Correlation and Contagion as Sources of Systemic Risk

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Abstract

We study systemic risk in a network model of the interbank market where the asset returns of the banks in the network are correlated. In this way we can study the interaction of two important channels for systemic risk (correlation of asset returns and contagion due direct financial linkages). We carry out a simulation study that determines the probability of a systemic crisis in the banking network as a function of both the asset correlation, and the connectivity and structure of the financial network. An important observation is the fact that the relation between asset correlation and the probability of a systemic crisis is hump-shaped; in particular, lowering the correlation between the asset returns of different banks does not always imply a lower probability of a systemic crisis. Moreover, in contrast to other studies we find that diversification at the level of individual banks may be beneficial for financial stability even if it does lead to a higher asset return correlation.

Keywords: Systemic risk, Contagion, Financial Networks, Asset Correlations

1 Introduction

The availability of modern risk-transfer tools enables banks to diversify away idiosyncratic risk concentrations in their portfolios. However, diversification at the level of individual banks might lead to more similar asset positions across banks and thus to a higher correlation of bank's asset returns. This has sparked a debate on the impact of increasing asset return correlations on financial stability. Prior to the financial crisis risk transfer between banks and diversification at the individual bank level was generally regarded as something positive. This view is for instance embodied in the following quote from a 2002-speech of Alan Greenspan (then chairman of the FED) to the council of foreign relations, see Greenspan (2002).

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> [In the past year] I, particularly, have been focusing on innovations in the management of risk and some of the implications of those innovations for our global economic and financial system. The development of our paradigms for containing risk has emphasized dispersion of risk to those willing, and presumably able, to bear it. If risk is properly dispersed, shocks to the overall economic systems will be better absorbed and less likely to create cascading failures that could threaten financial stability.

Note that Greenspan explicitly entertains the idea that the default of any given financial institution may result in "cascading failures" of other banks via a network of direct credit relationships. Reducing idiosyncratic risk concentrations may thus be beneficial as it reduces the likelihood that individual banks default in the first place.

After the financial crisis, diversification at the level of individual-bank level and the potential increase in the correlation of banks' asset portfolios were seen much more critical. For instance, Wagner (2010) argues that while diversification may indeed reduce the default probability of individual banks, the ensuing rise in asset correlations increases the likelihood of a systemic banking crisis (an event where many banks fail simultaneously). However, in his analysis network effects and direct business links between financial institutions are neglected. Other contributions criticize a high level of correlation between banks' asset portfolios on different grounds. In particular, Acharya and Yorulmazer (2007) argue that banks have an incentive to engage in *herding* to induce possible government bailouts.

Given these different views, in the present paper we study the impact of correlated asset positions on financial stability in a *network model* for financial institutions. The network represents direct business links between banks such as a borrower-lender relationship. This permits us to include two important sources for a systemic banking crisis in a single model: first we consider correlation between the asset positions of different banks (the so-called *correlation channel* for systemic risk); second we consider a contagious spreading of defaults through the financial network (the so-called *contagion channel* for systemic risk). We find that the correlation channel and the contagion channel are tightly connected; in particular, the impact of an increase in asset return correlation on financial stability is ambiguous and depends on the structure of the financial network. Moreover, in our setup diversification at the individual-bank level is typically beneficial for financial stability, in line with the informal argument of Greenspan mentioned earlier.

We use a simulation approach for our analysis. We randomly draw a financial network from a set of networks with given probabilistic characteristics that reflect stylized facts observed in real-world interbank networks. Subsequently we generate a set of asset returns for the banks in the network. We assume that a bank defaults if confronted with a large enough negative asset return. In that case all its creditor banks suffer a loss. If this loss is sufficiently large, some of the creditor banks default as well, which then leads to further losses and possibly to a whole cascade of contagious defaults. The use of randomly generated networks serves to robustify our analysis with respect to the details of the network topology. This is important since the exact structure of real-world financial networks is hard to observe due to a shortage of relevant data on financial linkages.

The present paper contributes also to the growing literature on network models and contagious defaults. The vast majority of papers in this area uses a two-step procedure. In the first step, they arrive at a network either by direct observation or by estimation on the basis of disclosed financial statements. Alternative approaches for network generation rely on micro-founded formation games (see Tardos and Wexler (2007)), asymptotic derivations for large and homogeneous networks (see Battiston et al. (2012)) or on simulation methods (see Hurd et al. (2014) or Hurd and Gleeson (2011)). In the second step, it is assumed that an exogenously chosen set of banks (called initially defaulting banks) fails, and the effect on the system is analyzed. Models of this type are frequently used by regulators. Examples include Elsinger et al. (2006) (Austria), Upper and Worms (2004) (Germany), Gai and Kapadia (2010) (UK) Degryse and Nguyen (2007) (Belgium), Blavarg and Nimander (2002) (Sweden), Mistrulli (2007) (Italy) or Lublóy (2004) (Hungary). Our setup differs from these contributions since we generate the set of initial defaulting banks by an economically relevant mechanism and since we study the interaction of the asset correlation channel and of the direct contagion channel.

