Risk Patterns and Correlated Brain Activities

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Risk Perception

- Which part is activated during *risk related decisions*?
- Can statistical analysis help to detect this area?
- Response curve (to stimuli)? classify “risky people”?
Risk Perception

- Survey conducted by Max Planck Institute

- 22 young, native German, right-handed and healthy volunteers
  - 3 subjects with extensive head movements (> 5 mm)
  - 2 subjects with different stimulus frequency
  - \( n = 22 - (3 + 2) = 17 \)

- Experiment
  - Risk Perception and Investment Decision (RPID) task (×81)
  - fMRI images every 2.5 sec.
  - Analysis of the first part (×45)
Risk Perception

Motivation

Returns  Pause  Decision

Risk Patterns and Correlated Brain Activities
Risk Perception – Thermodynamics

Theoretical framework

- Risk-return model
  Mohr et al., 2010

- Mechanical Equivalent of Heat
  1st law of thermodynamics
  Mayer, 1841

Empirical evidence

- fMRI analysis

- Experiments "Joule apparatus"
  Joule, 1843

Risk Patterns and Correlated Brain Activities
Motivation

Risk Perception

- functional Magnetic Resonance Imaging

- Measuring Blood Oxygenation Level Dependent (BOLD) effect every 2-3 sec
  High-dimensional, high frequency & large data set

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Figure 1: fMRI image observed every 2.5 sec, 12 horizontal slices of the brain’s scan, $91 \times 92 \times 71 \times (x, y, z)$ data points of size 22 MB; scan resolution: $2 \times 2 \times 2 \text{mm}^3$. 

Risk Patterns and Correlated Brain Activities
fMRI

Is there a significant reaction to specific stimuli in the hemodynamic response?

Voxel X

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fMRI methods

- **Voxel-wise GLM**
  - Linear model for each voxel separately
  - Strong a priori hypothesis necessary

- **Dynamic Semiparametric Factor Model (DSFM)**
  - Use a “time & space” dynamic approach
  - Separate low dim time dynamics from space functions
  - Low dim time series exploratory analysis
Outline

1. Motivation ✓
2. DSFM
3. Results vs. Subject’s Behaviour
4. Conclusion
5. Future Perspectives
6. References
7. Appendix
Notation

\[(X_{1,1}, Y_{1,1}), \ldots, (X_{J,1}, Y_{J,1}), \ldots, (X_{1,T}, Y_{1,T}), \ldots, (X_{J,T}, Y_{J,T})\], \quad t=1, \ldots, T

\[X_{j,t} \in \mathbb{R}^d, \ Y_{j,t} \in \mathbb{R}\]

\[T - \text{the number of observed time periods}\]

\[J - \text{the number of the observations in a period } t\]

\[E(Y_t|X_t) = F_t(X_t)\]

**Quantify** \(F_t(X_t)\). How does it move?
Dynamic Semiparametric Factor Model

\[
E(Y_t|X_t) = \sum_{l=0}^{L} Z_{t,l} m_l(X_t) = Z_t^T m(X_t) = Z_t^T A^* \Psi
\]

\[
Z_t = (1, Z_{t,1}, \ldots, Z_{t,L})^T, \text{low dim (stationary) time series}
\]

\[
m = (m_0, m_1, \ldots, m_L)^T, \text{tuple of functions}
\]

\[
\Psi = \{\psi_1(X_t), \ldots, \psi_K(X_t)\}^T, \psi_k(x) \text{ space basis}
\]

\[
A^*: (L + 1) \times K, \text{coefficient matrix}
\]
DSFM Estimation

\( Y_{t,j} = \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = Z_t^T A^* \psi(X_{t,j}) + \varepsilon_{t,j} \)

- \( \psi(x) = \{\psi_1(x), \ldots, \psi_K(x)\}^T \) tensor B spline basis

\[
(\hat{Z}_t, \hat{A}^*) = \arg \min_{Z_t, A^*} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - Z_t^T A^* \psi(X_{t,j}) \right\}^2 \tag{1}
\]

- Minimization by Newton-Raphson algorithm
B-Splines

Figure 2: $B$-splines basis functions; order of $B$-splines: quadratic; number of knots: $6 \times 6 = 36$
DSFM Estimation

