Measuring systemic risk: The Indirect Contagion Index

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Disclaimer

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Based on:
Rama Cont and Eric Schaanning (2016)
Measuring systemic risk: The Indirect Contagion Index
1. Endogenous risk and price-mediated contagion

2. Modeling fire sales

3. Monitoring systemic risk: The Indirect Contagion Index

4. Scenario design

5. Conclusion
Indirect exposures

Consider two institutions (A) and (B).

- A and B hold a common financial asset (1). A holds an illiquid asset (2) that B does not hold. Notional exposure of B to (2) is zero.
- However, in the event of a large shock to the value of the illiquid asset (2), A may be forced to sell some of its financial assets, pushing down its market price, resulting in a market loss for the bank B.
- So: B experiences a loss following a large shock to the illiquid asset: B has an (indirect) exposure to an asset it does not hold!
- Magnitude of this indirect exposure is directly linked to the overlap between B and institutions holding this asset.
- Large diversified institutions increase overlaps across system and become nodes for price-mediated contagion.
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Losses arising from indirect exposures

Figure: Losses of HSBC and Banco Santander as a function of losses in the Southern European real estate sector.
Objectives and questions

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• How can we **quantify** the notion of “interconnectedness" for Global systemically important banks (GSIBs)?
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- Is price-mediated contagion likely to be an important vector of contagion in the stress scenarios that we consider?
- Given institutions’ portfolio holdings, are the stress scenarios that we consider the right ones?
- How can we quantify the notion of “interconnectedness" for Global systemically important banks (GSIBs)?
- Can regulators disseminate a metric that would allow institutions to quantify their exposures to price-mediated contagion?
Bank stress tests and interconnectedness assessments

- Bank stress tests have become an essential component of bank supervision (EU-wide EBA stress tests, Dodd-Frank tests (DFAST, CCAR)).

- *Static balance sheet assumption:* Stress tests assume 'passive' behaviour by banks.

- BCBS 2015: “Stress tests conducted by bank supervisors still lack a genuine macro-prudential component”: “endogenous reactions to initial stress, loss amplification mechanisms and feedback effects” are missing.

- Currently “interconnectedness” in the GSIB methodology is based on (i) intra-financial system assets, (ii) intra-financial system liabilities, (iii) securities outstanding.
Channels of loss amplification in the financial system

1. Counterparty Risk: balance sheet contagion through asset devaluation

2. Funding channel: balance sheet contagion through withdrawal of funding (bank runs by depositors, institutional bank runs by lenders)
Channels of loss amplification in the financial system

1. **Counterparty Risk**: balance sheet contagion through asset devaluation

2. **Funding channel**: balance sheet contagion through withdrawal of funding (bank runs by depositors, institutional bank runs by lenders)

3. **Feedback effects from fire sales**: loss contagion through mark-to-market losses in common asset holdings

Research on financial networks and their use in macroprudential regulation has focused on direct contagion mechanisms (1+2). Regulatory measures have focused on 1 (large exposure limits, central clearing, CVA, ring-fencing) or 2 (LCR, NSFR).
Modeling fire sales
Systemic stress testing with endogenous effects

Ingredients:

1. Data: Portfolio holdings of financial institutions by asset class: 
   - $N$ institutions, $K$ illiquid asset classes, $M$ marketable asset classes
   - $N \times (M + K)$ portfolio matrix (network)
Systemic stress testing with endogenous effects

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5. Mark-to-market accounting: transmits market impact to all institutions $\rightarrow$ may lead to feedback if market losses large
## Balance sheets: illiquid and marketable assets

<table>
<thead>
<tr>
<th>Illiquid assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential mortgage exposures</td>
</tr>
<tr>
<td>Commercial real estate exposure</td>
</tr>
<tr>
<td>Retail exposures: Revolving credits, SME, Other</td>
</tr>
<tr>
<td>Indirect sovereign exposures in the trading book</td>
</tr>
<tr>
<td>Defaulted exposures</td>
</tr>
<tr>
<td>Residual exposures</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marketable assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate bonds</td>
</tr>
<tr>
<td>Sovereign debt</td>
</tr>
<tr>
<td>Derivatives</td>
</tr>
<tr>
<td>Institutional client exposures: interbank, CCPs,...</td>
</tr>
</tbody>
</table>