Influential early papers in the academic literature on contagion and financial networks include Allen and Gale (2000) and Eisenberg and Noe (2001). Moreover, network models are becoming increasingly more popular in other areas of economics; see for example Braumolle et al. (2014) or a paper by Elliott et al. (2014) where the authors model crossholdings via a network model applied to European sovereign debt data.

2 Model and methodology

2.1 The Model

The financial network. The network of interbank relationships is a central part of our model. In mathematical terms this network can be described by a directed graph $\mathbb{G}$ consisting of N nodes. Each node represents a financial institution, while edges between them represent interbank credit exposures. More precisely, an edge from bank i to bank j means that bank j has a credit exposure towards bank i. This convention ensures that the direction of edges corresponds to the direction in which losses due to defaults spread through the network. The most obvious example of a credit exposure is an interbank loan made by bank j to bank i; alternatively one might think of a counterparty-risk exposure incurred by bank j in an uncollateralized derivative transaction with bank i.

The graph[1] $\mathbb{G}$ is described by an adjacency matrix $E_{\mathbb{G}}$ with elements e_{ij} satisfying:

$$e_{ij} = \begin{cases} 1 & \text{if } i \text{ is a debtor of } j\,, \\ 0 & \text{if } i \text{ is not a debtor of } j\,. \end{cases} \tag{1}$$

In the sequel we use the following notation to describe the balance sheet of the banks in the network. The total asset value of bank k by is denoted A_k; the nominal value of the loans made to other banks in the system is denoted by A_k^{IB} (short for interbank); the external assets (e.g. loans to non-banks) are denoted by A_k^{EX}; finally, L_k^{IB} and L_k^{EX} represent the interbank liabilities and the external liabilities (e.g. customer deposits) of bank k, so that total liabilities are equal to $L_k = L_k^{IB} + L_k^{EX}$. The equity of bank

[1] Throughout the paper we use the terms *graph* and *network*. When talking about graph, we are concerned with the structure of financial linkages, whereas when referring to a network, we mean not just connections themselves but also balance sheet quantities of individual banks.

k is then given by $E_k = A_k - L_k$ and E_k/A_k is the *capital ratio* of the bank. These quantities are illustrated in Table 1.

Table 1: Balance sheet of bank k

assets	liabilities
interbank assets A_k^{IB}	interbank liabilities L_k^{IB}
external assets A_k^{EX}	external liabilities L_k^{EX}
	equity E_k
total assets A_k	total liabilities L_k

Next we introduce several assumptions that will permit us to create the financial network from a given adjacency matrix $E_{\mathbb{G}}$.

Assumption 1 *All loans in the system are of the same size, normalized to one.*

Under Assumption 1 bank k's interbank assets A_k^{IB} are given by the number of its debtors and the interbank liabilities L_k^{IB} are equal to the number of its creditors, that is

$$A_k^{IB} = \sum_{i=1}^{N} e_{ik} \;\text{ and }\; L_k^{IB} = \sum_{j=1}^{N} e_{kj}, \tag{2}$$

where e_{ij} are elements of $E_{\mathbb{G}}$ as described in equation (1).

The next assumption can be viewed as a stylized version of the risk capital requirements imposed under the current Basel regulations.

Assumption 2 *The capital ratio E_k/A_k of every bank is equal to an exogenously given constant $\gamma_k < 1$.*

Finally, we make an assumption on A_k^{IB}/A_k, the ratio of bank k's interbank assets and of its total assets; following Elliott et al. (2014) we refer to this ratio as the level of *integration* of bank k into the network. Loosely speaking we assume that the level of integration is equal to an exogenously given constant $\kappa > 0$ or, equivalently, that $A_k = \frac{1}{\kappa} A_k^{IB}$. However, under Assumption 2 this is not always consistent with the requirement that the external liabilities are nonnegative. In fact, the requirement that $L_k^{EX} \geq 0$ gives that

$$\gamma_k A_k = E_k = A_k - L_k^{IB} - L_k^{EX} \leq A_k - L_k^{IB},$$

and hence the inequality $A_k \geq L_k^{IB}/(1-\gamma_k)$. Motivated by these considerations we make the following assumption on A_k.