Selection of $L$ by explained variance

$$EV(L) = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - \sum_{l=0}^{L} Z_{t,l} m_l(X_{t,j}) \right\}^2}{\sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ Y_{t,j} - \bar{Y} \right\}^2}$$

number of $B$-splines (equally spaced) knots: $K = 12 \times 14 \times 14$

<table>
<thead>
<tr>
<th>$L$</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>20</th>
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<tr>
<td>EV</td>
<td>92.07</td>
<td>92.25</td>
<td>92.29</td>
<td>93.66</td>
<td>95.19</td>
</tr>
</tbody>
</table>

Table 1: $EV$ in percent of the model with different numbers of factors $L$, averaged over all 17 analyzed subjects.

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Panel DSFM

\[ Y_{t,j}^i = \sum_{l=0}^{L} (Z_{t,l}^i + \alpha_{t,l}^i) m_l(X_{t,j}) + \varepsilon_{t,j}^i, \quad 1 \leq j \leq J, \quad 1 \leq t \leq T, \]

- \( n = 17 \) weakly/strongly risk-averse subjects
- \( Y_{t,j} \) - BOLD signal; \( X_j \) voxel’s index
  \( \alpha_{t,l}^i \) - fixed individual effect;  
  Residual Analysis

- Identification condition: \( E \left\{ \sum_{i=1}^{n} \sum_{l=0}^{L} \alpha_{t,l}^i m_l(X_{t,j}) \middle| X_{t,j} \right\} = 0 \)
Panel DSFM Estimation

Feasible estimation algorithm:

1. Average $Y_{t,j}^i$ over subjects $i$ to obtain $\bar{Y}_{t,j}$
2. Estimate factors $m_l$ for the "average brain" [via (1)]
3. Given $\hat{m}_l$, for $i$, estimate $Z_{t,l}^i$

$$Y_{t,j}^i = \sum_{l=0}^{L} Z_{t,l}^i \hat{m}_l(X_{t,j}) + \varepsilon_{t,j}^i$$

- 26h - computing time; CPU - 2 × 2.8GHz; data set of size 24.31 GB
Estimated constant factor $\hat{m}_0(X) = \sum_{k=1}^{K} \hat{a}_{0,k} \psi_k(X)$ with $L = 20$
Estimated factor \( \hat{m}_5(X) = \sum_{k=1}^{K} \hat{a}_{5,k} \psi_k(X) \) with \( L = 20 \).

(MOFC = Medial orbitofrontal cortex)

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Estimated factor $\hat{m}_9(X) = \sum_{k=1}^{K} \hat{a}_{9,k} \psi_k(X)$ with $L = 20$
Estimated factor $\hat{m}_{12}(X) = \sum_{k=1}^{K} \hat{a}_{12,k} \psi_k(X)$ with $L = 20$

(PC = Paretial Cortex)
Estimated factor $\hat{m}_{16}(X) = \sum_{k=1}^{K} \hat{a}_{16,k} \psi_k(X)$ with $L = 20$

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Estimated factor $\hat{m}_{17}(X) = \sum_{k=1}^{K} \hat{a}_{17,k} \psi_k(X)$ with $L = 20$

Risk Patterns and Correlated Brain Activities
Estimated factor $\hat{m}_{18}(X) = \sum_{k=1}^{K} \hat{a}_{18,k} \psi_k(X)$ with $L = 20$
Estimated Factor Loading $\hat{Z}_5$

Figure 3: Estimated factor loading $\hat{Z}_5$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus.
Estimated Factor Loading $\hat{Z}_9$

Figure 4: Estimated factor loading $\hat{Z}_9$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus

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Estimated Factor Loading $\hat{Z}_{12}$

Figure 5: Estimated factor loading $\hat{Z}_{12}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus.
Estimated Factor Loading $\hat{Z}_{16}$

Figure 6: Estimated factor loading $\hat{Z}_{16}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus.
Estimated Factor Loading $\hat{Z}_{17}$

Figure 7: Estimated factor loading $\hat{Z}_{17}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus
Estimated Factor Loading $\hat{Z}_{18}$

Figure 8: Estimated factor loading $\hat{Z}_{18}$ for subjects within 30 minutes: 12 (upper panel) and 19 (lower panel) with $L = 20$; red dots denote stimulus.
Results vs. Subject’s Behaviour

Reaction to the stimulus

Figure 9: Detailed view of factor loading $\hat{Z}_1$ for subject 12 with vertical lines in time points of stimuli of 3 different task: decision (red), subjective expected return (green) and perceived risk (black).

