squeeze is suggested to speed up the algorithm if the domain of the distribution is very big or if only small samples are produced.
 Default: no squeeze.
- int unur_dari_set_tablesize** (UNUR_PAR* *parameters*, int *size*) —
 Set the size for the auxiliary table, that stores constants computed during generation. If *size* is set to 0 no table is used. The speed-up can be impressive if the PMF is expensive to evaluate and the “main part of the distribution” is concentrated in an interval shorter than the size of the table.
 Default is 100.
- int unur_dari_set_cpfactor** (UNUR_PAR* *parameters*, double *cp_factor*) —
 Set factor for position of the left and right construction point, resp. The *cp_factor* is used to find almost optimal construction points for the hat function. There is no need to change this factor in almost all situations.
 Default is 0.664.
- int unur_dari_set_verify** (UNUR_PAR* *parameters*, int *verify*) —
int unur_dari_chg_verify (UNUR_GEN* *generator*, int *verify*) —
 Turn verifying of algorithm while sampling on/off. If the condition is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
 Default is FALSE.
- int unur_dari_chg_pmparams** (UNUR_GEN* *generator*, double* *params*, int *n_params*) —
 Change array of parameters of the distribution in a given generator object. Notice that this call simply copies the parameters into the generator object. Thus if fewer parameters are provided then the remaining parameters are left unchanged. `unur_dari_reinit` must be executed before sampling from the generator again.
Important: The given parameters are not checked against domain errors; in opposition to the `unur_<distr>_new` calls.
- int unur_dari_chg_domain** (UNUR_GEN* *generator*, int *left*, int *right*) —
 Change the left and right border of the domain of the (truncated) distribution. If the mode changes when the domain of the (truncated) distribution is changed, then a correspondig `unur_dari_chg_mode` call is required. (There is no domain checking as in the `unur_init` call.) Use `INT_MIN` and `INT_MAX` for (minus) infinity. `unur_dari_reinit` must be executed before sampling from the generator again.
- int unur_dari_chg_mode** (UNUR_GEN* *generator*, int *mode*) —
 Change mode of distribution. `unur_dari_reinit` must be executed before sampling from the generator again.

- int unur_dari_upd_mode** (UNUR_GEN* *generator*) —
 Recompute the mode of the distribution. This call only works well when a distribution object from the UNURAN library of standard distributions is used (see [Chapter 7 \[Standard distributions\]](#), page 107). Otherwise a (slow) numerical mode finder is called. If no mode can be found, then 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`. `unur_dari_reinit` must be executed before sampling from the generator again.
- int unur_dari_chg_pmfsum** (UNUR_GEN* *generator*, double *sum*) —
 Change sum over the PMF of distribution. `unur_dari_reinit` must be executed before sampling from the generator again.
- int unur_dari_upd_pmfsum** (UNUR_GEN* *generator*) —
 Recompute sum over the PMF of the distribution. It only works when a distribution objects from the UNURAN library of standard distributions is used (see [Chapter 7 \[Standard distributions\]](#), page 107). Otherwise 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_DATA`. `unur_dari_reinit` must be executed before sampling from the generator again.

5.7.2 DAU – (Discrete) Alias-Urn method

- Required:* probability vector (PV)
Speed: Set-up: slow (linear with the vector-length), Sampling: very fast
reference: [WAa77]

DAU samples from distributions with arbitrary but finite probability vectors (PV) of length N . The algorithmus is based on an ingeneous method by A.J. Walker and requires a table of size (at least) N . It needs one random numbers and only one comparison for each generated random variate. The setup time for constructing the tables is $O(N)$.

By default the probability vector is indexed starting at 0. However this can be changed in the distribution object by a `unur_distr_discr_set_domain` call.

The method also works when no probability vector but a PMF is given. However then additionally a bounded (not too large) domain must be given or the sum over the PMF (see `unur_distr_discr_make_pv` for details).

Function reference

- UNUR_PAR* **unur_dau_new** (const UNUR_DISTR* *distribution*) —
 Get default parameters for generator.
- int unur_dau_set_urnfactor** (UNUR_PAR* *parameters*, double *factor*) —
 Set size of urn table relative to length of the probability vector. It must not be less than 1. Larger tables result in (slightly) faster generation times but require a more expensive setup. However sizes larger than 2 are not recommended.
 Default is 1.

5.7.3 DGT – (Discrete) Guide Table method (indexed search)

- Required:* probability vector (PV)
Speed: Set-up: slow (linear with the vector-length), Sampling: very fast
reference: [CAa74]

DGT samples from arbitrary but finite probability vectors. Random numbers are generated by the inversion method, i.e.,

1. Generate a random number $U \sim U(0,1)$.
2. Find largest integer I such that $F(I) = P(X \leq I) \leq U$.

Step (2) is the crucial step. Using sequential search requires $O(E(X))$ comparisons, where $E(X)$ is the expectation of the distribution. Indexed search however uses a guide table to jump to some $I' \leq I$ near I to find X in constant time. Indeed the expected number of comparisons is reduced to 2, when the guide table has the same size as the probability vector (this is the default). For larger guide tables this number becomes smaller (but is always larger than 1), for smaller tables it becomes larger. For the limit case of table size 1 the algorithm simply does sequential search. On the other hand the setup time for guide table is $O(N)$ (for size 1 no preprocessing is required). Moreover for very large guide tables memory effects might even reduce the speed of the algorithm. So we do not recommend to use guide tables that are more than three times larger than the given probability vector. If only a few random numbers have to be generated, (much) smaller table sizes are better. The size of the guide table relative to the length of the given probability vector can be set by a `unur_dgt_set_guidefactor` call.

There exist two variants for the setup step which can be set by a `unur_dgt_set_variant` call: Variants 1 and 2. Variant 2 is faster but more sensitive to roundoff errors when the guide table is large. By default variant 2 is used for short probability vectors ($N < 1000$) and variant 1 otherwise.

By default the probability vector is indexed starting at 0. However this can be changed in the distribution object by a `unur_distr_discr_set_domain` call.

The method also works when no probability vector but a PMF is given. However then additionally a bounded (not too large) domain must be given or the sum over the PMF (see `unur_distr_discr_make_pv` for details).

Function reference

`UNUR_PAR*` **unur_dgt_new** (`const UNUR_DISTR*` *distribution*)

Get default parameters for generator.

`int` **unur_dgt_set_guidefactor** (`UNUR_PAR*` *parameters*, `double` *factor*)

Set size of guide table relative to length of PV. Larger guide tables result in faster generation time but require a more expensive setup. Sizes larger than 3 are not recommended. If the relative size is set to 0, sequential search is used.

Default is 1.

`int` **unur_dgt_set_variant** (`UNUR_PAR*` *parameters*, `unsigned` *variant*)

Set variant for setup step. Possible values are 1 or 2. Variant 2 is faster but more sensitive to roundoff errors when the guide table is large. By default variant 2 is used for short probability vectors ($N < 1000$) and variant 1 otherwise.

5.7.4 DSROU – Discrete Simple Ratio-Of-Uniforms method

Required: T-concave PMF, mode, sum over PMF

Speed: Set-up: fast, Sampling: slow

reference: [LJa01]

DSROU is based on the ratio-of-uniforms method but uses universal inequalities for constructing a (universal) bounding rectangle. It works for all T-concave distributions with $T(x) = -1/\sqrt{x}$.

It requires the PMF, the (exact) location of the mode and the sum over the given PDF. The rejection constant is 4 for all T-concave distributions. Optionally the CDF at mode can be given to increase the performance of the algorithm by means of the `unur_dsrou_set_cdfatmode` call. Then the rejection constant is reduced to 2.

If the (exact) sum over the PMF is not known, then an upper bound can be used instead (which of course increases the rejection constant). But then `unur_dsrou_set_cdfatmode` must not be called.

It is possible to change the parameters and the domain of the chosen distribution without building a new generator object using the `unur_dsrou_chg_pmfparams` and `unur_dsrou_chg_domain` call, respectively. But then `unur_dsrou_chg_pmfsum`, `unur_dsrou_chg_mode` and `unur_dsrou_chg_cdfatmode` have to be used to reset the corresponding figures whenever they have changed.

If any of mode, CDF at mode, or the sum over the PMF has been changed, then `unur_dsrou_reinit` must be executed. (Otherwise the generator produces garbage).

There exists a test mode that verifies whether the conditions for the method are satisfied or not while sampling. It can be switched on or off by calling `unur_dsrou_set_verify` and `unur_dsrou_chg_verify`, respectively. Notice however that sampling is (a little bit) slower then.