Assumption 3 *The total asset value of the banks in the network is given by*

$$A_k = \max\left\{\frac{1}{\kappa_k}A_k^{IB}, \frac{1}{1-\gamma_k}L_k^{IB}, 1\right\}, \quad k = 1, \dots, N. \tag{3}$$

For typical parameterizations of the model $\frac{1}{\kappa_k}$ is significantly larger than $\frac{1}{1-\gamma_k}$. In that case if $A_k^{IB} \approx L_k^{IB}$ the first term from (3) is binding so that $\kappa_k A_k = A_k^{IB}$. If L_k^{IB} is much larger than A_k^{IB} the second inequality is binding and ensures that the total balance sheet size is not lower than the sum of interbank liabilities and equity. In the degenerate case where a bank has no connections at all, we simply set the A_k equal to one.

The external assets (liabilities) are finally given by the difference between total assets (liabilities) and interbank assets (liabilities plus capital buffer). This gives

$$A_k^{EX} = A_k - A_k^{IB} \qquad \text{and} \qquad L_k^{EX} = A_k - E_k - L_k^{IB} = (1-\gamma_k)L_k - L_k^{IB}\,. \tag{4}$$

To summarize, we have created a balance sheet structure from a given adjacency matrix $E_{\mathbb{G}}$ along the following steps:

1. Assign the value of interbank assets A_k^{IB} and liabilities L_k^{IB} of every bank in the network according to equation (2).
2. Determine the asset value A_k, $k = 1, \dots, N$, of the banks according to (3).
3. Define A_k^{EX} and L_k^{EX} according to equation (4).

Initial defaults. In our setup the return on bank ks external assets, denoted r_k, is random so that a large negative return shock can force a bank to default. We refer to this as an *initial default* and to the banks where this happens as *initial defaulters*, as a default due to a negative asset return happens at the start of a potential default cascade (see Section 2.2 below). Formally, an initial default occurs if the asset value after the return realization is lower than the liabilities of bank k, that is if

$$A_k^{IB} + A_k^{EX}(1 + r_k) < L_k\,.$$

Since $L_k = A_k - E_k = A_k(1-\gamma_k)$, an initial default thus occurs if $r_k < -\gamma A_k / A_k^{EX}$. For banks with level of integration equal to κ (the typical case) this can be rewritten as $r_k < -\gamma/(1-\kappa)$.

Correlation of asset returns. We assume that the random variables $r_1, \ldots, r_N$ follow the following simple one-factor model:

$$r_k = \mu + \sqrt{\beta} r^M + \sqrt{(1-\beta)}\epsilon_k, \quad 1 \leq k \leq N. \tag{5}$$

Here r^M is a *market return* that is common for all banks in the system and ϵ_k is an *idiosyncratic return* that differs across banks. We assume that r^M and $\epsilon_1, \ldots, \epsilon_N$ are independent and normally distributed with mean zero and standard deviation $\sigma = 0.2\sqrt{dt}$ with $dt = 1/252$. The parameter μ in equation (5) is set equal to $\mu = 0.05dt$

Under the factor model (5) the correlation between the asset value change r_k and r_l of two different banks is equal to β; in particular, for $\beta = 0$ the sensitivity of any bank to the market return is zero such that its solvency is only driven by its own idiosyncratic return. On the other hand, for $\beta = 1$ there are no individual shocks and every bank faces the same return on its external assets. Note that (5) implies that the marginal distribution of r_k and hence the probability of an initial default is not affected if the correlation parameter β is varied. This is in stark contrast to the analysis of Wagner (2010), where a higher level of correlation of different banks is associated with a lower volatility of banks' asset returns and hence with a lower probability of initial defaults. We come back to this issue in Section 3.3 below.

Table 2 illustrates the impact of β on the distribution of initial defaults. We see that lowering the value of β has two effects: the probability of observing at least one initial default is increased, while the probability that a large fraction of the banks in the system default decreases. These effects are well-known in the literature on portfolio credit risk models; see for instance Frey and McNeil (2003).[2]

[2]We have also conducted all of our simulations on a set of multivariate t-distributed returns to account for tail risk. The probability of a systemic crisis has a different magnitude in this case, but the qualitative results stay the same as in the normally distributed returns scenario.

	$\beta = 0$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.9$
P(k is in default)	0.024%	0.024%	0.024%	0.024%
P(at least one default)	2.40%	1.97%	1.39%	0.25%
P(at least 20% in default)	$4.9\ 10^{-13}$%	0.0043%	0.0077%	0.0088%

Table 2: Individual default probability of bank k, probability of observing at least one initial default and probability that more than 20% of the banks in the system default initially for varying correlation parameter β ($N = 100$ banks).

2.2 The Contagion Channel

The idea behind the contagion channel for systemic risk is simple. If a bank defaults in a financial network, it is unable to fulfill its obligations towards its creditors, which results in a reduction of the interbank assets of the creditor bank. If this loss is big enough it may cause the creditor bank to default, so that default can become contagious and spread through the system. For simplicity we assume zero recovery on defaulted interbank loans, such that one does not need to compute the value of recovery payments[3].

