On the Estimation of Credit Exposures Using Regression-Based Monte-Carlo Simulation

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Introduction

Exposure Framework
- Market Exposure
- Credit Exposure

The Exposure Algorithm
- Avoiding Simulations within Simulations
- American Option
- The Algorithm
- Decomposition and Truncation Scheme

Credit Exposure Examples
- Convertible Bond
- Cancelable Interest Rate Swap
- Variance Swap

Conclusion
Broader Background: Market & Credit Risk Aggregation

- General motivation: firmwide risk aggregation of market and credit risks - Top-Down vs. Bottom-Up Approach
- Top-Down approach is based on the concept of Copulas and specifies dependence at risk type level
Bottom-Up Approach

- Bottom-up methodology captures dependence at an elementary risk driver level
- Several steps that need to be considered:
  1. identification of risk factors;
  2. use of historical data to determine the relationships between risk drivers;
  3. simulation of the most appropriate model for the risk factors.
- Appropriate risk drivers
  - for market risk: equity prices, interest rates, swap rates, bond prices, FX rates, commodity prices, credit spreads, ...
  - for credit risk: default trends and rates, expected default frequencies (EDFs), credit spreads, recovery rates, macroeconomic variables, ...
Implications: consistent valuation for all type of products accounting for dependencies among risk drivers is needed; as well as a proper definition of a joint bottom-up loss variable for market and credit risk.
Risk management function has advanced significantly in recent years.

Three credit risk components: default indicator, exposure at default, loss given default.

Problem: to quantify credit exposure for complex products with no analytical (closed-form) solution in a scenario consistent way.


Our approach: modified version of least-squares Monte-Carlo technique by Longstaff and Schwartz (2001).
Notation

- Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ be a filtered probability space with underlying stochastic process $(X_t)_{t \geq 0}$, $X_t \in \mathbb{R}^d$, $\mathcal{F}_t = \sigma(X_s, 0 \leq s \leq t)$, $\mathbb{P}$ is physical prob. measure, assume there exists EMM $\mathbb{Q} \sim \mathbb{P}$
- $V_t, t \geq 0$: value of a financial product over time
- $NPV_t = \mathbb{E}^\mathbb{Q}[Cashflows(t, T)|\mathcal{F}_t]$: net present value of outstanding cash flows to some counterparty,
  $Cashflows(t, T), 0 \leq t \leq T$: a claim’s discounted net cash flow between time $t$ and $T$.
- Distinction between market and credit risk exposure
Definition (Market Exposure)

We define market exposure $ME_t$ at time $t$ as the future market value of the product, i.e.

$$ME_t = V_t, \quad t \geq 0.$$ 

Moreover, the time-$t$ maximum likely market exposure (MLME) at some confidence level $\alpha$ is given by

$$MLME_t^\alpha = \inf \{x : P[ME_t > x] \leq 1 - \alpha\}. \quad (1)$$

Then the expected market exposure (EME) at time $t$ as seen from time zero is given by

$$EME_t = E_P[ME_t] = E_P[ME_t | F_0]. \quad (2)$$
Credit Exposure

Definition (Credit Exposure)

We define the credit exposure $CE_t$ at time $t$ to a counterparty as its zero-floored expected discounted outstanding cash flows, i.e.

$$CE_t = \max(\NPV_t, 0), \quad t \geq 0. \quad (3)$$

Analogously to (1) and (2), we define

$$MLCE_t^\alpha = \inf\{x : \mathbb{P}[CE_t > x] \leq 1 - \alpha\} \quad t \geq 0,$$

and

$$ECE_t = \mathbb{E}^\mathbb{P}[CE_t] \quad t \geq 0.$$
MC-simulation has become the favorite tool for pricing complex financial instruments.

Calculation of future exposures: rather easy for products with closed-form solution; simulate underlying risk drivers $X_t$ under $\mathbb{P}$ (scenarios) and insert them into the corresponding formulas for $V_t$ or $NPV_t$ (pricing).

If there is no closed-form solution $\rightarrow$ computational infeasible, due to additional simulations (under $\mathbb{Q}$) for pricing.

Solution: approximate conditional expectations for $V_t$ or $NPV_t$.
Avoiding Simulations within Simulations

American Option
The Algorithm
Decomposition and Truncation Scheme

Pricing simulations
Scenario simulations
The American put option price $V_t$ at time $t$ is calculated according to

$$V_t = \sup_{\tau \geq t} \mathbb{E}^Q \left[ e^{-\int_t^\tau r_s ds} (K - S_{\tau}) | \mathcal{F}_t \right],$$

with optimal stopping times $\tau \in \mathcal{T}$

LSMC: Estimate optimal stopping time $\tau^*$ by backward induction comparing at each point in time the intrinsic value $I_t$ (exercise value) and the extrinsic value $F_t$ (continuation value),

For the purpose of modeling exposures, we also need to account for the change of measure $\frac{dQ}{dP}$
American Option
The Algorithm
Decomposition and Truncation Scheme

Figure: American put exposure comparison of LSMC pricing algorithm
Exposure Algorithm

- Additional Notation:
  \( C_{\text{Cashflows}}(t, T) = D(t, T)h(X_T) \) single cash-flow with discount factor: \( D(t, T) = e^{-\int_t^T r_s ds} \)
  payoff function: \( h(X_t) \)
  zero-coupon bond prices: \( B(t, T) = \mathbb{E}_Q^Q[D(t, T)|\mathcal{F}_t] \)
  change of measure density: \( M_t := \frac{dQ}{dP}\bigg|_{\mathcal{F}_t} \)

Algorithm (Exposure algorithm)

1. Simulate \( L \) independent paths \( X^{(l)}_{t_k},\ k = 0, \ldots, K,\ l = 1, \ldots, L, \) of the underlying process under the \( \mathbb{P} \) probability.
Algorithm (Exposure Algorithm cont.)