Function reference

UNUR_PAR* `unur_dsrou_new` (*const UNUR_DISTR** *distribution*) —
Get default parameters for generator.

int `unur_dsrou_reinit` (*UNUR_GEN** *generator*) —
Update an existing generator object after the distribution has been modified. It must be executed whenever the parameters or the domain of the distribution have been changed (see below). It is faster than destroying the existing object and building a new one from scratch. If reinitialization has been successful 1 is returned, in case of a failure 0 is returned.

int `unur_dsrou_set_cdfatmode` (*UNUR_PAR** *parameters*, *double Fmode*) —
Set CDF at mode. When set, the performance of the algorithm is increased by factor 2. However, when the parameters of the distribution are changed `unur_dsrou_chg_cdfatmode` has to be used to update this value. Notice that the algorithm detects a mode at the left boundary of the domain automatically and it is not necessary to use this call for a monotonically decreasing PMF.
Default: not set.

int `unur_dsrou_set_verify` (*UNUR_PAR** *parameters*, *int verify*) —
int `unur_dsrou_chg_verify` (*UNUR_GEN** *generator*, *int verify*) —
Turn verifying of algorithm while sampling on/off. If the condition $\text{squeeze}(x) \leq \text{PMF}(x) \leq \hat{\text{hat}}(x)$ is violated for some x then `unur_errno` is set to `UNUR_ERR_GEN_CONDITION`. However notice that this might happen due to round-off errors for a few values of x (less than 1%).
Default is FALSE.

int `unur_dsrou_chg_pmfparams` (*UNUR_GEN** *generator*, *double** *params*, *int n_params*) —
Change array of parameters of the distribution in a given generator object.
For standard distributions from the UNURAN library the parameters are checked. If these are invalid, then 0 is returned. Moreover the domain is updated automatically unless it has been changed before by a `unur_distr_discr_set_domain` call. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.
For other distributions *params* is simply copied into to distribution object. It is only checked that *n_params* does not exceed the maximum number of parameters allowed. Then 0 is returned and `unur_errno` is set to `UNUR_ERR_DISTR_NPARAMS`.

- int unur_dsrou_chg_domain** (UNUR_GEN* *generator*, int *left*, int *right*) –
 Change left and right border of the domain of the (truncated) distribution. If the mode changes when the domain of the (truncated) distribution is changed, then a corresponding `unur_dsrou_chg_mode` is required. (There is no checking whether the domain is set or not as in the `unur_init` call.)
- int unur_dsrou_chg_mode** (UNUR_GEN* *generator*, int *mode*) –
 Change mode of distribution. `unur_dsrou_reinit` must be executed before sampling from the generator again.
- int unur_dsrou_upd_mode** (UNUR_GEN* *generator*) –
 Recompute the mode of the distribution. See `unur_distr_cont_upd_mode` for more details. `unur_dsrou_reinit` must be executed before sampling from the generator again.
- int unur_dsrou_chg_cdfatmode** (UNUR_GEN* *generator*, double *Fmode*) –
 Change CDF at mode of distribution. `unur_dsrou_reinit` must be executed before sampling from the generator again.
- int unur_dsrou_chg_pmfsum** (UNUR_GEN* *generator*, double *sum*) –
 Change sum over PMF of distribution. `unur_dsrou_reinit` must be executed before sampling from the generator again.
- int unur_dsrou_upd_pmfsum** (UNUR_GEN* *generator*) –
 Recompute the sum over the the PMF of the distribution. It only works when a distribution objects from the UNURAN library of standard distributions is used (see [Chapter 7 \[Standard distributions\]](#), page 107). Otherwise `unur_errno` is set to `UNUR_ERR_DISTR_DATA`. `unur_dsrou_reinit` must be executed before sampling from the generator again.

5.7.5 DSTD – Discrete STandarD distributions

Required: standard distribution from UNURAN library (see [Chapter 7 \[Standard distributions\]](#), page 107).

Speed: Set-up: fast, Sampling: depends on distribution and generator

DSTD is a wrapper for special generators for discrete univariate standard distributions. It only works for distributions in the UNURAN library of standard distributions (see [Chapter 7 \[Standard distributions\]](#), page 107). If a distribution object is provided that is build from scratch, or no special generator for the given standard distribution is provided, the NULL pointer is returned.

For some distributions more than one special generator (*variants*) is possible. These can be chosen by a `unur_dstd_set_variant` call. For possible variants see [Chapter 7 \[Standard distributions\]](#), page 107. However the following are common to all distributions:

- UNUR_STDGEN_DEFAULT
 the default generator.
- UNUR_STDGEN_FAST
 the fasted available special generator.
- UNUR_STDGEN_INVERSION
 the inversion method (if available).

Notice that the variant `UNUR_STDGEN_FAST` for a special generator might be slower than one of the universal algorithms! Additional variants may exist for particular distributions.

Sampling from truncated distributions (which can be constructed by changing the default domain of a distribution by means of `unur_distr_discr_set_domain` call) is possible but requires the inversion method.

Function reference

UNUR_PAR* `unur_dstd_new` (`const UNUR_DISTR*` *distribution*)

Get default parameters for new generator. It requires a distribution object for a discrete univariant distribution from the UNURAN library of standard distributions (see [Chapter 7 \[Standard distributions\]](#), page 107).

Using a truncated distribution is allowed only if the inversion method is available and selected by the `unur_dstd_set_variant` call immediately after creating the parameter object. Use a `unur_distr_discr_set_domain` call to get a truncated distribution.

int `unur_dstd_set_variant` (`UNUR_PAR*` *parameters*, `unsigned` *variant*)

Set variant (special generator) for sampling from a given distribution. For possible variants see [Chapter 7 \[Standard distributions\]](#), page 107.

Common variants are `UNUR_STDGEN_DEFAULT` for the default generator, `UNUR_STDGEN_FAST` for (one of the) fastest implemented special generators, and `UNUR_STDGEN_INVERSION` for the inversion method (if available). If the selected variant number is not implemented, this call has no effect.

int `unur_dstd_chg_pmfparams` (`UNUR_GEN*` *gen*, `double*` *params*, `int` *n_params*)

Change array of parameters of the distribution in a given generator object. If the given parameters are invalid for the distribution, no parameters are set. Notice that optional parameters are (re-)set to their default values if not given for UNURAN standard distributions.

Important: Integer parameter must be given as doubles.

5.8 Methods for uniform univariate distributions

5.8.1 UNIF – wrapper for UNIFORM random number generator

UNIF is a simple wrapper that makes it possible to use a uniform random number generator as a UNURAN generator. There are no parameters for this method.

Function reference

UNUR_PAR* `unur_unif_new` (`const UNUR_DISTR*` *dummy*)

Get default parameters for generator. UNIF does not need a distribution object. *dummy* is not used and can (should) be set to `NULL`. It is used to keep the API consistent.

6 Using uniform random number generators

Each generator has a pointer to a uniform (pseudo-) random number generator (URNG). It can be set via the `unur_set_urng` call. It is also possible to read this pointer via `unur_get_urng` or change the URNG for an existing generator object by means of `unur_chg_urng`; By this very flexible concept it is possible that each generator has its own (independent) URNG or several generators can share the same URNG.

If no URNG is provided for a parameter or generator object a default generator is used which is the same for all generators. This URNG is defined in ‘`unuran_config.h`’ at compile time. A pointer to this default URNG can be obtained via `unur_get_default_urng`. Nevertheless, it is also possible to overwrite this default URNG by another one by means of the `unur_set_default_urng` call. However, this only takes effect for new parameter objects.

The pointer to a URNG is of type `UNUR_URNG*`. Its definition depends on the compilation switch `UNUR_URNG_TYPE` in ‘`unuran_config.h`’. Currently we have two possible switches (other values would result in a compilation error):

1. `UNUR_URNG_TYPE == UNUR_URNG_FVOID`

This uses URNGs of type `double uniform(void)`. If independent versions of the same URNG should be used, a copy of the subroutine has to be implemented in the program code (with different names, of course). UNURAN contains some build-in URNGs of this type in directory ‘`src/uniform/`’.

2. `UNUR_URNG_TYPE == UNUR_URNG_PRNG`

This uses the URNGs from the `prng` library. It provides a very flexible way to sample from arbitrary URNGs by means of an object oriented programming paradigm. Similarly to the UNURAN library independent generator objects can be built and used. Here `UNUR_URNG*` is simply a pointer to such a uniform generator object.

This library has been developed by the pLab group at the university of Salzburg (Austria, EU) and implemented by Otmar Lendl. It is available via anonymous ftp from <http://statistik.wu-wien.ac.at/prng/> or from the pLab site at <http://random.mat.sbg.ac.at/>.

3. `UNUR_URNG_TYPE == UNUR_URNG_RNGSTREAM`

Use Pierre L’Ecuyer’s `RngStream` library for multiple independent streams of pseudo-random numbers. It is available from <http://www.iro.umontreal.ca/~lecuyer/myftp/streams00/c/>.

4. `UNUR_URNG_TYPE == UNUR_URNG_GSL`

Use the URNG from the GNU Scientific Library (GSL). It is available from <http://www.gnu.org/software/gsl/>.

5. `UNUR_URNG_TYPE == UNUR_URNG_GENERIC`

This is a generic interface with limited support. It uses a structure to store both a function call of type `double urng(void*)` and a void pointer to the parameter list. Both pointers must be set directly using the structure `struct unur_urng_generic` (there are currently no calls that support this URNG type). It is defined as

```
struct unur_urng_generic {
    double (*getrand)(void *params);
    void *params;
};
```

All functions and parameters should be set at run time:

1. Allocate variable of type `struct unur_urng_generic` (or of type `UNUR_URNG`, which is the same):

```
UNUR_URNG *urng;
urng = malloc(sizeof(UNUR_URNG));
```

2. Set function of type `double (*rand)(void *)` for sampling from URNG:

```
urng->getrand = my_uniform_rng;
```

3. Set pointer to parameters of for this function (or NULL if no parameters are required):

```
urng->params = my_parameters;
```

4. Use this URNG:

```
unur_urng_set_default(urng);           (set default generator)
unur_urng_set_default_aux(urng);      (set default aux. generator)
```

Notice that this must be done before UNURAN generator object are created. Of course urng can also used just for a particular generator object. Use the following and similar calls:

```
unur_set_urng(par, urng);
unur_chg_urng(gen, urng);
```

It is possible to use other interfaces to URNGs without much troubles. All changes have to be done in file 'unuran_config.h'. If you need such a new interface please feel free to contact the authors of the UNURAN library.