For a financial network with given adjacency matrix $E_{\mathbb{G}}$, balance sheet quantities $A_k^{IB}, A_k^{EX}, L_k^{IB}, L_k^{EX}, A_k, \gamma_k$ and given return realization r_k for every bank k, the mechanism that (potentially) generates a default cascade is described as follows:

1. Perturb the external assets A_k^{EX} of each bank k by the return realisation r_k, that is let $A_k^{EX}(\text{new}) = A_k^{EX}(\text{old})(1 + r_k)$.

2. If any of the banks defaults, propagate the shock to the asset side of its creditors. The new amount of interbank assets satisfies: $A_k^{IB}(\text{new}) = A_k^{IB}(\text{old}) - \sum_i e_{ik} \mathbf{1}_{\{i \text{ is in default}\}}$

3. If the total value of bank k's assets falls below its liabilities, that is $A_k(\text{new}) < L_k$, bank k defaults.

4. Repeat the procedure until there is no further default.

[3] Thanks to this assumption, there is no need for a settlement algorithm in the spirit of Eisenberg and Noe (2001). Assuming non-zero recovery rate on distressed loans would however not change the overall quantitative nature of our results.

2.3 Simulation procedure and network generation

In order to assess the relative importance and the joint effect of both systemic risk channels, we conduct a two-layer Monte Carlo analysis. In the inner layer, we generate $K = 500$ return realizations that follow the factor model (5). This whole layer is embedded in the outer layer where 1000 random networks are created. The reason for using random graphs is the unobservable nature of real-world financial networks. As we are unable to observe the underlying financial network exactly[4], we need to resort to a probabilistic framework in which only particular stylized facts about the system are specified. These stylized facts are incorporated into the network generating process such that, in the end, each network can be seen as one realization of a random variable. In this way we robustify our analysis against misspecification of the underlying financial network while retaining the possibility to take *qualitative properties* of real-world financial networks into account.

In order to arrive at a proper network that describes mutual connections among financial institutions, we first need to sample an underlying adjacency matrix $E_{\mathbb{G}}$. For this we use two different probabilistic models, namely a homogeneous Erdos-Renyi random graph and a inhomogeneous model that generates graphs with a core-periphery structure.

2.3.1 Homogeneous (Erdos-Renyi) random graphs

The Erdos-Renyi model is a simple reference model that is a popular benchmark for more sophisticated networks. In the Erdos-Renyi model, a random graph is generated such that the probability that there is an edge between any two nodes in the graph is a constant number p_{ER}; or put differently, every Erdos-Renyi random graph is parameterized only by two numbers - the number of nodes in the graph N and the probability p_{ER} that any two of them are connected. Moreover, connections are formed independent of each other, that is the elements e_{ij}, $1 \leq i, j \leq N$ of $E_{\mathbb{G}}$ are iid Bernoulli random variables. For $p_{ER} = 1$ we get a complete directed graph in which every bank is connected to every other bank and vice versa, while for $p_{ER} = 0$ there are no links

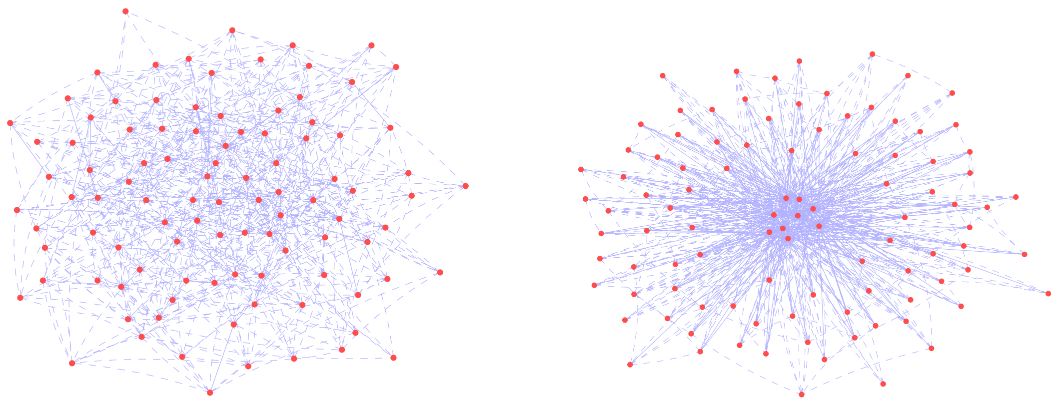
[4]There is just a handful of countries where regulators have a reasonably good idea about their own interbank market. These would include for example Austria, Mexico, Germany or Brazil.

between the banks in the system.