**Table**: Stylized representation of asset classes in bank balance sheets.  
(Data: European Banking Authority)
• Illiquid holdings of institution $i$: $\Theta^i := \sum_{\kappa=1}^{K} \Theta^{i\kappa}$.  
• Marketable Securities held by $i$: $\Pi^i := \sum_{\mu=1}^{M} \Pi^{i\mu}$.  
• Equity (Tier 1 capital): $C^i$
• Illiquid holdings of institution $i$: $\Theta^i := \sum_{\kappa=1}^{K} \Theta^{i\kappa}$.

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• Financial institutions are subject to various one-sided portfolio constraints: leverage ratio, capital ratio, liquidity ratio.

• Leverage ratio of $i$:

$$\lambda^i = \frac{\text{Assets}(i)}{C^i} = \frac{\Theta^i + \Pi^i}{C^i} \leq \lambda_{\text{max}}$$
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\]

• A stress scenario is defined by a vector $\epsilon \in [0, 1]^K$ whose components $\epsilon_{\kappa}$ are the percentage shocks to asset class $\kappa$.
• Initial/Direct loss of portfolio $i$: $L^i(\epsilon) = \epsilon.\Theta^i = \sum_{\kappa} \Theta^i_{\kappa} \epsilon_{\kappa}$
Deleveraging

**Deleveraging assumption:** if following a loss $L^i$ in asset values the leverage of bank $i$ exceeds the constraint

$$\lambda^i = \frac{\Theta^i + \Pi^i - L^i}{C^i - L^i} > \lambda_{\text{max}}$$

bank deleverages by selling a proportion $\Gamma^i \in [0, 1]$ of assets in order to restore a leverage ratio $\lambda_b^i \leq \lambda_{\text{max}}$:

$$\left(1 - \Gamma^i\right)\Pi^i + \Theta^i - L^i \leq \lambda_b^i \leq \lambda_{\text{max}} \Rightarrow \Gamma^i = \frac{C^i(\lambda^i - \lambda_b^i)}{\Pi^i}1_{\lambda^i > \lambda_{\text{max}}''}$$
Deleveraging in response to a loss

Figure: Percentage of marketable asset deleveraged in response to a shock to assets (circles) for a leverage constraint of 20. Leverage targeting (dotted blue) would lead to a linear response.
Market impact function
Market impact function and market depth

The impact of a total distressed liquidation volume $q$ is modelled by a *level-dependent market impact function*

$$
\Psi_{\mu}(q, S) = \left(1 - \frac{B_{\mu}}{S}\right) \left(1 - \exp \left(-\frac{q}{D_{\mu}}\right)\right),
$$

• $S \geq B_{\mu}$ where $B_{\mu}$ is the price-floor
• $D_{\mu} = cADV_{\mu} \sigma_{\mu} \sqrt{\tau}$,
• $c \approx 0.25$, a coefficient to make $\Psi_{\mu}$ consistent with empirical estimates of the linear impact model for small volumes $q$.
• $\tau$ is the liquidation horizon

Measuring systemic risk R. Cont and E. Schaanning
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- \( S \geq B_\mu \) where \( B_\mu \) is the price-floor
- \( ADV \): average daily volume, \( \sigma_\mu \): daily volatility of asset
- \( c \approx 0.25 \), a coefficient to make \( \Psi_\mu \) consistent with empirical estimates of the linear impact model for small volumes \( q \).
- \( \tau \) is the liquidation horizon
Estimated market depth
Market impact and feedback effects

Total liquidation in asset $\mu$ at k-th round: $q^\mu = \sum_{j=1}^{N} \prod_{k}^{j,\mu} \Gamma^j_{k+1}$

Market impact: $\frac{\Delta S^\mu}{S^\mu} = -\psi^\mu(q^\mu)$,

Impact/ inverse demand function: $\psi^\mu > 0, \psi^\mu' > 0, \psi^\mu(0) = 0$. 
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Price move at k-th iteration of fire sales:

$$S_{k+1}^\mu = S_k^\mu \left( 1 - \psi_\mu \left( \sum_{j=1}^{N} \Pi_{k}^j,\mu \Gamma_{k+1}^j \right) \right)$$,
Market impact and feedback effects