Some generating methods provide the possibility of correlation induction. To use this feature a second auxiliary URNG is required. It can be set and changed by the `unur_set_urng_aux` and `unur_chg_urng_aux` call, respectively. Since the auxiliary generator is by default the same as the main generator, the auxiliary URNG must be set after any `unur_set_urng` or `unur_chg_urng` call! Since in special cases mixing of two URNG might cause problems, we supply a default auxiliary generator that can be used by the `unur_use_urng_aux_default` call (after the main URNG has been set). This default auxiliary generator can be changed with analogous calls as the (main) default uniform generator.

Function reference

Default uniform RNGs

`UNUR_URNG*` **unur_get_default_urng** (void) —
Get the pointer to the default URNG. The default URNG is used by all generators where no URNG was set explicitly by a `unur_set_urng` call.

`UNUR_URNG*` **unur_set_default_urng** (`UNUR_URNG*` *urng_new*) —
Change the default URNG for new parameter objects.

`UNUR_URNG*` **unur_set_default_urng_aux** (`UNUR_URNG*` *urng_new*) —
`UNUR_URNG*` **unur_get_default_urng_aux** (void) —
Analogous calls for default auxiliary generator.

Uniform RNGs for generator objects

`int` **unur_set_urng** (`UNUR_PAR*` *parameters*, `UNUR_URNG*` *urng*) —
Use the URNG *urng* for the new generator. This overwrite the default URNG. It also sets the auxiliary URNG to *urng*.

`UNUR_URNG*` **unur_chg_urng** (`UNUR_GEN*` *generator*, `UNUR_URNG*` *urng*) —
Change the URNG for the given generator. It returns the pointer to the old URNG that has been used by the generator. It also changes the auxiliary URNG to *urng* and thus overwrite the last `unur_chg_urng_aux` call.

`UNUR_URNG*` **unur_get_urng** (`UNUR_GEN*` *generator*) —
Get the pointer to the URNG that is used by the generator. This is usefull if two generators should share the same URNG.

- int `unur_set_urng_aux`** (UNUR_PAR* *parameters*, UNUR_URNG* *urng_aux*) –
Use the auxiliary URNG `urng_aux` for the new generator. (Default is the default URNG or the URNG from the last `unur_set_urng` call. Thus if the auxiliary generator should be different to the main URNG, `unur_set_urng_aux` must be called after `unur_set_urng`. The auxiliary URNG is used as second stream of uniform random number for correlation induction. It is not possible to set an auxiliary URNG for a method that does not use one (i.e. the call returns 0).
- int `unur_use_urng_aux_default`** (UNUR_PAR* *parameters*) –
Use the default auxiliary URNG. (It must be set after `unur_get_urng`.) It is not possible to set an auxiliary URNG for a method that does not use one (i.e. the call returns 0).
- int `unur_chgto_urng_aux_default`** (UNUR_GEN* *generator*) –
Switch to default auxiliary URNG. (It must be set after `unur_get_urng`.) It is not possible to set an auxiliary URNG for a method that does not use one (i.e. the call returns 0).
- UNUR_URNG* `unur_chg_urng_aux`** (UNUR_GEN* *generator*, UNUR_URNG* *urng_aux*) –
Change the auxiliary URNG for the given generator. It returns the pointer to the old auxiliary URNG that has been used by the generator. It has to be called after each `unur_chg_urng` when the auxiliary URNG should be different from the main URNG. It is not possible to change the auxiliary URNG for a method that does not use one (i.e. the call NULL).
- UNUR_URNG* `unur_get_urng_aux`** (UNUR_GEN* *generator*) –
Get the pointer to the auxiliary URNG that is used by the generator. This is useful if two generators should share the same URNG.

7 UNURAN Library of standard distributions

Although it is not its primary target, many distributions are already implemented in UNURAN. This section presents these available distributions and their parameters.

The syntax to get a distribution object for distributions `<dname>` is:

```
UNUR_DISTR* unur_distr_<dname> (double* params, int n_params)
```

params is an array of doubles of size *n_params* holding the parameters.

E.g. to get an object for the gamma distribution (with shape parameter) use

```
unur_distr_gamma( params, 1 );
```

Distributions may have default parameters with need not be given explicitly. E.g. The gamma distribution has three parameters: the shape, scale and location parameter. Only the (first) shape parameter is required. The others can be omitted and are then set by default values.

```
/* alpha = 5; default: beta = 1, gamma = 0 */
double fpar[] = {5.};
unur_distr_gamma( fpar, 1 );
```

```
/* alpha = 5, beta = 3; default: gamma = 0 */
double fpar[] = {5., 3.};
unur_distr_gamma( fpar, 2 );
```

```
/* alpha = 5, beta = 3, gamma = -2
double fpar[] = {5., 3., -2.};
unur_distr_gamma( fpar, 3 );
```

Important: Naturally the computational accuracy limits the possible parameters. There shouldn't be problems when the parameters of a distribution are in a "reasonable" range but e.g. the normal distribution $N(10^{15}, 1)$ won't yield the desired results. (In this case it would be better generating $N(0, 1)$ and *then* transform the results.)

Of course computational inaccuracy is not specific to UNURAN and should always be kept in mind when working with computers.

Important: The routines of the standard library are included for non-uniform random variate generation and not to provide special functions for statistical computations.

Remark

The following keywords are used in the tables:

<i>PDF</i>	probability density function, with variable <i>x</i> .
<i>PMF</i>	probability mass function, with variable <i>k</i> .
<i>constant</i>	normalization constant for given PDF and PMF, resp. They must be multiplied by <i>constant</i> to get the "real" PDF and PMF.
<i>CDF</i>	gives information whether the CDF is implemented in UNURAN.
<i>domain</i>	domain PDF and PMF, resp.
<i>parameters</i>	<i>n_std</i> (<i>n_total</i>): list list of parameters for distribution, where <i>n_std</i> is the number of parameters for the standard form of the distribution and <i>n_total</i> the total number for the (non-standard form of the) distribution. <i>list</i> is the list of parameters in the order as they are stored in the array of parameters. Optional parameter that can be omitted are enclosed in square brackets [...].

A detailed list of these parameters gives then the range of valid parameters and defaults for optional parameters that are used when these are omitted.

reference gives reference for distribution (see [Appendix B \[Bibliography\]](#), page 131).

special generators

lists available special generators for the distribution. The first number is the variant that to be set by `unur_cstd_set_variant` and `unur_dstd_set_variant` call, respectively. If no variant is set the default variant DEF is used. In the table the respective abbreviations DEF and INV are used for UNUR_STDGEN_DEFAULT and UNUR_STDGEN_INVERSION. Also the references for these methods are given (see [Appendix B \[Bibliography\]](#), page 131).

Notice that these generators might be slower than universal methods.

If DEF is omitted, the first entry is the default generator.

7.1 UNURAN Library of continuous univariate distributions

7.1.1 beta – Beta distribution

PDF: $(x - a)^{p-1} (b - x)^{q-1}$

constant: $1/(Beta(p, q) (b - a)^{p+q-1})$

domain: $a < x < b$

parameters 2 (4): p, q [, a, b]

No.	name		default	
[0]	<i>p</i>	> 0		(<i>scale</i>)
[1]	<i>q</i>	> 0		(<i>scale</i>)
[2]	<i>a</i>		0	(<i>location, scale</i>)
[3]	<i>b</i>	> <i>a</i>	1	(<i>location, scale</i>)

reference: [JKBc95] ch.25, p.210

7.1.2 cauchy – Cauchy distribution

PDF: $\frac{1}{1 + ((x - \theta) / \lambda)^2}$

constant: $\frac{1}{\pi \lambda}$

domain: $-\infty < x < \infty$

parameters 0 (2): [theta [, lambda]]

No.	name		default	
[0]	<i>θ</i>		0	(<i>location</i>)
[1]	<i>λ</i>	> 0	1	(<i>scale</i>)

reference: [JKBb94] ch.16, p.299

special generators:

INV Inversion method

7.1.3 chi – Chi distribution

PDF: $x^{\nu-1} \exp(-x^2/2)$

constant: $1/(2^{(\nu/2)-1} \Gamma(\nu/2))$

domain: $0 \leq x < \infty$

parameters 1 (1): nu

No.	name		default	
[0]	<i>ν</i>	> 0		(<i>shape</i>)

reference: [JKBb94] ch.18, p.417

special generators:

DEF Ratio of Uniforms with shift (only for $\nu \geq 1$) [MJ87]

7.1.4 chisquare – Chisquare distribution

PDF: $x^{(\nu/2)-1} \exp(-x/2)$

constant: $1/(2^{\nu/2} \Gamma(\nu/2))$

domain: $0 \leq x < \infty$

parameters 1 (1): nu

No.	name	default
[0]	$\nu > 0$	(shape (degrees of freedom))

reference: [JKBb94] ch.18, p.416

7.1.5 exponential – Exponential distribution

PDF: $\exp(-\frac{x-\theta}{\sigma})$

constant: $\frac{1}{\sigma}$

domain: $\theta \leq x < \infty$

parameters 0 (2): [sigma [, theta]]