2.3.2 Inhomogeneous (core-periphery) random graphs

According to Soramäki et al. (2006), Bech and Atalay (2010), Iori et al. (2008) and others, a typical financial network exhibits a significant degree of so-called *disassortativity*, that is small banks tend to be connected to large ones and vice versa. An interpretation of this finding is that large banks act as intermediaries for smaller ones. This structure is in contrast to the structure of social networks that tend to be assortative (people with few friends tend to be connected with other people having a small number of friends). To account for the observed disassortativity, we extend the Erdos-Renyi setting by making each bank belong either to a group called *core* with probability p_{core} or to a group called *periphery* with a probability $1 - p_{core}$. The difference between these two groups of institutions lies in the probability of forming connections with other banks. A core bank has a large probability of establishing a connection both with other core banks and with other peripherals while a connection between two peripherals is less likely. In this paper we take the probability of a connection between two core banks equal to $p_{CC} = 0.9$; the probability of a connection between two peripheral banks is set to $p_{PP} = 0.01$; the probability of a connection between a core bank and a peripheral in either direction is set to $p_{CP} = p_{PC} = 0.5$. Given the type of the banks in the system, connections are formed independent of each other. In this way we end up with an assortative network that we refer to as a *core-periphery* structure. The resulting network exhibits a star shape with few banks tightly connected in the center and the rest on the periphery. In financial terms core banks can be interpreted as (large) dealer banks that act as an intermediary for the other banks in the network. The difference between an Erdos-Renyi and a core-periphery network is illustrated in Figure 1.

Note that since a core bank has on average more connections than a peripheral bank a higher value of p_{core} leads to a higher density of the ensuing network. In particular, for $p_{core} = 1$, the whole network is formed by core banks so we actually get a very dense Erdos-Renyi setting (identical to the case where $p_{ER} = 0.9$) whereas for $p_{core} = 0$, every bank is peripheral. Since peripherals are connected with probability $p_{PP} = 0.01$, we get



(a) Erdos-Renyi random graph (b) Core-periphery random graph

Figure 1: One realisation of a random graph for $N = 100$ banks; left panel Erdos Renyi network; right panel core-periphery network.

a sparse Erdos-Renyi setting (corresponding to $p_{ER} = 0.01$). Therefore, an intermediate level of p_{core} corresponds to a network which lies between two homogeneous Erdos-Renyi extremes. In the Monte Carlo simulations, the probability p_{core} of belonging to the core is varied between 0 and 20%.

3 Results

We now present the results of a simulation study that illustrates the impact of the asset return correlation and of the density/connectivity of the network on financial stability. We measure the density of a given network by the expected number of counterparties of a randomly chosen bank in the system.[5] From now on, we will call this quantity *connectivity* and denote it by C. In the Erdos-Renyi random graph, connectivity is given by $C = p_{ER}(N-1)$; in the case of a core-periphery network, connectivity is easily seen to be

$$C = (N-1)(p_{core}^2 p_{CC} + p_{core}(1-p_{core})(p_{CP} + p_{PC}) + (1-p_{core})^2 p_{PP}).$$

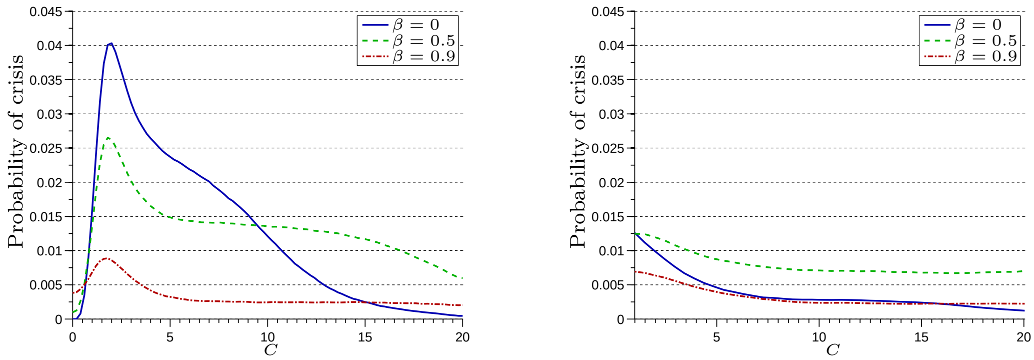
The output variable in our analysis is the relative frequency of scenarios in the simulation in which a systemic crisis occurred. Here a scenario is viewed as one realization of random network together with one realization of random returns, and a systemic crisis is defined

[5]In graph theoretic literature, this is known as the average graph degree.

as a scenario where more than 20% of all banks in the network are in default at the end of the default cascade. In the sequel we will call this relative frequency simply the *probability of a systemic crisis.* Note that the exact value of the threshold in the definition of a systemic crisis (20% or different) is irrelevant. In fact, for all but very small values of the connectivity parameter C we observed a *dichotomous* behavior: in a given scenario there are either very few defaults or the network is wiped out (almost) entirely This behaviour was observed for both network types and for all values of β.