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Reaction to the stimulus

Figure 10: Reaction to stimulus $\Delta \hat{Z}_{s,l} = \frac{1}{3} \sum_{\tau=1}^{3} \Delta \hat{Z}_{s+\tau,l}$, where $\Delta \hat{Z}_{t,l} \overset{\text{def}}{=} \hat{Z}_{s+t,l} - \hat{Z}_{s,l}$, $t = 1, 2, 3$, $s$ is the time of stimulus for factors loadings $\hat{Z}_{t,12}$, for subjects 12 (left) and 19 (right) during the experiment (45 stimuli).
Risk attitude

- Subject’s risk perception $\tilde{R}_{i,s}$ -
  - standard deviation
  - empirical frequency of loss (negative return)
  - difference between highest and lowest return (range)
  - coefficient of range (range/mean)
  - empirical frequency of ending below 5%
  - coefficient of variation (standard deviation/mean)

- Different subject - different risk perception
  fitted by correlation between risk metrics of return streams and $R_{i,j,s}$ - answers for ”perceived risk” task Q1, $N = 27$
Risk attitude

- Subjective expected return $\tilde{m}_{i,s}$
  - recency (higher weights on later returns)
  - primacy (higher weights on earlier returns)
  - below 0% (higher weights on returns below 0%)
  - below 5% (higher weights on returns below 5%)
  - mean

- Selecting return ratings for each subject individually
  best model selected by prediction power of one-leave-out cross validation procedure, $N = 27$
Risk attitude

- Each subject $i$ has $(R_i, m_i)$
- Risk-return choice model

$$V_i(x_s) = m_i(x_s) - \beta_i R_i(x_s), \quad 1 \leq i \leq n, 1 \leq s \leq 27$$

$x_s$ - return stream, $m_i$-subjective expected return, $R_i$ - perceived risk, $V_i$ - subjective value (unobserved), 5% - risk free return

- $\beta$ Risk attitude parameter
Risk attitude

Estimation of individual risk attitude by logistic regression

\[
P \{ \text{risky choice}|(m, R) \} = \frac{1}{1 + \exp(m - \beta R - 5)}
\]

\[
P \{ \text{sure choice}|(m, R) \} = 1 - \frac{1}{1 + \exp(m - \beta R - 5)}
\]

risky choice - unknown return, sure choice - fixed, 5% return

\hat{\beta} derived by maximum likelihood method
Figure 11: Risk attitude $\hat{\beta}_i$ for 17 subjects; modeled by the softmax function from individuals’ decisions, estimated by ML method. Mohr et. al. Risk Patterns and Correlated Brain Activities.
SVM Classification Analysis

- **Support Vector Machines (SVM)**
  17 subjects, 20 factor loading time series per subject

- **Leave-one-out method** to train and estimate classification rate
  SVM with Gaussian kernel; $(R, C)$ chosen to maximize classification rate

- **Weakly/strongly** risk-averse subjects differ in reaction to stimulus $\Delta \hat{Z}_{i,t,l}$

  ![Reaction to Stimulus](Image)
SVM Classification Analysis

1. factors attributed to risk patterns: \( l = 5, 9, 12, 16, 17, 18 \)
2. only “Decision under Risk” (Q3) stimulus
3. average reaction to s stimulus \( \Delta \hat{Z}_{s,l} = \frac{1}{3} \sum_{\tau=1}^{3} \Delta \hat{Z}_{s+\tau,l} \)

SVM input data: volatility of \( \Delta \hat{Z}_{s,l} \) over all Q3

<table>
<thead>
<tr>
<th>Std</th>
<th>Estimated</th>
<th></th>
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<tr>
<td></td>
<td>Strongly</td>
<td>Weakly</td>
</tr>
<tr>
<td>Data</td>
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<td>0.00</td>
</tr>
<tr>
<td>Weakly</td>
<td>0.14</td>
<td>0.86</td>
</tr>
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</table>

Table 2: Classification rates of the SVM method, **without** knowing the subject’s estimated risk attitude
Figure 12: Normalized Principal Component Analysis on volatility of $\Delta \hat{Z}_{i,l}$ after stimulus for weakly/strongly risk-averse subjects; variance explained by the first and second components: 72%, 85%, respectively.
Conclusion

- Factors \( \hat{m} \) identify activated areas, neurological reasonable.