No.	name	default
[0]	$\sigma > 0$	1 (scale)
[1]	θ	0 (location)

reference: [JKBb94] ch.19, p.494

special generators:

INV Inversion method

7.1.6 extremeI – Extreme value type I (Gumbel-type) distribution

PDF: $\exp(-\exp(-\frac{x-\zeta}{\theta}) - \frac{x-\zeta}{\theta})$

constant: $\frac{1}{\theta}$

domain: $-\infty < x < \infty$

parameters 0 (2): [zeta [, theta]]

No.	name	default
[0]	ζ	0 (location)
[1]	$\theta > 0$	1 (scale)

reference: [JKBc95] ch.22, p.2

special generators:

INV Inversion method

7.1.7 extremeII – Extreme value type II (Frechet-type) distribution

PDF: $\exp(-(\frac{x-\zeta}{\theta})^{-k}) (\frac{x-\zeta}{\theta})^{-k-1}$

constant: $\frac{k}{\theta}$

domain: $\zeta < x < \infty$

parameters 1 (3): k [, zeta [, theta]]

No.	name	default
[0]	$k > 0$	(shape)
[1]	ζ	0 (location)
[2]	$\theta > 0$	1 (scale)

reference: [JKBc95] ch.22, p.2

special generators:

INV Inversion method

7.1.8 gamma – Gamma distribution

PDF: $(\frac{x-\gamma}{\beta})^{\alpha-1} \exp(-\frac{x-\gamma}{\beta})$

constant: $1/(\beta\Gamma(\alpha))$

domain: $\gamma < x < \infty$

parameters 1 (3): alpha [, beta [, gamma]]

No.	name	default	
[0]	α > 0		(<i>shape</i>)
[1]	β > 0	1	(<i>scale</i>)
[2]	γ	0	(<i>location</i>)

reference: [JKBb94] ch.17, p.337

special generators:

DEF	Acceptance Rejection combined with Acceptance Complement [ADa74] [ADa82]
2	Rejection from log-logistic envelopes [CHa77]

7.1.9 laplace – Laplace distribution

PDF: $\exp(-\frac{|x-\theta|}{\phi})$

constant: $\frac{1}{2\phi}$

domain: $-\infty < x < \infty$

parameters 0 (2): [theta [, phi]]

No.	name	default	
[0]	θ	0	(<i>location</i>)
[1]	ϕ > 0	1	(<i>scale</i>)

reference: [JKBc95] ch.24, p.164

special generators:

INV	Inversion method
-----	------------------

7.1.10 logistic – Logistic distribution

PDF: $\exp(-\frac{x-\alpha}{\beta} (1 + \exp(-\frac{x-\alpha}{\beta}))^{-2}$

constant: $\frac{1}{\beta}$

domain: $-\infty < x < \infty$

parameters 0 (2): [alpha [, beta]]

No.	name	default	
[0]	α	0	(<i>location</i>)
[1]	β > 0	1	(<i>scale</i>)

reference: [JKBc95] ch.23, p.115

special generators:

INV	Inversion method
-----	------------------

7.1.11 lomax – Lomax distribution (Pareto distribution of second kind)

PDF: $(x + C)^{-(a+1)}$

constant: $a C^a$

domain: $0 \leq x < \infty$

parameters 1 (2): a [, C]

No.	name	default
[0]	a > 0	(<i>shape</i>)
[1]	C > 0	1 (<i>scale</i>)

reference: [JKBb94] ch.20, p.575

special generators:

INV Inversion method

7.1.12 normal – Normal distribution

PDF: $\exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$

constant: $\frac{1}{\sigma\sqrt{2\pi}}$

domain: $-\infty < x < \infty$

parameters 0 (2): [mu [, sigma]]

No.	name	default
[0]	μ	0 (<i>location</i>)
[1]	$\sigma > 0$	1 (<i>scale</i>)

reference: [JKBb94] ch.13, p.80

special generators:

DEF ACR method (Acceptance-Complement Ratio) [HDa90]
 1 Box-Muller method [BMa58]
 2 Polar method with rejection [MGa62]
 3 Kindermann-Ramage method [KRa76]
 INV Inversion method (slow)

7.1.13 pareto – Pareto distribution (of first kind)

PDF: $x^{-(a+1)}$

constant: $a k^a$

domain: $k < x < \infty$

parameters 2 (2): k, a

No.	name	default
[0]	k > 0	(<i>shape, location</i>)
[1]	a > 0	(<i>shape</i>)

reference: [JKBb94] ch.20, p.574

special generators:

INV Inversion method

7.1.14 powerexponential – Powerexponential (Subbotin) distribution

PDF: $\exp(-|x|^\tau)$

constant: $1/(2\Gamma(1 + 1/\tau))$

domain: $-\infty < x < \infty$

parameters 1 (1): tau

No.	name	default	
[0]	τ	> 0	(shape)

reference: [JKBc95] ch.24, p.195

special generators:

DEF	Transformed density rejection (only for $\tau \geq 1$) [DLa86]
-----	---

7.1.15 rayleigh – Rayleigh distribution

PDF: $x \exp(-1/2 (\frac{x}{\sigma})^2)$

constant: $\frac{1}{\sigma^2}$

domain: $0 \leq x < \infty$

parameters 1 (1): sigma

No.	name	default	
[0]	σ	> 0	(scale)

reference: [JKBb94] ch.18, p.456

7.1.16 student – Student's t distribution

PDF: $(1 + \frac{t^2}{\nu})^{-(\nu+1)/2}$

constant: $\frac{1}{\sqrt{\nu} B(1/2, \nu/2)}$

CDF: not implemented!

domain: $-\infty < x < \infty$

parameters 1 (1): nu

No.	name	default	
[0]	ν	> 0	(shape)

reference: [JKBc95] ch.28, p.362

7.1.17 triangular – Triangular distribution

PDF: $2x/H$, for $0 \leq x \leq H$
 $2(1-x)/(1-H)$, for $H \leq x \leq 1$

constant: 1

domain: $0 \leq x \leq 1$

parameters 0 (1): [H]

No.	name	default	
[0]	H	$0 \leq H \leq 1$	1/2 (shape)

reference: [JKBc95] ch.26, p.297

special generators:

INV	Inversion method
-----	------------------

7.1.18 uniform – Uniform distribution

PDF: $\frac{1}{b-a}$

constant: 1

domain: $a < x < b$

parameters 0 (2): [a, b]

No.	name	default	
[0]	<i>a</i>	0	(<i>location</i>)
[1]	<i>b</i> > <i>a</i>	1	(<i>location</i>)

reference: [JKBc95] ch.26, p.276

special generators:

INV Inversion method

7.1.19 weibull – Weibull distribution

PDF: $(\frac{x-\zeta}{\alpha})^{c-1} \exp(-(\frac{x-\zeta}{\alpha})^c)$

constant: $\frac{c}{\alpha}$

domain: $\zeta < x < \infty$

parameters 1 (3): c [, alpha [, zeta]]

No.	name	default	
[0]	<i>c</i> > 0		(<i>shape</i>)
[1]	α > 0	1	(<i>scale</i>)
[2]	ζ	0	(<i>location</i>)

reference: [JKBb94] ch.21, p.628

special generators:

INV Inversion method

7.2 UNURAN Library of continuous multivariate distributions

7.3 UNURAN Library of discrete univariate distributions

At the moment there are no CDFs implemented for discrete distribution. Thus `unur_distr_discr_upd_pmfsum` does not work properly for truncated distribution.

7.3.1 binomial – Binomial distribution

PMF: $\binom{n}{k} p^k (1-p)^{n-k}$

constant: 1

domain: $0 \leq k \leq n$

parameters 2 (2): n, p

No.	name	default	
[0]	<i>n</i> ≥ 1		(<i>no. of elements</i>)
[1]	<i>p</i> $0 < p < 1$		(<i>shape</i>)

reference: [JKKa92] ch.3, p.105

special generators:

DEF Ratio of Uniforms/Inversion [STa89]

7.3.2 geometric – Geometric distribution

PMF: $p(1-p)^k$

constant: 1

domain: $0 \leq k < \infty$

parameters 1 (1): p

No.	name	default	
[0]	p	$0 < p < 1$	(shape)

reference: [JKKa92] ch.5.2, p.201

special generators:

INV	Inversion method
-----	------------------

7.3.3 hypergeometric – Hypergeometric distribution

PMF: $\binom{M}{k} \binom{N-M}{n-k} / \binom{N}{n}$

constant: 1

domain: $\max(0, n - N + M) \leq k \leq \min(n, M)$

parameters 3 (3): N, M, n

No.	name	default	
[0]	N	≥ 1	(no. of elements)
[1]	M	$1 \leq M \leq N$	(shape)
[2]	n	$1 \leq n \leq N$	(shape)

reference: [JKKa92] ch.6, p.237

special generators:

DEF	Ratio of Uniforms/Inversion [STa89]
-----	-------------------------------------

7.3.4 logarithmic – Logarithmic distribution

PMF: θ^k / k

constant: $-\log(1 - \theta)$

domain: $1 \leq k < \infty$

parameters 1 (1): theta

No.	name	default	
[0]	θ	$0 < \theta < 1$	(shape)

reference: [JKKa92] ch.7, p.285

special generators:

DEF	Inversion/Transformation [KAa81]
-----	----------------------------------

7.3.5 negativebinomial – Negative Binomial distribution

PMF: $\binom{k+r-1}{r-1} p^r (1-p)^k$

constant: 1

domain: $0 \leq k < \infty$

parameters 2 (2): p, r

No.	name	default	
[0]	p	$0 < p < 1$	(shape)
[1]	r	> 0	(shape)

reference: [JKKa92] ch.5.1, p.200

7.3.6 poisson – Poisson distribution

PMF: $\theta^k/k!$

constant: $\exp(\theta)$

domain: $0 \leq k < \infty$

parameters 1 (1): theta

No.	name	default
[0]	$\theta > 0$	(<i>shape</i>)

reference: [JKKa92] ch.4, p.151

special generators:

DEF	Tabulated Inversion combined with Acceptance Complement [ADb82]
2	Tabulated Inversion combined with Patchwork Rejection [ZHa94]

8 Error handling

This chapter describes the way that UNURAN routines report errors.