(2) At terminal date $t_K = T$, set $V^{(l)}_T = h(X^{(l)}_T)$ for $l = 1, \ldots, L$, and define the stopping time $\tau_K := T$.

(3) Apply backward induction, i.e. $k + 1 \rightarrow k$ for $k = K - 1, \ldots, 1$

(a) estimate extrinsic or continuation values

$$F_{t_k}^{(l)} = B^{(l)}(t_k, \tau_{k+1})\tilde{F}_{t_k}^{(l)} \text{ for } l = 1, \ldots, L,$$

where $\tilde{F}_{t_k}^{(l)}$ is estimated by regressing discounted measure-rebased values

$$\frac{D^{(l)}(t_k, \tau_{k+1})}{B^{(l)}(t_k, \tau_{k+1})} \frac{M^{(l)}_{\tau_{k+1}}}{M^{(l)}_{t_k}} V^{(l)}_{\tau_{k+1}}$$

on appropriate basis functions;
Algorithm (Exposure Algorithm cont.)

(3) (b) define a new stopping time \( \tau_{k}^{(l)} \) according to the stopping rule; e.g. for the American option
\[
\tau_{k}^{(l)} = \min\{ m \in \{ k, k + 1, \ldots, K \} | I_{t_m}^{(l)} \geq F_{t_m}^{(l)} , I_{t_m}^{(l)} > 0 \};
\]
(c) for each path \( l = 1, \ldots, L \) set \( NPV_{t_k}^{(l)} = \max\{ I_{t_k}^{(l)} , F_{t_k}^{(l)} \} \) and \( NPV_{t_m}^{(l)} = 0 \) for \( t_m > \tau_{k}^{(l)} \).

(4) Calculate the estimated exposure at time \( t = 0 \) by
\[
NPV_{0} = \sum_{1 \leq l \leq L} D^{(l)}(0, \tau_{1}) M_{\tau_{1}}^{(l)} V_{\tau_{1}}^{(l)} / L .
\]

(5) Set \( CE_{t}^{(l)} = \max( NPV_{t}^{(l)} , 0 ) \) for the estimated credit exposures.
Deomposition and Truncation

- Let $D_1, \ldots, D_d$ be a partition of the state-space of $X_T$
- Assuming interest rates to be zero, the continuation value at time $t$ is given by $F_t = \mathbb{E}^\mathbb{P}[\frac{M_T}{M_t} h(X_T) | \mathcal{F}_t]$.
- Applying Bayes theorem we can decompose the continuation value:

$$F_t = \sum_{i=1}^{d} \mathbb{E}^\mathbb{P}[\frac{M_T}{M_t} h(X_T) | X_T \in D_i, \mathcal{F}_t] \cdot \mathbb{P}[X_T \in D_i | \mathcal{F}_t].$$

- Truncation of $I$ by setting minimum and maximum values for this partition
- Decomposition of $II$ can be achieved by multi-nominal regression techniques
Convertible Bond

- Setup: Convertible bond with multiple exercise decisions (voluntary and forced conversion, put and call options)
- Underlying stochastic driver: dividend paying stock with dynamics

\[ dS_t = (\mu - \delta)S_t dt + \sigma S_t dW_t^P \quad S_0 = 100 \]

- Exercise can take place at each point in time; maturity: 2 years.
2Y Convertible Bond Exposures

Convertible Bond Exposure

Cumulative Distributions of 3M, 6M, 1Y and 2Y Exposure
Cancelable Interest Rate Swaps

- A callable (putable) swap is an interest rate swap (IRS), where the fixed rate payer (receiver) has the right, but not the obligation to terminate the contract at pre-determined dates during the swaps lifetime.

- Specification: short-rate $r_t$ is governed by a two-factor Vasicek model.

- 5-year cancelable IRS contract, which can be exercised each half a year.
5Y Cancelable Interest Rate Swaps

Callable Interest Rate Swap

Putable Interest Rate Swap

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Variance Swap

- A standard variance swap pays off the difference between the annualized realized variance $\sigma^2_R$ and a pre-specified strike $K$.
- Additional feature: capped by $\sigma^2_{Cap} = 0.075$; i.e. the payoff becomes
  $$\min \left( \frac{1}{T} \int_0^T \nu_s ds, \sigma^2_{Cap} \right) - K,$$
  where $\nu_t$ denotes the instantaneous variance rate, $K = 0.05$.
- $\nu_t$ is modelled by Heston model with stochastic central tendency.
2Y Capped Variance Swap

![Diagram showing 2Y Capped Variance Swap Exposure]

Cumulative Distributions of 3M, 6M, 1Y and 2Y Exposure

![Diagram showing cumulative distributions]

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Estimation of Credit Exposures
Conclusion

- Presented approach allows to estimate credit exposures without closed-form formulas by backward induction.
- The algorithm incorporates change of measure technique, and partitions the state space of the payoff function.
- Practical feature: simple and easy to implement.
- However, comparison of performance with other techniques such as non-parametric approaches using Malliavin calculus are desirable.
References


