Introduction

Learning Outcomes

- Fundamental mathematical methods

Depending on your prior knowledge:

- *Repetition* of mathematical notions and methods.
- Learning of new methods.

Static Analysis of Equilibria

- At which price do we have market equilibrium?

  Find a price where demand and supply function coincide.

- Which amounts of goods have to be produced in a national economy such that consumers' needs are satisfied?

  Find the inverse of the matrix in a Leontief input-output model.

- How can a consumer optimize his or her utility?

  Find the absolute maximum of a utility function.

- What is the optimal production program for a company?

  Find the absolute maximum of a revenue function.

Comparative-Statistic Analysis

- When market equilibrium is distorted, what happens to the price?

  Determine the derivative of the price as a function of time.

- What is the marginal production vector when demand changes in a Leontief model?

  Compute the derivative of a vector-valued function.

- How does the optimal utility of a consumer change, if income or prices change?

  Compute the derivative of the maximal utility w.r.t. exogenous parameters.

Dynamic Analysis

- Assume we know the rate of change of a price w.r.t. time.
  How does the price evolve?

  Solve a difference equation or differential equation, resp.

- Which political program optimizes economic growth of a state?

  Determine the parameters of a differential equation, such that the terminal point of a solution curve is maximal.

- What is the optimal investment and consumption strategy of a consumer who wants to maximize her intertemporal utility?

  Determine the rate of savings (as a function of time) which maximizes the sum of discounted consumption.

Learning Outcomes – Basic Concepts

- Linear Algebra:

  matrix and vector · matrix algebra · vector space · rank and linear dependency · inverse matrix · determinant · eigenvalues · quadratic form · definiteness and principle minors

- Univariate Analysis:

  function · graph · one-to-one and onto · limit · continuity · differential quotient and derivative · monotonicity · convex and concave

- Multivariate Analysis:

  partial derivative · gradient and Jacobian matrix · total differential · implicit and inverse function · Hessian matrix · Taylor series

Learning Outcomes – Optimization

- Static Optimization:

  local and global extremum · saddle point · convex and concave · Lagrange function · Kuhn-Tucker conditions · envelope theorem

- Dynamic Analysis:

  integration · differential equation · difference equation · stable and unstable equilibrium point · difference equations · cobweb diagram · control theory · Hamiltonian and transversality condition

Course Organization

- Course based on *slides*.
  Download for handouts available.
- *Reading* and *preparation* of new chapters in self-study (handouts).
- *Presentation* of new concepts and examples.
- *Homework problems*.
- *Discussion* of students' solutions of homework problems.
- *Short online quizzes* in each course unit.
- *Question time* for final test.
- *Final test*.

Course Material

All information and course materials can be found and downloaded via the the CANVAS (see *Downloads*).

Literature

- ALPHA C. CHIANG, KEVIN WAINWRIGHT
  Fundamental Methods of Mathematical Economics
  McGraw-Hill, 2005.

- KNUT SYDSÆTER, PETER HAMMOND
  Essential Mathematics for Economics Analysis
  Prentice Hall, 3rd ed., 2008.

- KNUT SYDSÆTER, PETER HAMMOND, ATLE SEIERSTAD, ARNE STRØM
  Further Mathematics for Economics Analysis
  Prentice Hall, 2005.

- JOSEF LEYDOLD
  Mathematik für Ökonomen
  3. Auflage, Oldenbourg Verlag, München, 2003 (in German).

Further Exercises

Books from *Schaum's Outline Series* (McGraw Hill) offer many example problems with detailed explanations. In particular:

- SEYMOUR LIPSCHUTZ, MARC LIPSON
  Linear Algebra, 4th ed., McGraw Hill, 2009.
- RICHARD BRONSON
  Matrix Operations, 2nd ed., McGraw Hill, 2011.
- ELLIOT MENDELSON
  Beginning Calculus, 3rd ed., McGraw Hill, 2003.
- ROBERT WREDE, MURRAY R. SPIEGEL
  Advanced Calculus, 3rd ed., McGraw Hill, 2010.
- ELLIOTT MENDELSON
  3,000 Solved Problems in Calculus, McGraw Hill, 1988.

Prerequisites*

Knowledge about fundamental concepts and tools (like terms, sets, equations, sequences, limits, univariate functions, derivatives, integration) is obligatory for this course. These are (should have been) already known from high school and mathematical courses in your Bachelor program.

For the case of knowledge gaps we refer to the *Bridging Course Mathematics*. A link to learning materials for that course can be found on the web page.

Some slides still cover these topics and are marked by symbol * in the title of the slide.
However, we will discuss these slide only on request.

Prerequisites – Issues*

The following problems may cause issues:

- Drawing (or sketching) of graphs of functions.
- Transform equations into equivalent ones.
- Handling inequalities.
- Correct handling of fractions.
- Calculations with exponents and logarithms.
- Obstructive multiplying of factors.
- Usage of mathematical notation.

Presented *“solutions”* of such calculation subtasks are surprisingly often *wrong*.

Über die mathematische Methode

*Man **kann** also gar nicht prinzipieller Gegner der mathematischen Denkformen sein, sonst müßte man das Denken auf diesem Gebiete überhaupt aufgeben. Was man meint, wenn man die mathematische Methode ablehnt, ist vielmehr die höhere Mathematik. Man hilft sich, wo es absolut nötig ist, lieber mit schematischen Darstellungen und ähnlichen primitiven Behelfen, als mit der angemessenen Methode.*
Das ist nun aber natürlich unzulässig.

Joseph Schumpeter (1906)

Über die mathematische Methode der theoretischen Ökonomie, Zeitschrift für Volkswirtschaft, Sozialpolitik und Verwaltung Bd. 15, S. 30–49 (1906).

About the Mathematical Method

*One **cannot** be an opponent of mathematical forms of thought as a matter of principle, since otherwise one has to stop thinking in this field at all. What one means, if someone refuses the mathematical method, is in fact higher mathematics. One uses a schematic representation or other primitive makeshift methods where absolutely required rather than the appropriate method.*
However, this is of course not allowed.

Joseph Schumpeter (1906)

Über die mathematische Methode der theoretischen Ökonomie, Zeitschrift für Volkswirtschaft, Sozialpolitik und Verwaltung Bd. 15, S. 30–49 (1906).
Translation by JL.

Science Track

- Discuss basics of mathematical reasoning.
- Extend our tool box of mathematical methods for static optimization and dynamic optimization.
- For more information see the corresponding web pages for the courses *Mathematics I* and *Mathematics II*.

Computer Algebra System (CAS)

Maxima is a so called **Computer Algebra System** (CAS), i.e., one can

- manipulate algebraic expressions,
- solve equations,
- differentiate and integrate functions symbolically,
- perform abstract matrix algebra,
- draw graphs of functions in one or two variables,
- ...

wxMaxima is an IDE for this system:

`http://wxmaxima.sourceforge.net/`

You find an *Introduction to Maxima for Economics* on the web page of this course.

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May you do well!

Viel Erfolg!