8.1 Error reporting

UNURAN routines report an error whenever they cannot perform the task requested of them. For example, apply transformed density rejection to a distribution that violates the T-concavity condition, or trying to set a parameter that is out of range. It might also happen that the setup fails for transformed density rejection for a T-concave distribution with some extreme density function simply because of round-off errors that makes the generation of a hat function numerically impossible. Situations like this may happen when using black box algorithms and you should check the return values of all routines.

All `..._set_...`, and `..._chg_...` calls return 0 if it was not possible to set or change the desired parameters, e.g. because the given values are out of range, or simply because you have changed the method but not the corresponding set call and thus an invalid parameter or generator object is used.

All routines that return a pointer to the requested object will return a NULL pointer in case of error. (Thus you should always check the pointer to avoid possible segmentation faults. Sampling routines usually do not check the given pointer to the generator object. However you can switch on checking for NULL pointer defining the compiler switch `UNUR_ENABLE_CHECKNULL` in `'unuran_config.h'` to avoid nasty segmentation faults.)

The library distinguishes between two major classes of error:

(fatal) errors:

The library was not able to construct the requested object.

warnings: Some problems encounters while constructing a generator object. The routine has tried to solve the problem but the resulting object might not be what you want. For example, chosing a special variant of a method does not work and the initialization routine might switch to another variant. Then the generator produces random variates of the requested distribution but correlation induction is not possible. However it also might happen that changing the domain of a distribution has failed. Then the generator produced random variates with too large/too small range, i.e. their distribution is not correct

It is obvious from the example that this distinction between errors and warning is rather crude and sometimes arbitrary.

UNURAN routines use the global variable `unur_errno` to report errors, completely analogously to C library's `errno`. (However this approach is not thread-safe. There can be only one instance of a global variable per program. Different threads of execution may overwrite `unur_errno` simultaneously). Thus when an error occurs the caller of the routine can examine the error code in `unur_errno` to get more details about the reason why a routine failed. You get a short description of the error by a `unur_get_strerror` call. All the error code numbers have prefix `UNUR_ERR_` and expand to non-zero constant unsigned integer values. Error codes are divided into six main groups.

List of error codes

- Errors that occurred while handling distribution objects.

`UNUR_ERR_DISTR_SET`
set failed (invalid parameter).

`UNUR_ERR_DISTR_GET`
get failed (parameter not set).

`UNUR_ERR_DISTR_NPARAMS`
invalid number of parameters.

- UNUR_ERR_DISTR_DOMAIN
parameter(s) out of domain.
- UNUR_ERR_DISTR_GEN
invalid variant for special generator.
- UNUR_ERR_DISTR_REQUIRED
incomplete distribution object, entry missing.
- UNUR_ERR_DISTR_UNKNOWN
unknown distribution, cannot handle.
- UNUR_ERR_DISTR_INVALID
invalid distribution object.
- UNUR_ERR_DISTR_DATA
data are missing.
- Errors that occurred while handling parameter objects.
 - UNUR_ERR_PAR_SET
set failed (invalid parameter)
 - UNUR_ERR_PAR_VARIANT
invalid variant -> using default
 - UNUR_ERR_PAR_INVALID
invalid parameter object
- Errors that occurred while handling generator objects.
 - UNUR_ERR_GEN
error with generator object.
 - UNUR_ERR_GEN_DATA
(possibly) invalid data.
 - UNUR_ERR_GEN_CONDITION
condition for method violated.
 - UNUR_ERR_GEN_INVALID
invalid generator object.
 - UNUR_ERR_GEN_SAMPLING
sampling error.
- Errors that occurred while parsing strings.
 - UNUR_ERR_STR
error in string.
 - UNUR_ERR_STR_UNKNOWN
unknown keyword.
 - UNUR_ERR_STR_SYNTAX
syntax error.
 - UNUR_ERR_STR_INVALID
invalid parameter.
 - UNUR_ERR_FSTR_SYNTAX
syntax error in function string.
 - UNUR_ERR_FSTR_DERIV
cannot derivate function.
- Other run time errors.

<code>UNUR_ERR_DOMAIN</code>	argument out of domain.
<code>UNUR_ERR_ROUNDOFF</code>	(serious) round-off error.
<code>UNUR_ERR_MALLOCC</code>	virtual memory exhausted.
<code>UNUR_ERR_NULL</code>	invalid NULL pointer.
<code>UNUR_ERR_COOKIE</code>	invalid cookie.
<code>UNUR_ERR_GENERIC</code>	generic error.
<code>UNUR_ERR_COMPILE</code>	Requested routine requires different compilation switches. Recompilation of library necessary.
<code>UNUR_ERR_SHOULD_NOT_HAPPEN</code>	Internal error, that should not happen. Please report this bug!

Function reference

`extern unsigned unur_errno` Variable
 Global variable for reporting diagnostics of error.

8.2 Output streams

In addition to reporting error via the `unur_errno` mechanism the library also provides an (optional) error handler. The error handler is called by the library functions when they are about to report an error. Then a short error diagnostics is written via two output streams. Both can be switched on/off by compiler flag `UNUR_WARNINGS_ON` in `'unuran_config.h'`.

The first stream is `stderr`. It can be enabled by defining the macro `UNUR_ENABLE_STDERR` in `'unuran_config.h'`.

The second stream can be set arbitrarily by the `unur_set_stream` call. If no such stream is given by the user a default stream is used by the library: all warnings and error messages are written into the file `unuran.log` in the current working directory. The name of this file defined by the macro `UNUR_LOG_FILE` in `'unuran_config.h'`. If the `stdout` should be used, define this macro by `"stdout"`.

This output stream is also used to log descriptions of build generator objects and for writing debugging information. If you want to use this output stream for your own programs use `unur_get_stream` to get its file handler. This stream is enabled by the compiler switch `UNUR_ENABLE_LOGFILE` in `'unuran_config.h'`.

All warnings, error messages and all debugging information are written onto the same output stream. To distinguish between the messages for different generators define the macro `UNUR_ENABLE_GENID` in `'unuran_config.h'`. Then every generator object has a unique identifier that is used for every message.

Function reference

`const char* unur_get_strerror (const int unur_errno)`
 Get a short description for error code value.

FILE* unur_set_stream (FILE* *new_stream*) —

Set new file handle for output stream; the old file handle is returned. The NULL pointer is not allowed. (If you want to disable logging of debugging information use `unur_set_default_debug(UNUR_DEBUG_OFF)` instead.)

The output stream is used to report errors and warning, and debugging information. It is also used to log descriptions of build generator objects (when this feature is switched on; see also ?).

FILE* unur_get_stream (void) —

Get the file handle for the current output stream.

9 Debugging

The UNURAN library has several debugging levels which can be switched on/off by debugging flags. This debugging feature can be enabled by defining the macro `UNUR_ENABLE_LOGGING` in `'unuran_config.h'`. The debugging levels range from print a short description of the build generator object to a detailed description of hat functions til tracing the sampling routines. The output is print onto the output stream obtained by `unur_get_stream` (see also ?). These flags can be set or changed by the respective calls `unur_set_debug` and `unur_chg_debug` independently for each generator. The default debugging flags are given by the macro `UNUR_DEBUGFLAG_DEFAULT` in `'unuran_config.h'`. This default can be overwritten at run time by a `unur_set_default_debug` call.

Off course these debugging flags depend on the chosen method. Since most of these are merely for debugging the library itself, a description of the flags are given in the corresponding source files of the method. Nevertheless the following flags can be used with all methods.

Common debug flags:

```
UNUR_DEBUG_OFF
    switch off all debugging information

UNUR_DEBUG_ALL
    all avaivable information

UNUR_DEBUG_INIT
    parameters of generator object after initialization

UNUR_DEBUG_SETUP
    data created at setup

UNUR_DEBUG_ADAPT
    data created during adaptive steps

UNUR_DEBUG_SAMPLE
    trace sampling
```

Almost all routines check a given pointer they read from or write to the given adress. This does not hold for time-critical routines like all sampling routines. Then your are responsible for checking a pointer that is returned from a `unur_init` call. However it is possible to turn on checking for invalid NULL pointers even in such time-critical routines by defining `UNUR_ENABLE_CHECKNULL` in `'unuran_config.h'`.