- Estimated factor loadings show differences for individuals with different risk attitudes (e.g., 12 vs. 19).

- SVM classification analysis of measurements in \( \hat{Z}_{t,l} \), \( l = 5, 9, 12, 16, 17, 18 \) after stimulus, can distinguish weakly/strongly risk-averse individuals with high classification rate, without knowing the subject’s answers.
Future Perspectives

- Comparison with the PCA/ICA (PARAFAC) approach
- Analysis of the second part of the experiment (under assumption of independency) to "generate" larger number of subjects
- Improvement of the classification criterion
- Penalized DSFM with seasonal effects
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Woolrich, M., Ripley, B., Brady, M., Smith, S.
Temporal Autocorrelation in Univariate Linear Modelling of FMRI Data
*NeuroImage*, 21: 2245-2278, 2010
Voxel-wise GLM

- FEAT - FMRI Expert Analysis Tool by Department of Clinical Neurology, University of Oxford

- GLM framework
  \[ Y = XB + \eta, \]
  
  \( Y \) - single voxel BOLD time series, \( X \) - design matrix (regressors, i.e. visual, auditory)

- Significant, active areas \( \mathbf{B} \) selected by z-scores\( \equiv \frac{B_i - 0}{\sqrt{\text{Var}(B_i)}} \)
  
  and grouping (20 neighbors) scheme
B-Splines

Univariate B-spline basis $\Psi = \{\psi_1(X), \ldots, \psi_K(X)\}^\top$ is a series of $\psi_k(X)$ functions defined by $x_0 \leq x_2 \leq \ldots \leq x_{K-1}$, $K$ knots and order $p$, i.e. for $p = 2$ (quadratic)

$$
\psi_j(x) = \begin{cases} 
\frac{1}{2} (x - x_j)^2 & \text{if } x_j \leq x < x_{j+1} \\
\frac{1}{2} - (x - x_{j+1})^2 + (x - x_{j+1}) & \text{if } x_{j+1} \leq x < x_{j+2} \\
\frac{1}{2} \{1 - (x - x_{j+2})^2\} & \text{if } x_j \leq x < x_{j+1} \\
x & \text{otherwise}
\end{cases}
$$
B-Splines

- Knots $K$ and order $p$ has to be specified in advance ($EV$ criterion); $K$ corresponds to bandwidth

- In higher dimensions, for $\text{dim}(X) = d > 1$

$$
\Psi = \{\psi_1(X_1), \ldots, \psi_{K_1}(X_1)\} \times \ldots \times \{\psi_1(X_d), \ldots, \psi_{K_d}(X_d)\}
$$

- Flexible and computationally efficient approach to capture various spatial structures
Residual Analysis

Figure 13: Boxplots of random subsets (size $3 \times 10^7$) from $\varepsilon_{t,j}^i$ ($4.3 \times 10^9$ points) for all 17 analyzed subjects. Kurtosis exceeds 10.
Residual Analysis

Figure 14: Histograms of random subsets (size $3 \times 10^7$) from $\varepsilon_{t,j}^i$ ($4.3 \times 10^9$ points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively. Normality hypothesis (KS test) for standardized $\varepsilon_{t,j}^i$ rejected for all subjects, $\alpha = 5\%$.
Residual Analysis

Figure 15: Histograms of random subsets (size $3 \times 10^7$) from $\varepsilon_{i,j}^t$ ($4.3 \times 10^9$ points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively.
Figure 16: QQplots of random subsets (size $3 \times 10^7$) from $\varepsilon_{t,j}^i$ ($4.3 \times 10^9$ points) for subjects $i = 1, 2, 3, 4, 5, 6, 8, 9$, respectively.
Residual Analysis

Figure 17: QQplots of random subsets (size $3 \times 10^7$) from $\varepsilon^i_{t,j}$ ($4.3 \times 10^9$ points) for subjects $i = 10, 11, 12, 15, 16, 17, 18, 19$ respectively.
Figure 18: Averaged reaction $\Delta \hat{Z}_{s,9}^i$ to stimulus for all 15 Q3 questions for weakly/strongly risk-averse individuals
Figure 19: Averaged reaction $\Delta \hat{Z}_{s,12}$ to stimulus for all 15 Q3 questions for weakly/strongly risk-averse individuals.
Return Ratings

\( r_i, \ i = 1, \ldots, 10 \) denotes sequence of random returns in each trial.