3.1 Erdos-Renyi networks.

The results for Erdos-Renyi random networks are illustrated in Figures 2a and 3a. Figure 2a gives the probability of a systemic crisis for fixed β and varying C; Figure 3a depicts sections for fixed C and varying β.

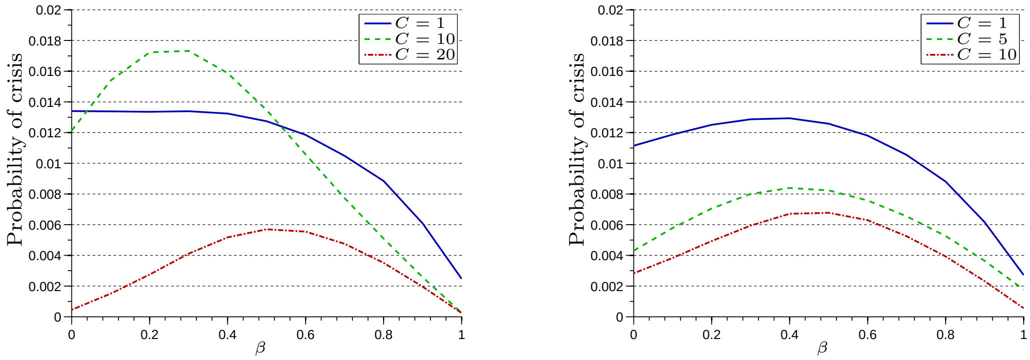


(a) Erdos-Renyi network

(b) Core-periphery network

Figure 2: Probability of systemic crisis for both network structures on $N = 100$ nodes as a function of network connectivity C for particular levels of correlation β. Bank equity ratio $\gamma = 0.035$.

Connectivity. First we discuss the impact of variations in network connectivity C (see Figure 2a). Here we observe a hump-shaped behavior: for small values of C, the probability of a systemic crisis is small. Intuitively, this is due to the fact that in a very sparse network the contagion channel is inactive since there is almost no opportunity for shock propagation. As C increases the likelihood of a systemic crisis increases up to a maximum at which the system is most vulnerable. Beyond that maximum the probability of a systemic crisis decreases again, and banking networks with a high connectivity



(a) Erdos-Renyi network (b) Core-periphery network

Figure 3: Share of scenarios with systemic crisis for both network structures on $N = 100$ nodes as a function of asset correlation β for particular levels of connectivity C. Bank equity ratio $\gamma = 0.035$.

appear to be fairly resilient. This resilience is due to enhanced hedging opportunities of the institutions in the system (if a bank has more counterparties, the loss caused by its default is borne by more banks). Finally we see that for β close to one the probability of a crisis is relatively insensitive with respect to network connectivity. This is due to the fact that for β large the occurrence of a systemic crisis is determined to a large extent by the realisation of the common return factor r^M, independent of the structure of the financial network.

Note finally that the hump-shaped form of the relation between C and the probability of a systemic crisis in Erdos Renyi graphs is in line with findings from other recent papers in the network literature, see for instance Hurd et al. (2014), Gai and Kapadia (2010), or Elliott et al. (2014).

Correlation. Next we consider the impact of varying the asset return correlation β (see Figure 3a.) For medium and high values of C we observe a hump-shaped behavior. For β close to zero the probability of a systemic crisis is *increasing* in β. This is of course due to the fact that by increasing β we increase the probability that a large part of the system defaults initially, see Table 2. However, if β exceeds a certain threshold $\bar{\beta}$, the probability of a systemic crisis is *decreasing* in β. In order to understand this behavior we make recourse to the argument of Greenspan (2002) mentioned in the introduction of the paper: with direct links between banks there can be default cascades during which the initial default of a few financial institutions spreads through a large part of the financial system. Such a cascade is more likely if many of the "initial survivors" are also close to default because they were hit by a negative shock on their asset returns. This is in turn more likely for β large since in that case a negative market-return shock substantially weakens all banks in the system. In fact, for β sufficiently high a single initial default may be enough to generate a systemic crisis via the contagion channel. Moreover, we know from Table 2 that the probability of observing *at least one* initial default is decreasing in β. Taken together, these arguments explain the hump-shaped nature of the relation between β and the probability of a systemic crisis. For low values of C the network is very fragile (recall our discussion of Figure 2a), so that the "Greenspan effect" (the fact that a higher β may decrease the probability of a systemic crisis) kicks in already for relatively low values of β; in the extreme case $C = 1$ the probability of a systemic crisis is even decreasing in β for all values of β.