Another debugging tool used in the library are magic cookies that validate a given pointer. It produces an error whenever a given pointer points to an object that is invalid in the context. The usage of magic cookies can be switched on by defining `UNUR_COOKIES` in `'unuran_config.h'`.

Function reference

```
int unur_set_debug (UNUR_PAR* parameters, unsigned debug)           —
    Set debugging flags for generator.

int unur_chg_debug (UNUR_GEN* generator, unsigned debug)           —
    Change debugging flags for generator.

int unur_set_default_debug (unsigned debug)                           —
    Overwrite the default debugging flag.
```


10 Testing

The following routines can be used to test the performance of the implemented generators and can be used to verify the implementations. They are declared in ‘`unuran_tests.h`’ which has to be included.

Function reference

void unur_run_tests (UNUR_PAR* *parameters*, unsigned *tests*)

Run a battery of tests. The following tests are available (use | to combine these tests):

UNUR_TEST_ALL

run all possible tests.

UNUR_TEST_TIME

estimate generation times.

UNUR_TEST_N_URNG

count number of uniform random numbers

UNUR_TEST_CHI2

run χ^2 test for goodness of fit

UNUR_TEST_SAMPLE

print a small sample.

All these tests can be started individually (see below).

void unur_test_printsample (UNUR_GEN* *generator*, int *n_rows*, int *n_cols*, FILE* *out*)

Print a small sample with *n_rows* rows and *n_cols* columns. *out* is the output stream to which all results are written.

UNUR_GEN* unur_test_timing (UNUR_PAR* *parameters*, int *log_sample_size*, double* *time_setup*, double* *time_sample*, int *verbosity*, FILE* *out*)

Timing. *parameters* is an parameter object for which setup time and marginal generation times have to be measured. The results are written into *time_setup* and *time_sample*, respectively. *log_sample_size* is the common logarithm of the sample size that is used for timing.

If *verbosity* is TRUE then a small table is printed to the `stdout` with setup time, marginal generation time and average generation times for generating 10, 100, . . . random variates. All times are given in micro seconds and relative to the generation times for the underlying uniform random number (using the UNIF interface) and an exponential distributed random variate using the inversion method.

The created generator object is returned. If a generator object could not be created successfully, then NULL is returned.

If *verbosity* is TRUE the result is written to the output stream *out*.

Notice: All timing results are subject to heavy changes. Reruning timings usually results in different results. Minor changes in the source code can cause changes in such timings up to 25 percent.

double unur_test_timing_uniform (const UNUR_PAR* *parameters*, int *log_sample_size*)

double unur_test_timing_exponential (const UNUR_PAR* *parameters*, int *log_sample_size*)

Marginal generation times for the underlying uniform random number (using the UNIF interface) and an exponential distributed random variate using the inversion method. These times are used in `unur_test_timing` to compute the relative timings results.

double unur_test_timing_total (const UNUR_PAR* *parameters*, int *samplesize*, double *max_duration*)

Timing. *parameters* is an parameter object for which average times a sample of size *samplesize* (including setup) are estimated. Thus sampling is repeated and the median of these timings is returned (in micro seconds). The number of iterations is computed automatically such that the total amount of time necessary for the test does not exceed *max_duration* (given in seconds).

If an error occurs then -1 is returned.

Notice: All timing results are subject to heavy changes. Reruning timings usually results in different results. Minor changes in the source code can cause changes in such timings up to 25 percent.

int unur_test_count_urn (UNUR_GEN* *generator*, int *samplesize*, int *verbosity*, FILE* *out*)

Count used uniform random numbers. It returns the total number of uniform random numbers required for a sample of non-uniform random variates of size *samplesize*. Counting uniform random numbers might not work for the chosen UNUR_URNG_TYPE in 'unuran_config.h'. In this case -1 is returned.

If *verbosity* is TRUE the result is written to the output stream *out*.

double unur_test_chi2 (UNUR_GEN* *generator*, int *intervals*, int *samplesize*, int *classmin*, int *verbosity*, FILE* *out*)

Run a Chi² test with the *generator*. The resulting p-value is returned.

It works with discrete and continuous univariate distributions. For the latter the CDF of the distribution is required.

intervals is the number of intervals that is used for continuous univariate distributions. *samplesize* is the size of the sample that is used for testing. If it is set to 0 then a sample of size *intervals*² is used (bounded to some upper bound).

classmin is the minimum number of expected entries per class. If a class has too few entries then some classes are joined.

verbosity controls the output of the routine. If it is set to 1 then the result is written to the output stream *out*. If it is set to 2 additionally the list of expected and observed data is printed. There is no output when it is set to 0.

int unur_test_moments (UNUR_GEN* *generator*, double* *moments*, int *n_moments*, int *samplesize*, int *verbosity*, FILE* *out*)

Computes the first *n_moments* central moments for a sample of size *samplesize*. The result is stored into the array *moments*. *n_moments* must be an integer between 1 and 4.

If *verbosity* is TRUE the result is written to the output stream *out*.

double unur_test_correlation (UNUR_GEN* *generator1*, UNUR_GEN* *generator2*, int *samplesize*, int *verbosity*, FILE* *out*)

Compute the correlation coefficient between streams from *generator1* and *generator2* for two samples of size *samplesize*. The resulting correlation is returned.

If *verbosity* is TRUE the result is written to the output stream *out*.

int unur_test_quartiles (UNUR_GEN* *generator*, double* *q0*, double* *q1*, double* *q2*, double* *q3*, double* *q4*, int *samplesize*, int *verbosity*, FILE* *out*)

Estimate quartiles of sample of size *samplesize*. The resulting quantiles are stored in the variables *q*:

q0 minimum

q1 25%

q2 median (50%)

q3 75%

q4 maximum

If *verbosity* is **TRUE** the result is written to the output stream *out*.

11 Miscellaneous

11.1 Mathematics

The following macros have been defined

`UNUR_INFINITY`

indicates infinity for floating point numbers (of type `double`). Internally `HUGE_VAL` is used.

`INT_MAX`

`INT_MIN` indicate infinity and minus infinity, resp., for integers (defined by ISO C standard).

`TRUE`

`FALSE` boolean expression for return values of `set` functions.

Appendix A Glossary

CDF	cumulative distribution function
HR	hazard rate (or failure rate)
PDF	probability density function
dPDF	derivative (gradient) of probability density function
PMF	probability mass function
PV	(finite) probability vector

T-concave**T_c-concave**

a function $f(x)$ is called T-concave if the transformed function $T(f(x))$ is concave. We only deal with transformations T_c , where

$$c = 0 \quad T(x) = \log(x)$$

$$c = -0.5 \quad T(x) = -1/\sqrt{x}$$

$$c \neq 0 \quad T(x) = \text{sign}(x) * x^c$$

Appendix B Bibliography

Standard Distributions

- [JKKa92] N.L. JOHNSON, S. KOTZ, AND A.W. KEMP (1992). *Univariate Discrete Distributions*, 2nd edition, John Wiley & Sons, Inc., New York.
- [JKBb94] N.L. JOHNSON, S. KOTZ, AND N. BALAKRISHNAN (1994). *Continuous Univariate Distributions*, Volume 1, 2nd edition, John Wiley & Sons, Inc., New York.
- [JKBc95] N.L. JOHNSON, S. KOTZ, AND N. BALAKRISHNAN (1995). *Continuous Univariate Distributions*, Volume 2, 2nd edition, John Wiley & Sons, Inc., New York.
- [JKBd97] N.L. JOHNSON, S. KOTZ, AND N. BALAKRISHNAN (1997). *Discrete Multivariate Distributions*, John Wiley & Sons, Inc., New York.
- [KBJe00] S. KOTZ, N. BALAKRISHNAN, AND N.L. JOHNSON (2000). *Continuous Multivariate Distributions*, Volume 1: Models and Applications, John Wiley & Sons, Inc., New York.

Universal methods

- [AJa93] J.H. AHRENS (1993). *Sampling from general distributions by suboptimal division of domains*, Grazer Math. Berichte 319, 30pp.
- [AJa95] J.H. AHRENS (1995). *An one-table method for sampling from continuous and discrete distributions*, Computing 54(2), pp. 127-146.
- [CAa74] H.C. CHEN AND Y. ASAU (1974). *On generating random variates from an empirical distribution*, AIIE Trans. 6, pp. 163-166.
- [DLa86] L. DEVROYE (1986). *Non-Uniform Random Variate Generation*, Springer Verlag, New York.
- [GWa92] W.R. GILKS AND P. WILD (1992). *Adaptive rejection sampling for Gibbs sampling*, Applied Statistics 41, pp. 337-348.
- [HWa95] W. HOERMANN (1995). *A rejection technique for sampling from T-concave distributions*, ACM Trans. Math. Software 21(2), pp. 182-193.
- [HDa96] W. HOERMANN AND G. DERFLINGER (1996). *Rejection-inversion to generate variates from monotone discrete distributions*, ACM TOMACS 6(3), 169-184.
- [HLa00] W. HOERMANN AND J. LEYDOLD (2000). *Automatic random variate generation for simulation input*. In: J.A. Joines, R. Barton, P. Fishwick, K. Kang (eds.), Proceedings of the 2000 Winter Simulation Conference, pp. 675-682.
- [LJa00] J. LEYDOLD (2000). *Automatic Sampling with the Ratio-of-Uniforms Method*, ACM Trans. Math. Software 26(1), pp. 78-98.
- [LJa01] J. LEYDOLD (2001). *A simple universal generator for continuous and discrete univariate T-concave distributions*, ACM Trans. Math. Software 27(1), pp. 66-82.
- [LJa02] J. LEYDOLD (2002). *Short universal generators via generalized ratio-of-uniforms method*, Preprint
- [WAa77] A.J. WALKER (1977). *An efficient method for generating discrete random variables with general distributions*, ACM Trans. Math. Software 3, pp. 253-256.