Subjective Expected Return (SER) models:

- **Mean**
  \[
  SER = \frac{\sum_{i=10-m}^{10} r_i}{m}
  \]
  \( m \)-number of returns remembered, \( 2 \leq m \leq 10 \)

- **Recency**
  \[
  SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10-m}^{10} p}, \quad p = (i - 9 + m)^g
  \]
  \( g \) - weighting parameter of returns, \( 0 < g < 1 \)

Risk Patterns and Correlated Brain Activities
Return Ratings

[] Primacy

\[
SER = \frac{\sum_{i=10-m}^{10} ri \cdot p}{\sum_{i=10-m}^{10} p}, \quad p = (11 - i)^g
\]

- \( m \)-number of returns remembered, \( 2 \leq m \leq 10 \)
- \( g \) - weighting parameter of returns, \( 0 < g < 1 \)

[] Overweight < 0%

\[
SER = \frac{\sum_{i=10-m}^{10} ri \cdot p}{\sum_{i=10-m}^{10} p}, \quad p = \begin{cases} 
1, & \text{if } r_i \geq 0 \\
1 + w, & \text{otherwise}
\end{cases}
\]

- \( w \) - additional weight of returns, \( 0 < w < 1 \); \( 1 \leq m \leq 9 \)
Return Ratings

- Overweight < 5%

\[ SER = \frac{\sum_{i=10-m}^{10} r_i p}{\sum_{i=10}^{10-m} p}, \quad p = \begin{cases} 
1, & \text{if } r_i \geq 5 \\
1 + w, & \text{otherwise}
\end{cases} \]

- Parameters fitted by Cross Validation over all 27 trials

\( w \) - additional weight of returns, \( 0 < w < 1; 1 \leq m \leq 9 \)
Return Ratings

Figure 20: Distribution of return ratings over analyzed subjects

Risk Patterns and Correlated Brain Activities
Risk Metrics

Risk perception - risk metrics used by individuals

- Standard deviation of a return sequence
- Empirical frequency of loss (negative returns / all returns)
- Range - difference between highest and lowest return in a sequence
- Coefficient of range (range / mean)
- Empirical frequency of ending below 5% (returns < 5% / all returns)
- Coefficient of variation (standard deviation / mean)
Risk Metrics

Figure 21: Distribution of risk metrics over analyzed subjects
### SVM Scores

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<td>1</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>β</td>
<td></td>
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<td>5.6</td>
<td>11.3</td>
<td>5.0</td>
<td>6.3</td>
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<td>8.6</td>
<td>5.4</td>
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<tr>
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<td>0.43</td>
<td>0.32</td>
<td>0.58</td>
<td>0.40</td>
<td>0.44</td>
<td>0.23</td>
</tr>
</tbody>
</table>

|     | Weakly   |     |     |     |     |     |     |     |     |
|     | i        | 2   | 5   | 6   | 9   | 11  | 12  | 21  |     |
| β  |          | 4.8 | 4.1 | 3.7 | 4.7 | 3.8 | 1.3 | 1.8 |     |
| Score |         | 0.32| −1.03| −0.32| −0.44| −0.79| −0.04| −0.08|     |

**Table 3:** Estimated risk attitude and SVM scores (obtained **without** knowing the subject’s answers)
Figure 22: Scatter plot of $\hat{\beta}_i$ vs SVM scores
Risk Metrics

Figure 23: Scatter plot of $\hat{\beta}_i$ vs risk perception models (vertical line). 1 - Standard deviation, 2 - Coefficient of variation, 3 - Empirical frequency of loss; 4 - Empirical frequency of ending below 5%, 5 - Coefficient of range, 6 - Coefficient of variation.