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$$\Pi_{k+1}^i,^\mu = \left( 1 - \Gamma_{k+1}^i \right)$$ 

Non-liquidated assets

$$\left( 1 - \Psi_\mu \left( \sum_{j=1}^{N} \Pi_{k}^j \Gamma_{k+1}^j \right) \right)$$

Price impact on remaining holdings

Previous value

Measuring systemic risk

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Fire sales losses

- Mark to market loss:

\[ M_{k+1}^i := \sum_{\mu=1}^{M} \left( (1 - r_{k+1}) \prod_{k}^{i\mu} - \prod_{k+1}^{i\mu} \right) \]

\[ = (1 - r_{k+1}) \sum_{\mu=1}^{M} \prod_{k}^{i\mu} \psi_{\mu} \left( \sum_{j=1}^{N} \prod_{k}^{j\mu} \Gamma_{k+1}^{j} \right) \]
Fire sales losses

- Mark to market loss:

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\]

- Realised loss (implementation shortfall / slippage):

\[
R^i_{k+1} := \alpha \Gamma_{k+1}^i \sum_{\mu=1}^{M} \Pi_k^i \psi_\mu \left( \sum_{j=1}^{N} \Pi_k^j \Gamma_{k+1}^j \right)
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Fire sales losses

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- Fire sales loss:

\[
L_{k}^i = (1 - (1 - \alpha) \Gamma_{k+1}^i) \sum_{\mu=1}^{M} \prod_{k}^{i,\mu} \psi_{\mu} \left( \sum_{j=1}^{N} \prod_{k}^{j,\mu} \Gamma_{k+1}^j \right)
\]
Estimated fire-sales losses EBA scenario
Monitoring systemic risk: The Indirect Contagion Index
Bipartite network of asset holdings

Indirect exposures across institutions through common asset holdings
Portfolio overlaps as drivers of price-mediated contagion

For $\alpha = 1$ and $\Psi_{\mu}(x) = \frac{x}{D_{\mu}}$ with $D_{\mu} = c \frac{ADV_{\mu}}{\sigma_{\mu}} \sqrt{\tau}$, the indirect loss of bank $i$ resulting from deleveraging by other banks becomes:

$$L^i = \sum_{j=1}^{N} \sum_{\mu=1}^{M} \frac{\Pi_{i\mu} \Pi_{j\mu}}{D_{\mu}} \Gamma^j = \sum_{j=1}^{N} \Omega_{ij} \Gamma^j,$$

where $\Omega_{ij}$ is the \textit{liquidity-weighted overlap} between portfolios $i$ and $j$ (Cont & Wagalath 2013):

$$\Omega_{ij} = \sum_{\mu=1}^{M} \frac{\Pi_{i\mu} \Pi_{j\mu}}{D_{\mu}}$$

$D_{\mu} = $ market depth for asset $\mu$

$\Omega_{ij} = $ exposure of marketable assets of $i$ to deleveraging by $j$.

$\Rightarrow$ loss contagion = contagion process on network defined by $[\Omega_{ij}]$
Indirect contagion

The first round fire-sales losses across the banking system are thus given by

$$ Floss = \Omega \Gamma. $$

When the liquidity-weighted overlap network is close to a 1-factor model

$$ \Omega \approx \lambda_1 uu^\top, $$

then the first round fire sales loss of $i$ is

$$ \log(Floss^i) = \log(\lambda_1 u_i \sum_{j=1}^{N} u_j \Gamma_j(\epsilon)), $$

and we expect a slope 1 when regressing the log fire-sales losses on the log ICI:

$$ \log(Floss^i) = 1 \times \log(u_i) + \log(\lambda_1) + \log(< u, \Gamma(\epsilon) >). $$
Indirect Contagion Index Construction

1. Collect portfolio holdings $\Pi_{i,\mu}$ by asset class for each financial institution in the network, at the granularity level corresponding to bank stress tests.