3.2 Core-periphery networks.

We repeat the same analysis for core-periphery networks. The results are depicted in Figures 2b and 3b. As in the Erdos Renyi case the probability of a systemic crisis is a nonlinear function of β and C. The relation between β and the probability of a systemic crisis is of the same hump-shaped form as in the Erdos-Renyi network, with a similar interpretation (compare Figures 3a and 3b.) However, we observe a different behavior with respect to variations in the connectivity C: for core-periphery networks the probability of a systemic crisis is decreasing in C (Figure 2b) whereas in the Erdos Renyi networks this relation was hump-shaped. Moreover, in the core periphery networks the probability of a systemic crisis is generally lower than in the Erdos-Renyi case. These findings are in line with the general claim that heterogeneous network structures are relatively resilient, see for instance Gai et al. (2011) and Simon (1962). Moreover, they lend support to regulatory attempts to generate networks with a high degree of connectivity, for instance by limiting the amount of direct lending between any two financial institutions.

We also considered a variant of the model where the equity capital ratio γ_{core} of core institutions (institutions that have many links and that can therefore be regarded as systemically important) is higher than the equity ratio of peripherals. We found that this modification significantly reduces the probability of a systemic crisis, which obviously supports proposals to regulate systemically important institutions more tightly.

3.3 An alternative asset model and diversification.

In order to contribute to the debate on the merits of diversification in the banking sector mentioned in the introduction we now consider an alternative model for asset returns where banks can diversify their external asset position. More precisely, we assume that there are N correlated investment opportunities for banks; investment opportunity (project) i has a return of the form

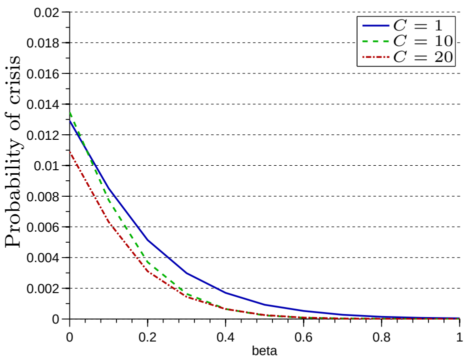
$$p_i = \mu + \sqrt{\rho} r^{\text{sys}} + \sqrt{1-\rho}\,\delta_i\,, \quad 1 \le i \le N \tag{6}$$

where r^{sys} and δ_i, $1 \le i \le N$ are independent, $N(0, \sigma^2)$-distributed random variables and where $0 < \rho < 1$. Equation (6) implies that the return on different projects has a common factor which might be a natural assumption for the banking sector. The actual return of bank k is then modelled as a convex combination of p_k and of the "market portfolio" $\frac{1}{N}\sum_{i=1}^N p_i$, that is

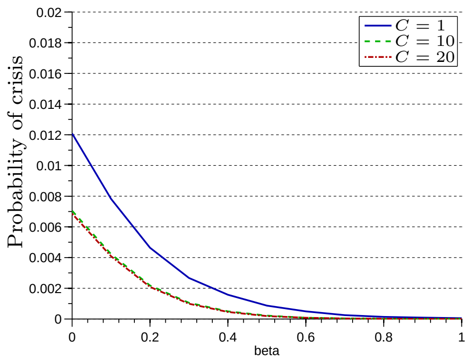
$$r_k = \beta p_k + \frac{1-\beta}{N}\sum_{i=1}^{N} p_i \quad \text{for some } \beta \in [0,1]. \tag{7}$$

For $\beta = 0$ (no diversification) we have $r_k = p_k$ whereas for $\beta = 1$ (perfect diversification) every bank holds the market portfolio given by $\mu + \sqrt{\beta} r^{\text{sys}} + \frac{\sqrt{1-\beta}}{N}\sum_{i=1}^N \delta_i$. It is easily seen that under (7) the variance of r_k (and hence the initial default probability) is decreasing in β, essentially since the variance of the idiosyncratic part is reduced. In particular, we get that for $\beta = 0$ the variance of r_k is equal to σ^2 whereas for $\beta = 1$ the variance of r_k is equal to $\sigma^2\big(\rho + \frac{1-\rho}{N}\big)$. Hence β can be viewed as a measure of the diversification of banks' external asset portfolios.

In Figures 4a and 4b we plot the probability of a systemic crisis in the modified setup (6), (7) for varying levels of diversification β and for both network types. We unambiguously find that an increase in the level of diversification lowers the probability of a systemic crisis. This supports the informal argument of Greenspan (2002) on the merits of diversification in banking networks.



(a) Erdos-Renyi network



(b) Core-periphery network

Figure 4: Probability of systemic crisis for both network structures on $N = 100$ nodes as a function of network connectivity C for particular levels of correlation β according to the alternative model specification from Section (3.3). Bank equity ratio $\gamma = 0.035$.