Special generators

- [ADa74] J.H. AHRENS, U. DIETER (1974). *Computer methods for sampling from gamma, beta, Poisson and binomial distributions*, Computing 12, 223-246.
- [ADa82] J.H. AHRENS, U. DIETER (1982). *Generating gamma variates by a modified rejection technique*, Communications of the ACM 25, 47-54.

- [ADb82] J.H. AHRENS, U. DIETER (1982). *Computer generation of Poisson deviates from modified normal distributions*, ACM Trans. Math. Software 8, 163-179.
- [BMa58] G.E.P. BOX AND M.E. MULLER (1958). *A note on the generation of random normal deviates*, Annals Math. Statist. 29, 610-611.
- [CHa77] R.C.H. CHENG (1977). *The Generation of Gamma Variables with Non-Integral Shape Parameter*, Appl. Statist. 26(1), 71-75.
- [HDa90] W. HOERMANN AND G. DERFLINGER (1990). *The ACR Method for generating normal random variables*, OR Spektrum 12, 181-185.
- [KAa81] A.W. KEMP (1981). *Efficient generation of logarithmically distributed pseudo-random variables*, Appl. Statist. 30, 249-253.
- [KRa76] A.J. KINDERMAN AND J.G. RAMAGE (1976). *Computer Generation of Normal Random Variables*, J. Am. Stat. Assoc. 71(356), 893 - 898.
- [MJa87] J.F. MONAHAN (1987). *An algorithm for generating chi random variables*, ACM Trans. Math. Software 13, 168-172.
- [MGa62] G. MARSAGLIA (1962). *Improving the Polar Method for Generating a Pair of Random Variables*, Boeing Sci. Res. Lab., Seattle, Washington.
- [STa89] E. STADLOBER (1989). *Sampling from Poisson, binomial and hypergeometric distributions: ratio of uniforms as a simple and fast alternative*, Bericht 303, Math. Stat. Sektion, Forschungsgesellschaft Joanneum, Graz.
- [ZHa94] H. ZECHNER (1994). *Efficient sampling from continuous and discrete unimodal distributions*, Pd.D. Thesis, 156 pp., Technical University Graz, Austria.

Appendix C Function Index

-		unur_dgt_set_guidefactor.....	99
_unur_tdr_is_ARS_running.....	81	unur_dgt_set_variant.....	99
F		unur_distr_<dname>.....	107
FALSE.....	127	unur_distr_beta.....	108
I		unur_distr_binomial.....	113
INT_MAX.....	127	unur_distr_cauchy.....	108
INT_MIN.....	127	unur_distr_cemp_get_data.....	49
T		unur_distr_cemp_new.....	48
TRUE.....	127	unur_distr_cemp_read_data.....	49
U		unur_distr_cemp_set_data.....	48
unur_arou_chg_verify.....	63	unur_distr_chi.....	108
unur_arou_get_hatarea.....	62	unur_distr_chisquare.....	109
unur_arou_get_sqratio.....	62	unur_distr_cont_eval_cdf.....	43
unur_arou_get_squeezearea.....	62	unur_distr_cont_eval_dpdf.....	43
unur_arou_new.....	62	unur_distr_cont_eval_hr.....	45
unur_arou_set_center.....	63	unur_distr_cont_eval_pdf.....	43
unur_arou_set_cpoints.....	62	unur_distr_cont_get_cdf.....	43
unur_arou_set_guidefactor.....	63	unur_distr_cont_get_cdfstr.....	44
unur_arou_set_max_segments.....	62	unur_distr_cont_get_domain.....	44
unur_arou_set_max_sqratio.....	62	unur_distr_cont_get_dpdf.....	43
unur_arou_set_pedantic.....	63	unur_distr_cont_get_dpdfstr.....	44
unur_arou_set_usecenter.....	63	unur_distr_cont_get_hr.....	45
unur_arou_set_verify.....	63	unur_distr_cont_get_hrstr.....	45
unur_auto_new.....	58	unur_distr_cont_get_mode.....	46
unur_auto_set_logss.....	58	unur_distr_cont_get_pdf.....	43
unur_chg_debug.....	121	unur_distr_cont_get_pdfarea.....	46
unur_chg_urng.....	104	unur_distr_cont_get_pdfparams.....	44
unur_chgto_urng_aux.....	105	unur_distr_cont_get_pdfstr.....	44
unur_cstd_chg_pdfparams.....	64	unur_distr_cont_get_truncated.....	44
unur_cstd_chg_truncated.....	64	unur_distr_cont_new.....	42
unur_cstd_new.....	64	unur_distr_cont_set_cdf.....	42
unur_cstd_set_variant.....	64	unur_distr_cont_set_cdfstr.....	43
unur_dari_chg_domain.....	97	unur_distr_cont_set_domain.....	44
unur_dari_chg_mode.....	97	unur_distr_cont_set_dpdf.....	42
unur_dari_chg_pmfparams.....	97	unur_distr_cont_set_hr.....	45
unur_dari_chg_pmfsum.....	98	unur_distr_cont_set_hrstr.....	45
unur_dari_chg_verify.....	97	unur_distr_cont_set_mode.....	45
unur_dari_new.....	96	unur_distr_cont_set_pdf.....	42
unur_dari_reinit.....	96	unur_distr_cont_set_pdfarea.....	46
unur_dari_set_cpfactor.....	97	unur_distr_cont_set_pdfparams.....	44
unur_dari_set_squeeze.....	97	unur_distr_cont_set_pdfstr.....	43
unur_dari_set_tablesize.....	97	unur_distr_cont_upd_mode.....	45
unur_dari_set_verify.....	97	unur_distr_cont_upd_pdfarea.....	46
unur_dari_upd_mode.....	98	unur_distr_corder_eval_cdf.....	47
unur_dari_upd_pmfsum.....	98	unur_distr_corder_eval_dpdf.....	47
unur_dau_new.....	98	unur_distr_corder_eval_pdf.....	47
unur_dau_set_urnfactor.....	98	unur_distr_corder_get_cdf.....	47
UNUR_DEBUG_ADAPT.....	121	unur_distr_corder_get_distribution.....	46
UNUR_DEBUG_ALL.....	121	unur_distr_corder_get_domain.....	47
UNUR_DEBUG_INIT.....	121	unur_distr_corder_get_dpdf.....	47
UNUR_DEBUG_OFF.....	121	unur_distr_corder_get_mode.....	48
UNUR_DEBUG_SAMPLE.....	121	unur_distr_corder_get_pdf.....	47
UNUR_DEBUG_SETUP.....	121	unur_distr_corder_get_pdfarea.....	48
unur_dgt_new.....	99	unur_distr_corder_get_pdfparams.....	47
		unur_distr_corder_get_rank.....	47
		unur_distr_corder_get_truncated.....	48
		unur_distr_corder_new.....	46
		unur_distr_corder_set_domain.....	47
		unur_distr_corder_set_mode.....	48
		unur_distr_corder_set_pdfarea.....	48
		unur_distr_corder_set_pdfparams.....	47
		unur_distr_corder_set_rank.....	47
		unur_distr_corder_upd_mode.....	48