2. Estimate a market depth parameter $D_{\mu} \propto \frac{ADV_{\mu}}{\sigma_{\mu}}$ for each asset class.
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4. Compute the “Perron eigenvector" $u = (u_i, i = 1...N)$ of the matrix of liquidity-weighted overlaps $\Omega(\Pi) = \Pi D^{-1}\Pi^\top$ (SVD of $\Pi \sqrt{D^{-1}}$).

The Indirect Contagion Index is the Perron eigenvector, $\text{ICI} = u$, whose component $\text{ICI}(i) = u_i$ provides a measure of centrality of the node $i$ in the network whose links are weighted by the overlap matrix $\Omega$. 

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Principal component analysis of portfolio holdings

*Figure*: European banking system: Eigenvalues of matrix of liquidity-weighted overlaps. Source: EBA (public)
The Indirect Contagion Index (EBA 2016)
The EU indirect contagion network (2016)
Portfolio overlaps, $\Omega_{ij}$, across EU banks (EBA 2016)
Figure: Bank-level fire-sales losses regressed on the ICI.
**Figure:** Bank-level fire-sales losses regressed on the ICI.
Table: Regression of bank-level fire-sales losses on the Indirect Contagion Index for all banks.

<table>
<thead>
<tr>
<th></th>
<th>Round 1</th>
<th>Round 2</th>
<th>Round 3</th>
<th>Round 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>0.684***</td>
<td>0.762***</td>
<td>0.594***</td>
<td>0.100</td>
<td>0.490***</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.052)</td>
<td>(0.047)</td>
<td>(0.168)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Intercept</td>
<td>10.85***</td>
<td>11.39***</td>
<td>11.12***</td>
<td>9.06***</td>
<td>11.4***</td>
</tr>
<tr>
<td></td>
<td>(0.190)</td>
<td>(0.130)</td>
<td>(0.128)</td>
<td>(0.411)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>n</td>
<td>51</td>
<td>49</td>
<td>32</td>
<td>16</td>
<td>51</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.64</td>
<td>0.82</td>
<td>0.83</td>
<td>-0.04</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table: Regressing fire-sales losses on the $ICI$. *** denotes significance $p < 10^{-4}$. 
**Figure:** Slope of the regression of fire-sales losses on the ICI, as a function of the shock size and market depth.
Figure: $R^2$ of the regression of fire-sales losses on the ICI, as a function of the shock size and market depth.
Robustness checks

**Nominal overlaps.** Perron eigenvector of

$$\Omega_{Nominal} = \Pi \Pi^\top.$$
Robustness checks

Nominal overlaps. Perron eigenvector of

$$\Omega_{Nominal} = \Pi \Pi^\top.$$  

Cosine Similarity. [Getmansky et al., 2016], Portfolio weights:

$$w_i := \frac{1}{\sum_{\mu=1}^{M} \Pi_{i,\mu} (\Pi_{i,1}, \ldots, \Pi_{i,M})^\top}.$$  

Cosine similarity: Perron eigenvector of $\Omega_{C.S.}$ given by

$$\Omega_{C.S.}^{ij} = \frac{\langle w_i, w_j \rangle}{\|w_i\|_2 \|w_j\|_2} \in [-1, 1].$$
Robustness checks

**Nominal overlaps.** Perron eigenvector of

\[
\Omega_{Nominal} = \Pi \Pi^T.
\]

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\]

**Size.**

\[
size = \frac{(\Pi^1, \ldots, \Pi^N)}{\|(\Pi^1, \ldots, \Pi^N)\|_2},
\]

where $\Pi^i := \sum_{\mu=1}^{M} \Pi_{i,\mu}$. 