4 Conclusion

We have presented a simulation study that is concerned with the joint effect of correlated asset positions and of the network structure of a banking system on financial stability. Both a simple case of a homogeneous Erdos-Renyi network and a more realistic scenario of inhomogeneous core-periphery network structure were examined in the process.

We conclude that in order to judge the implications of correlation on the magnitude of systemic risk, one needs to take the underlying network structure into account. Most dangerous are homogeneous networks of intermediate density since they are dense enough to propagate shocks but not dense enough to hedge off potential risk. Moreover, we found that lower values of asset correlation do not always reduce the probability of a systemic crisis Furthermore, we present results for the more realistic case of core-periphery networks. There, the probability of a crisis is generally lower than in the homogeneous Erdos-Renyi network, which indicates high resiliency of such networks. Finally, we found that in banking networks diversification of banks' external assets can be beneficial for financial stability.

References

Viral V. Acharya and Tanju Yorulmazer. Too many to fail - an analysis of time-inconsistency in bank closure policies. *Journal of Financial Intermediation*, 16:1–31, January 2007.

Franklin Allen and Douglas Gale. Financial contagion. *The Journal of Political Economy*, 108:1–33, February 2000.

Stefano Battiston, Domenico Delli Gatti, Mauro Gallegati, Bruce C. N. Greenwald, and Joseph E. Stiglitz. Credit default cascades: When does risk diversification increase stability? *Journal of Financial Stability*, 8(3):138–149, 2012.

Morten L. Bech and Enghin Atalay. The topology of the federal funds market. *Physica A: Statistical Mechanics and its Application*, 389(22):5223–5246, November 2010.

Martin Blavarg and Patrick Nimander. Interbank exposures and systemic risk. *Sveriges Riksbank. Economic Review*, 2:19–45, 2002.

Yann Braumolle, Rachel Kranton, and Martin D'Amours. Strategic interaction and networks. *American Economic Review*, 104(3):898–930, 2014.

Hans Degryse and Gregory Nguyen. Interbank exposures: An empirical examination of contagion risk in the belgian banking system. *International Journal of Central Banking*, 3(2):123–171, 2007.

Larry Eisenberg and Thomas H. Noe. Systemic risk in financial systems. *Management Science*, 47(2):236–249, February 2001.

Matthew Elliott, Benjamin Golub, and Matthew O. Jackson. Financial networks and contagion. *American Economic Review*, 104(10):3115–3153, 2014.

Helmut Elsinger, Alfred Lehar, and Martin Summer. Using market information for banking system risk assessment. *International Journal of Central Banking*, 2(1), March 2006.

Rüdiger Frey and Alexander J. McNeil. Dependent defaults in models of portfolio credit risk. *The Journal of Risk*, 6(1):59–92, 2003.

Prasanna Gai and Sujit Kapadia. Contagion in financial networks. *Proceedings of the Royal Society*, 2010.

Prasanna Gai, Andrew Haldane, and Sujit Kapadia. Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470, July 2011.

Alan Greenspan. Speech before the council of foreign relations. *International Financial Risk Management, Washington D.C.*, November 2002.

Tom Hurd, Davide Cellai, Huibin Cheng, Sergey Melnik, and Quentin Shao. Illiquidity and insolvency: a double cascade model of financial crises. arXiv:1310.6873 [q-fin.GN], 2014.

Tom R. Hurd and James P. Gleeson. A framework for analyzing contagion in banking networks. arXiv:1110.4312 [q-fin.GN], 2011.

Giulia Iori, Giulia De Masi, Ovidiu Vasile Precup, Giampaolo Gabbi, and Guido Caldarelli. A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1):259–278, 2008.

Ágnes Lublóy. Domino effect in the hungarian interbank market. Working Paper, 2004.

Paolo Emilio Mistrulli. Assessing nancial contagion in the interbank market: a comparison between estimated and observed bilateral exposures. *Temi di Discussione No. 641. Banca dItalia, Servizio Studi*, 2007.

Herbert Simon. The architecture of complexity. *Proceedings of the American Philosophical Society*, 106(6):467–482, 1962.

Kimmo Soramäki, Morten L. Bech, Jeffrey Arnold, Robert J. Glass, and Walter E. Beyeler. The topology of interbank payment flows. *Staff Reports, Federal Research Bank of New York*, 243, 2006.

Éva Tardos and Tom Wexler. *Algorithhmic Game Theory*, chapter Network formation games and potential function method. Cambridge University Press, 2007.

Christian Upper and Andreas Worms. Estimating bilateral exposures in the german interbank market: Is there a danger of contagion? *European Economic Review*, 48 (4):827–849, August 2004.

Wolf Wagner. Diversification at financial institutions and systemic crises. *Journal of*

Financial Intermediation, 19(3):373–386, July 2010.