unur_distr_corder_upd_pdfarea	48	unur_distr_poisson	115
unur_distr_cvec_eval_dpdpf	50	unur_distr_powerexponential	111
unur_distr_cvec_eval_pdf	50	unur_distr_rayleigh	112
unur_distr_cvec_get_covar	50	unur_distr_set_name	41
unur_distr_cvec_get_dpdpf	50	unur_distr_student	112
unur_distr_cvec_get_mean	50	unur_distr_triangular	112
unur_distr_cvec_get_mode	51	unur_distr_uniform	112
unur_distr_cvec_get_pdf	49	unur_distr_weibull	113
unur_distr_cvec_get_pdfparams	51	unur_dsrou_chg_cdfatmode	101
unur_distr_cvec_get_pdfvol	51	unur_dsrou_chg_domain	101
unur_distr_cvec_new	49	unur_dsrou_chg_mode	101
unur_distr_cvec_set_covar	50	unur_dsrou_chg_pmfparams	100
unur_distr_cvec_set_dpdpf	49	unur_dsrou_chg_pmfsum	101
unur_distr_cvec_set_mean	50	unur_dsrou_chg_verify	100
unur_distr_cvec_set_mode	51	unur_dsrou_new	100
unur_distr_cvec_set_pdf	49	unur_dsrou_reinit	100
unur_distr_cvec_set_pdfparams	51	unur_dsrou_set_cdfatmode	100
unur_distr_cvec_set_pdfvol	51	unur_dsrou_set_verify	100
unur_distr_cvemp_get_data	52	unur_dsrou_upd_mode	101
unur_distr_cvemp_new	52	unur_dsrou_upd_pmfsum	101
unur_distr_cvemp_read_data	52	unur_dstd_chg_pmfparams	102
unur_distr_cvemp_set_data	52	unur_dstd_new	102
unur_distr_discr_eval_cdf	53	unur_dstd_set_variant	102
unur_distr_discr_eval_pmf	53	unur_empk_chg_smoothing	89
unur_distr_discr_eval_pv	53	unur_empk_chg_varcor	89
unur_distr_discr_get_cdfstr	54	unur_empk_new	88
unur_distr_discr_get_domain	54	unur_empk_set_beta	89
unur_distr_discr_get_mode	55	unur_empk_set_kernel	88
unur_distr_discr_get_pmfparams	54	unur_empk_set_kernelgen	89
unur_distr_discr_get_pmfstr	54	unur_empk_set_positive	89
unur_distr_discr_get_pmfsum	55	unur_empk_set_smoothing	89
unur_distr_discr_get_pv	53	unur_empk_set_varcor	89
unur_distr_discr_make_pv	53	unur_empl_new	90
unur_distr_discr_new	52	UNUR_ERR_COMPILE	119
unur_distr_discr_set_cdf	53	UNUR_ERR_COOKIE	119
unur_distr_discr_set_cdfstr	54	UNUR_ERR_DISTR_DATA	118
unur_distr_discr_set_domain	54	UNUR_ERR_DISTR_DOMAIN	118
unur_distr_discr_set_mode	55	UNUR_ERR_DISTR_GEN	118
unur_distr_discr_set_pmf	53	UNUR_ERR_DISTR_GET	117
unur_distr_discr_set_pmfparams	54	UNUR_ERR_DISTR_INVALID	118
unur_distr_discr_set_pmfstr	54	UNUR_ERR_DISTR_NPARAMS	117
unur_distr_discr_set_pmfsum	55	UNUR_ERR_DISTR_REQUIRED	118
unur_distr_discr_set_pv	53	UNUR_ERR_DISTR_SET	117
unur_distr_discr_upd_mode	55	UNUR_ERR_DISTR_UNKNOWN	118
unur_distr_discr_upd_pmfsum	55	UNUR_ERR_DOMAIN	119
unur_distr_exponential	109	UNUR_ERR_FSTR_DERIV	118
unur_distr_extremeI	109	UNUR_ERR_FSTR_SYNTAX	118
unur_distr_extremeII	109	UNUR_ERR_GEN	118
unur_distr_free	41	UNUR_ERR_GEN_CONDITION	118
unur_distr_gamma	110	UNUR_ERR_GEN_DATA	118
unur_distr_geometric	113	UNUR_ERR_GEN_INVALID	118
unur_distr_get_dim	41	UNUR_ERR_GEN_SAMPLING	118
unur_distr_get_name	41	UNUR_ERR_GENERIC	119
unur_distr_get_type	42	UNUR_ERR_MALLOC	119
unur_distr_hypergeometric	114	UNUR_ERR_NULL	119
unur_distr_is_cemp	42	UNUR_ERR_PAR_INVALID	118
unur_distr_is_cont	42	UNUR_ERR_PAR_SET	118
unur_distr_is_cvec	42	UNUR_ERR_PAR_VARIANT	118
unur_distr_is_cvemp	42	UNUR_ERR_ROUNDOff	119
unur_distr_is_discr	42	UNUR_ERR_SHOULD_NOT_HAPPEN	119
unur_distr_laplace	110	UNUR_ERR_STR	118
unur_distr_logarithmic	114	UNUR_ERR_STR_INVALID	118
unur_distr_logistic	110	UNUR_ERR_STR_SYNTAX	118
unur_distr_lomax	110	UNUR_ERR_STR_UNKNOWN	118
unur_distr_negativebinomial	114	unur_errno	119
unur_distr_normal	111	unur_free	57
unur_distr_pareto	111	unur_get_default_urng	104

unur_get_default_urng_aux	104	unur_srou_set_r	72
unur_get_dimension	57	unur_srou_set_usemirror	73
unur_get_distr	57	unur_srou_set_usesqueeze	73
unur_get_genid	57	unur_srou_set_verify	73
unur_get_stream	120	unur_srou_upd_mode	74
unur_get_streerror	119	unur_srou_upd_pdfarea	74
unur_get_urng	104	unur_ssr_chg_cdfatmode	76
unur_get_urng_aux	105	unur_ssr_chg_domain	76
unur_hinv_chg_truncated	67	unur_ssr_chg_mode	76
unur_hinv_estimate_error	67	unur_ssr_chg_pdfarea	76
unur_hinv_eval_approxinvcdf	66	unur_ssr_chg_pdfatmode	76
unur_hinv_get_n_intervals	66	unur_ssr_chg_pdfparams	75
unur_hinv_new	65	unur_ssr_chg_verify	75
unur_hinv_set_boundary	66	unur_ssr_new	75
unur_hinv_set_cpoinst	66	unur_ssr_reinit	75
unur_hinv_set_guidefactor	66	unur_ssr_set_cdfatmode	75
unur_hinv_set_max_intervals	66	unur_ssr_set_pdfatmode	75
unur_hinv_set_order	65	unur_ssr_set_usesqueeze	75
unur_hinv_set_u_resolution	65	unur_ssr_set_verify	75
unur_hrb_chg_verify	68	unur_ssr_upd_mode	76
unur_hrb_new	67	unur_ssr_upd_pdfarea	76
unur_hrb_set_upperbound	67	unur_str2distr	29
unur_hrb_set_verify	68	unur_str2gen	29
unur_hrd_chg_verify	68	unur_tabl_chg_verify	79
unur_hrd_new	68	unur_tabl_get_hatarea	78
unur_hrd_set_verify	68	unur_tabl_get_sqratio	78
unur_hri_chg_verify	69	unur_tabl_get_squeezearea	78
unur_hri_new	69	unur_tabl_new	77
unur_hri_set_p0	69	unur_tabl_set_areafraction	78
unur_hri_set_verify	69	unur_tabl_set_boundary	78
UNUR_INFINITY	127	unur_tabl_set_guidefactor	78
unur_init	57	unur_tabl_set_max_intervals	78
unur_ninv_chg_max_iter	70	unur_tabl_set_max_sqratio	77
unur_ninv_chg_pdfparams	71	unur_tabl_set_nstp	78
unur_ninv_chg_start	70	unur_tabl_set_slopes	78
unur_ninv_chg_table	71	unur_tabl_set_variant_setup	77
unur_ninv_chg_truncated	71	unur_tabl_set_variant_splitmode	77
unur_ninv_chg_x_resolution	70	unur_tabl_set_verify	79
unur_ninv_new	70	unur_tdr_chg_truncated	81
unur_ninv_set_max_iter	70	unur_tdr_chg_verify	82
unur_ninv_set_start	70	unur_tdr_eval_invcdfhat	83
unur_ninv_set_table	70	unur_tdr_get_hatarea	81
unur_ninv_set_usenewton	70	unur_tdr_get_sqratio	81
unur_ninv_set_useregula	70	unur_tdr_get_squeezearea	81
unur_ninv_set_x_resolution	70	unur_tdr_new	80
unur_run_tests	123	unur_tdr_set_c	80
unur_sample_cont	57	unur_tdr_set_center	82
unur_sample_discr	57	unur_tdr_set_cpoinst	82
unur_sample_vec	57	unur_tdr_set_darsfactor	81
unur_set_debug	121	unur_tdr_set_guidefactor	82
unur_set_default_debug	121	unur_tdr_set_max_intervals	81
unur_set_default_urng	104	unur_tdr_set_max_sqratio	81
unur_set_default_urng_aux	104	unur_tdr_set_pedantic	82
unur_set_stream	120	unur_tdr_set_usecenter	82
unur_set_urng	104	unur_tdr_set_usedars	80
unur_set_urng_aux	105	unur_tdr_set_usemode	82
unur_srou_chg_cdfatmode	74	unur_tdr_set_variant_gw	80
unur_srou_chg_domain	73	unur_tdr_set_variant_ia	80
unur_srou_chg_mode	73	unur_tdr_set_variant_ps	80
unur_srou_chg_pdfarea	74	unur_tdr_set_verify	82
unur_srou_chg_pdfatmode	74	unur_test_chi2	124
unur_srou_chg_pdfparams	73	unur_test_correlation	124
unur_srou_chg_verify	73	unur_test_count_urn	124
unur_srou_new	72	unur_test_moments	124
unur_srou_reinit	72	unur_test_printsample	123
unur_srou_set_cdfatmode	72	unur_test_quartiles	124
unur_srou_set_pdfatmode	73	unur_test_timing	123

unur_test_timing_exponential	123	unur_utdr_set_cpfactor	84
unur_test_timing_total	124	unur_utdr_set_deltafactor	84
unur_test_timing_uniform	123	unur_utdr_set_pdfatmode	84
unur_unif_new	102	unur_utdr_set_verify	84
unur_use_urng_aux_default	105	unur_utdr_upd_mode	84
unur_utdr_chg_domain	84	unur_utdr_upd_pdfarea	85
unur_utdr_chg_mode	84	unur_vempk_chg_smoothing	93
unur_utdr_chg_pdfarea	85	unur_vempk_chg_varcor	93
unur_utdr_chg_pdfatmode	85	unur_vempk_new	93
unur_utdr_chg_pdfparams	84	unur_vempk_set_smoothing	93
unur_utdr_chg_verify	84	unur_vempk_set_varcor	93
unur_utdr_new	83	unur_vmt_new	91
unur_utdr_reinit	83	unur_vmt_set_marginalgen	91