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Similarity between overlap measures

<table>
<thead>
<tr>
<th></th>
<th>ICI</th>
<th>Nom. Ov.</th>
<th>Cos. Sim.</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICI</td>
<td>1</td>
<td>0.68 (0.85)</td>
<td>-0.13 (-0.22)</td>
<td>0.60 (0.80)</td>
</tr>
<tr>
<td>Nom. Ov.</td>
<td>1</td>
<td>-0.14 (-0.22)</td>
<td>0.78 (0.92)</td>
<td></td>
</tr>
<tr>
<td>Cos. Sim.</td>
<td></td>
<td></td>
<td></td>
<td>-0.17 (-0.26)</td>
</tr>
<tr>
<td>Size</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

**Table:** Similarity between the various overlap measures: The bold numbers are rank-correlations (Kendall’s $\tau$), while the numbers in brackets are linear correlations (Spearman’s $\rho$).
Price-mediated contagion

Modeling fire sales

Indirect Contagion Index

Scenario design

Conclusion

Measuring systemic risk

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Liquidity-weighted overlaps
Nominal overlaps
\[ \log_{10}(FSLoss^i) = b_1 \log_{10}(X) + b_0 + \epsilon \]

**Table**: Regression of bank losses on the Indirect Contagion Index and other measures \((X)\) for all banks. First round only.
\[ \log_{10}(FSLoss^i) = b_1 \log_{10}(ICI) + b_2 \log_{10}(N.Ov.) + b_3 \log_{10}(Size) + b_0 + \epsilon. \]

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Estimate</th>
<th>Std. dev.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICI</td>
<td>0.22***</td>
<td>(0.062)</td>
<td>9.34E-4</td>
</tr>
<tr>
<td>Nominal Overlap</td>
<td>0.22***</td>
<td>(0.080)</td>
<td>5.97E-3</td>
</tr>
<tr>
<td>Size</td>
<td>22***</td>
<td>(7.09)</td>
<td>3.20E-3</td>
</tr>
<tr>
<td>Intercept</td>
<td>-147***</td>
<td>(51)</td>
<td>5.90E-3</td>
</tr>
</tbody>
</table>

n = 51 adj. \( R^2 = 0.84 \)
Global systemically important banks

<table>
<thead>
<tr>
<th>Category (and weighting)</th>
<th>Individual indicator</th>
<th>Indicator weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-jurisdictional activity (20%)</td>
<td>Cross-jurisdictional claims</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>Cross-jurisdictional liabilities</td>
<td>10%</td>
</tr>
<tr>
<td>Size (20%)</td>
<td>Total exposures as defined for use in the Basel III leverage ratio</td>
<td>20%</td>
</tr>
<tr>
<td>Interconnectedness (20%)</td>
<td>Intra-financial system assets</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Intra-financial system liabilities</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Securities outstanding</td>
<td>6.67%</td>
</tr>
<tr>
<td>Substitutability/financial institution infrastructure (20%)</td>
<td>Assets under custody</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Payments activity</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Underwritten transactions in debt and equity markets</td>
<td>6.67%</td>
</tr>
<tr>
<td>Complexity (20%)</td>
<td>Notional amount of over-the-counter (OTC) derivatives</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Level 3 assets</td>
<td>6.67%</td>
</tr>
<tr>
<td></td>
<td>Trading and available-for-sale securities</td>
<td>6.67%</td>
</tr>
</tbody>
</table>

**Figure:** BCBS GSIB Indicator measurement approach. Source: Basel Committee on Banking Supervision (2013).
“Spillover"-ICI: Discount self-inflicted losses

Consider a portfolio network given by:

\[
\begin{pmatrix}
1000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
100 & 1100 & 100 & 100 & 100 & 100 & 100 & 100
\end{pmatrix}
\]

\[\Pi = \Pi^\top\]

\[D = (1000, 2000)^\top.\]

- Compute \( \Omega = \Pi D^{-1} \Pi^\top \), as before.
- Compute the principal (largest) eigenvalue and the corresponding eigenvector (the "Perron eigenvector") of \( \Omega_0 := \Omega - \text{diag}(\Omega_{11}, \ldots, \Omega_{NN}) \).
**ICI** and **ICI**$_0$

Figure: Illustrative example showing how the **ICI**$_0$ discounts self-inflicted losses compared to the losses caused for other participants relative to the **ICI**.
$ICI_0$
Scenario design
Motivation

• Currently, the starting point for stress scenario design is often based on macroeconomic- and broader financial developments.
• The stress test scenario is often defined in terms of macroeconomic variables, which banks map to specific risk factors.
• Portfolio holdings and exposures do not play a large role, if any, in constructing the scenario.
Motivation

- Currently, the starting point for stress scenario design is often based on macroeconomic- and broader financial developments.
- The stress test scenario is often defined in terms of macroeconomic variables, which banks map to specific risk factors.
- Portfolio holdings and exposures do not play a large role, if any, in constructing the scenario.

Reverse stress testing and scenario design: First collect portfolio holdings and identify the main exposures/vulnerabilities. This has two advantages:

- For a given scenario, we can assess how “close" it is to a worst-case scenario in terms of contagion effects.
- The scenario can be designed such that particular weaknesses of the system are tested. This ensures that the scenario is “relevant".
Worst-case contagion scenario

Assume that the deleveraging of institutions is proportional to their resilience $R_i \in [0, 1]$. The weakest bank has resilience $R_i = 1$; a bank which is “fully" resilient and generates no fire sales has $R_i = 0$. 
Worst-case contagion scenario

Assume that the deleveraging of institutions is proportional to their resilience $R_i \in [0, 1]$. The weakest bank has resilience $R_i = 1$; a bank which is “fully” resilient and generates no fire sales has $R_i = 0$. View $\Omega$ as a map from deleveraging proportions/shock to fire-sales losses:

$$\Omega : [0, 1]^N \mapsto \mathbb{R}_+^N.$$

We want to find the scenario which maximizes

$$\max_{||x||_2 \leq 1} \left\{ 1^T \Omega x \right\} = \max_{||x||_2 \leq 1} \left\{ f^T x \right\},$$

where $f := 1^T \Omega R$. The worst-case scenario, which follows immediately from Cauchy-Schwarz, is

$$x^* = \frac{f}{||f||_2}.$$
Estimated fire-sales losses EBA scenario
Worst-case fire-sales losses

- Price-mediated contagion
- Modeling fire sales
- Indirect Contagion Index
- Scenario design
- Conclusion

Measuring systemic risk

R. Cont and E. Schaanning
Ratio of EBA FSLoss to worst-case FSLoss

![Graph showing the ratio of EBA FSLoss to worst-case FSLoss as a function of liquidation horizon and shock size.](image)
Further work

The problem

\[
\max_{\|x\|_2 \leq 1} \left\{ 1^T \Omega x \right\}
\]  \hspace{1cm} (1)

only looks at the fire-sales losses. It (i) ignores losses suffered on illiquid assets, and (ii) implicitly assumes a leverage targeting behaviour instead of a threshold behaviour.
Further work

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\max_{\|x\|_2 \leq 1} \left\{ 1^T \Omega x \right\}
\]  \hspace{1cm} (1)

only looks at the fire-sales losses. It (i) ignores losses suffered on illiquid assets, and (ii) implicitly assumes a leverage targeting behaviour instead of a threshold behaviour.

Ideally, we would like to find scenarios \( \epsilon \in [0, 1]^{M+K} \) as shocks to asset classes, that maximize

\[
\max_{\|\epsilon\|_2 \leq 1} 1^T A\epsilon + 1^T \Omega \Gamma(A\epsilon),
\]  \hspace{1cm} (2)

where \( A = (\Theta, \Pi) \), \( \Gamma : \mathbb{R}^N \rightarrow \mathbb{R}^N \) is the threshold deleveraging function, and \( \epsilon \) is potentially subject to further restrictions. This is a concave minimization.
Conclusions

• Overlapping portfolios give rise to an indirect contagion network. Under stress, the risk of a portfolio thus depends on the distress that similar portfolio-holders suffer.
• The indirect contagion index predicts fire-sales losses well, and can be used to quantify the systemicness of institutions.
Conclusions

- Overlapping portfolios give rise to an indirect contagion network. Under stress, the risk of a portfolio thus depends on the distress that similar portfolio-holders suffer.
- The indirect contagion index predicts fire-sales losses well, and can be used to quantify the systemicness of institutions.
- From the liquidity-weighted overlap network, we can derive a "worst-case" contagion scenario via a simple optimisation problem. This can be used both for benchmarking current stress scenarios, and for designing relevant future scenarios.
- The worst-case contagion scenario leads to a "perfect-storm" contagion, where the weaknesses of the system are specifically targeted.
References

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Simulating fire-sales in banking and shadow banking system.
*Mimeo*. 


